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On heat transfer through a solid slab heated uniformly and periodically: determination of thermal properties

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Abstract

In this paper, some heat transfer characteristics through a sample that is uniformly heated on one of its surfaces by a power density modulated by a periodical square wave are discussed. The solution of this problem has two contributions, comprising a transient term and an oscillatory term, superposed to it. The analytical solution is compared to the experimental results obtained by using the approach first proposed by Ångström, which has become a wellknown thermal wave experimental procedure used for the determination of thermal diffusivity. A number of conclusions are drawn from this comparison, which highlight the need to carefully consider the experimental setup employed when carrying out this type of measurement. The results may be of interest to those dealing with heat transfer problems, thermal characterization techniques and/or involved in the teaching of partial differential equations at undergraduate or graduate level.

1. Introduction

Heat transfer problems are of considerable importance due to their relevance in daily life and scientific research. Therefore, it becomes very important to deal with them in introductory as well as advanced courses on physics and engineering. A physical situation often encountered is transient heating of a solid slab by a continuous heat source placed on one of its surfaces. This situation has been discussed recently in this journal [1, 2]. Another interesting example is that of a sample subjected to periodical harmonic uniform heating. This type of heating appears, for example, in the field of photothermal (PT) and related techniques [3], a group of well established methods that are useful, among other things, for the thermal characterization of materials. They are based on measurements of the harmonic temperature oscillations (the

so-called thermal waves) generated in a sample by periodical heating, which can be induced by light (photons) or by other means, e.g., by Joule's effect [4]. One of the simplest and most inexpensive experimental setups that can be used to teach these techniques is the one proposed by Ångström [5], in which a metallic bar is alternately heated and cooled on one extreme, while the other is kept at fixed (e.g. room) temperature. In this way, temperature oscillations with time are generated that are measured at different distances away from the heat source. From the temperature amplitude attenuation with distance, and from their phase lag with respect to the heat source, the thermal diffusivity of a sample can be obtained. Although the Ångström method has the advantage of being simple, accurate and inexpensive, it also has some drawbacks that limit its application in a general way. Amongst these are the relatively large volume of material required, and the fact that its application is limited to solid, homogeneous samples made of high thermal conductivity materials. Nevertheless, due to its technical simplicity and the straightforward nature of the physical-mathematical formalism involved, experiments based on Ångström's method are very often conducted in undergraduate physics laboratories, as described by Bodas et al [6] and Bouchard [7]. Other authors [8-10] have also provided details on data processing which are based on a very similar method inspired by Fourier's early proposals of measurements of thermal properties at the earth crust using the daily periodical temperature oscillations [11, 12]. For different reasons, such as the use of synchronous detection [13-17] in PT methods and the attention paid to the long-term temperature distribution, once the system has lost information regarding its initial conditions, the presence of a transient contribution is often ignored, with most studies focusing on the oscillatory component alone. This paper is focused on the effects of both the transient and the oscillatory contributions to the temperature field when a square wave temperature modulation is applied to the surface of a solid. The heat diffusion equation is solved-under Robin boundary conditions-to calculate the temperature field, and its solution is compared with the results of an experiment inspired by the Ångström method. The experiment was prepared to show the importance of an appropriate selection of the temperature testing device as well its correct use. For the particular case of a thermally thin sample for transient heat conduction [2], which can experimentally be achieved in an easier manner than a thermally thick sample [1, 2], we show that the solution of the problem can be achieved in a much simpler way using the here-called calorimetric approximation based on the energy balance equation. The implications of these results should be helpful in framing the experiment setup in teaching laboratories.

2. Periodical heating

PT techniques, in all the existing variants, require a controlled excitation source to produce a temporally periodic thermal response in the sample from which heat transfer parameters such as thermal diffusivity and thermal effusivity can be obtained. For this purpose, the sample's heating power density is frequently modulated by a periodical rectangular wave that can be described, without loss of generality, by the following function:

$$T(t) = \frac{1}{2} \sum_{m} \operatorname{sinc}(m/2) \exp(i(\omega_{m}t - m\pi/2)) \qquad \text{for } t > 0.$$
(1)

Figure 1 shows a schematic representation of the amplitude of the modulation function given by equation (1), which will be used in theoretical calculations.

In equation (1) $\omega_m \equiv 2\pi mf$, where f is the modulation frequency, m is an integer and sinc is the cardinal sine function. Note that this type of modulation can be implemented in most laboratories using simple and relatively inexpensive equipment.



Figure 1. Schematic representation of the modulation function (amplitude) used for heating the sample probe.



Figure 2. Schematics of the heat source and the thermometric region of the physical system to be measured.

In the experimental setup used here, a rod of length L made up of some solid material (surrounded by an insulating material to minimize and stabilize heat loss by convection) is heated by an external source in thermal contact to one of its ends (see figure 2). Heating is affected by Joule's effect using an electrical resistance introduced into a small portion of the rod to guarantee best possible thermal contact. This heating configuration is designed so that it can be assumed that the region of the rod surrounding the resistance becomes part of the heat source (see inset of the figure). One can further assume that the remaining portion of the rod



Figure 3. Schematic representation of the physical system used for the mathematical model.

lies in the thermometric region of thickness $l_s < L$, along which the temperature distribution can be measured by a set of thermocouples. Note that part of the electrical power supplied is lost to the surroundings, so that the effective heating power density is actually smaller than the generated by Joule's effect.

The simplicity and low-cost of the experimental setup makes it suitable for use in teaching laboratories.

3. Heat diffusion model

Consider the following one-dimensional heat diffusion equation for the homogeneous sample probe with cylindrical symmetry, schematically represented in figure 3:

$$\frac{\partial^2}{\partial z^2}\Theta(t,z) - \alpha_{\rm s}^{-1}\frac{\partial}{\partial t}\Theta(t,z) = \kappa_{\rm s}^{-1}G(t,z).$$
⁽²⁾

In this equation, z is the spatial coordinate, t > 0 is the time, α_s and κ_s are the thermal diffusivity and thermal conductivity of the sample respectively, $\Theta(t, z)$ is the temperature variation of the sample around the ambient temperature and $G_s(t, z)$ is the surface heat source. The solution of equation (2) is constrained by the following Robin type boundary conditions:

$$-\kappa_{s} \left. \frac{\partial}{\partial z} \Theta(t, z) \right|_{z=0} + h \Theta(t, 0) = 0,$$

$$\kappa_{s} \left. \frac{\partial}{\partial z} \Theta(t, z) \right|_{z=l_{s}} + h \Theta(t, l_{s}) = 0.$$
(3)

In the above equation, h is the thermal exchange coefficient (that takes into account convective and radiative heat losses [2]) which is assumed to be the same at both sample ends. Our interest lies in the case where the time dependence of G_s is given by a periodic function in time, i.e.

$$G_{\rm s}(t,z) = -I_0 \delta(z) T(t), \tag{4}$$

where I_0 is the heating power density applied to the sample at z = 0 and T is a periodic modulation function, e.g. with the form given by equation (1).

3.1. Transient response

Since T(t) is a periodic function, it can be expanded on a Fourier basis. By means of the Laplace transform, we obtain the following boundary value problem in the spatial coordinate:

Table 1. Thermal regimes as a function of the modulation frequency.

Thermal regimes	
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$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\hat{\Theta}(\varsigma,z) - q_s^2(\varsigma)\hat{\Theta}(\varsigma,z) = -\frac{I_0\delta(z)}{\kappa_s} \sum_m \frac{C_m}{\varsigma - \mathrm{i}\omega_m},$$
$$-\kappa_s \left. \frac{\mathrm{d}}{\mathrm{d}z}\hat{\Theta}(\varsigma,z) \right|_{z=0} + h\hat{\Theta}(\varsigma,0) = 0, \qquad \kappa_s \left. \frac{\mathrm{d}}{\mathrm{d}z}\hat{\Theta}(\varsigma,z) \right|_{z=l_s} + h\hat{\Theta}(\varsigma,l_s) = 0,$$
(5)

where $q_s^2 \equiv \zeta \alpha_s^{-1}$ with ζ being the complex angular frequency from the Laplace transform and C_m the coefficients of the Fourier expansion of T(*t*). The solution of equation (5) represents the thermal response of the sample to the excitation and is given by the following expression:

$$\hat{\Theta}(\varsigma, z) = \sum_{m} \hat{T}_{m}(\varsigma, z),$$

$$\hat{T}_{m}(\varsigma, z) = \frac{I_{0}C_{m}}{\kappa_{s}(\varsigma - i\omega_{m})} \left\{ \frac{(1 - r_{s})\exp(q_{s}(z - l_{s})) + (1 + r_{s})\exp(-q_{s}(z - l_{s}))}{q_{s}[(1 + r_{s})^{2}\exp(q_{s}l_{s}) - (1 - r_{s})^{2}\exp(-q_{s}l_{s})]} \right\}.$$
(6)

In the previous expression, $r_s = h(\kappa_s q_s)^{-1}$. The temperature distributions due to the modulated heat source show a heavily damped behaviour in the spatial dependence: these distributions are often referred as thermal waves. Despite the discussion about the correctness of the term [18, 19], the temperature distributions are in fact damped by a factor called the thermal diffusion length, which depends on the modulation frequency [3]. The particular value of the modulation frequency at which the thermal diffusion length equals the sample thickness is called the characteristic frequency $f_c = \alpha_s (\pi l_s^2)^{-1}$, which defines the thermal regimes in the PT techniques (see table 1).

It is convenient to define the following dimensionless numbers:

$$q_{\rm s}l_{\rm s} = \sqrt{\frac{\zeta}{\pi f_{\rm c}}} \equiv \sqrt{\xi}, \qquad z^* \equiv \frac{z}{l_{\rm s}}, \qquad r_{\rm s} = \frac{h}{\kappa_{\rm s}} \sqrt{\frac{\pi f_{\rm c}}{\zeta}} \equiv \frac{Bi}{\sqrt{\xi}},$$
 (7)

where $Bi \equiv h \cdot l_s \cdot \kappa_s^{-1}$ is the Biot number, z^* is the normalized spatial coordinate and ξ calledhere the relative complex frequency. With these definitions, the term \hat{T}_m in equation (6) can be rewritten as

$$\hat{T}_{m}(\xi, z) = \frac{I_{0}l_{s}^{3}C_{m}\exp(-\sqrt{\xi} \cdot z^{*})}{\kappa_{s}\alpha_{s}(\sqrt{\xi} + Bi)(\xi - 2i\nu_{m})} \left\{ \frac{1 + \left(\frac{1 - r_{s}}{1 + r_{s}}\right)\exp(-\sqrt{\xi}(2 - z^{*}))}{1 - \left(\frac{1 - r_{s}}{1 + r_{s}}\right)^{2}\exp(-2\sqrt{\xi})} \right\} = \cdots = \frac{I_{0}l_{s}^{3}C_{m}\exp(-\sqrt{\xi} \cdot z^{*}) \cdot F(\xi, z)}{\kappa_{s}\alpha_{s}(\sqrt{\xi} + Bi)(\xi - 2i\nu_{m})} \equiv \frac{A_{m} \cdot \exp(-\sqrt{\xi} \cdot z^{*}) \cdot F(\xi, z)}{(\sqrt{\xi} + Bi)(\xi - 2i\nu_{m})}.$$
(8)

In this equation $v_m \equiv \omega_m (2\pi f_c)^{-1}$ will be called the relative modulation frequency, and the terms A_m are amplitude factors (with units of K \cdot s) depending on the harmonic index *m*.

Any thermal regime can then be obtained from the asymptotic behaviour of function $F(\xi, z)$ and the values of *Bi*. Notice that

$$\left|\frac{1-r_{\rm s}}{1+r_{\rm s}}\right| = \left|\frac{\sqrt{\xi}-Bi}{\sqrt{\xi}+Bi}\right| < 1.$$
⁽⁹⁾

As a consequence of equation (9) function F, although sensitive to the combination of values of ξ and Bi, is bounded and decays asymptotically to zero. This implies that F is exponential in nature, which is a necessary condition (using Post's inversion formula) for the existence of the inverse Laplace transform. However, the inversion might not be possible in terms of elementary functions in a closed form. In general, the inversion of equation (8) is only possible by numerical procedures. In the next subsections we will perform this inversion only for two limiting situations for which analytical expressions become available.

3.2. Thermally thick sample approximation

For $\exp(-q_s l_s) \approx 0$, the sample can be considered as very thick in the sense that $\xi \gg 1$ corresponds to the thermally thick regime. In this case, equation (8) becomes

$$\hat{T}_m(\xi, z) \approx \frac{A_m \cdot \exp(-\sqrt{\xi} \cdot z^*)}{(\sqrt{\xi} + Bi)(\xi - 2i\nu_m)}.$$
(10)

There are no available analytical expressions in terms of elementary functions for the inverse Laplace transform of equation (10) but for $z^* = 0$ one finds that equation (6) leads to

$$\Theta(t,0) \approx \frac{I_0}{\varepsilon_s} \sum_m \frac{C_m}{(b_s^2 + i\omega_m)} \{ b_s - b_s \operatorname{erfc}(b_s \sqrt{t}) \exp\left(b_s^2 t\right) + \cdots + \exp(i\omega_m t) [b_s - \sqrt{i\omega_m} \operatorname{erf}(\sqrt{i\omega_m t})] \}.$$
(11)

In the previous expression, $\varepsilon_s = \kappa_s \cdot \alpha_s^{-1/2}$ is the thermal effusivity of the sample and $b_s \equiv Bi \cdot (\pi f_c)^{1/2}$. Equation (11) is shown in figure 4 for a 40 cm thickness aluminum sample with $\alpha_s = 1 \text{ cm}^2 \text{ s}^{-1}$, $\kappa_s = 2.38 \text{ W cm}^{-1} \text{ K}^{-1}$ and $h = 1.75 \text{ mW cm}^{-2} \text{ K}^{-1}$, heated by a power density of 300 mW cm⁻². Calculations were performed using the modulation function defined by equation (1).

Even if closed forms for the inverse Laplace transform of equation (10) cannot be obtained for $z^* > 0$, it can be argued, since equation (10) tends to zero for sufficient great values of $\Re\{q_s l_s\}$ (which is precisely the starting point assumption), that the temperature Θ tends to zero for $z^* > 0$. This is the expected behaviour in the thermally thick approximation, already discussed by Salazar *et al* [1] for the case of continuous illumination.

3.3. Thermally thin sample approximation

Now we deal with the opposite case, $\xi \ll 1$, for which $\exp(\pm q_s l_s) \approx 1 \pm q_s l_s$. The general expression for \hat{T}_m (equation (6)) can be approximated by

$$\hat{T}_m(\xi, z) \approx \frac{A_m}{(\xi - i\nu_m)} \left[\frac{1 + Bi(1 - z^*)}{\xi + Bi + Bi^2} \right].$$
 (12)

Using this approximation from equation (12) we find that

$$\Theta(t,z) \approx \frac{I_0}{C_s l_s} \sum_m \frac{C_m (1 + Bi(1 - z^*))}{(b_s^2 + u_s + i\omega_m)} \Big[\exp(i\omega_m t) - \exp\left(-(b_s^2 + u_s)t\right) \Big],$$
(13)

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Figure 4. Theoretical calculation of the transient temperature profile at z = 0, in the thick sample approximation.

where $u_s \equiv 2Bi \cdot \pi f_c$. Now, let us consider that the Biot number is much smaller than unity. Then we have

$$\hat{T}_m \approx \frac{A_m}{(\xi - i\nu_m)(\xi + Bi)} \tag{14}$$

and

$$\Theta \approx \frac{I_0}{C_s l_s} \sum_m \frac{C_m}{(u_s + i\omega_m)} [\exp(i\omega_m t) - \exp(-u_s t)].$$
(15)

Figure 5 shows the theoretical predictions of the sample's temperature at z = 0. It can be seen that the time dependence shows a similar behaviour as that reported by Salazar *et al* [1] for a transient source but displaying the influence of the periodicity of the heat source. The temperature of the sample rises slowly from the ambient temperature following small oscillations around a central value. This central value can be obtained from the solution of equation (2), under the boundary conditions given by equation (3), when the heat source does not vary with time [1].

Marin *et al* [2] demonstrated that, for continuous heating, the thermally thin approximation is achieved for a value of the Biot number much smaller than unity. However, because in the present case the heating is modulated in amplitude, this is no longer a necessary and sufficient condition. Instead it is necessary to consider the influence of the characteristic frequency as well. From the analysis of expressions (8–10) and (12), the thermal response resolved in time is defined not only through the Biot number but also through the combination of the values of Biot number and the characteristic frequency.

3.4. Temperature profile for long times

Once the transient state is reached, the temperature distribution along the sample will vary periodically around a central value, given by the mean asymptotic value of the transient temperature distribution expressed by expression (11) or (13), depending on the particular case.



Figure 5. Theoretical calculation of the transient variations temperature profile at z = 0, in the thin sample approximation. The dotted line represents the central value calculated for continuous heating.

For this state—that will be designed here as the stationary state—the temperature variations are calculated by solving equation (2) under the boundary conditions (3), by means of the unitary Fourier transform. In this way, the following equivalent boundary value problem is obtained:

$$\frac{\mathrm{d}^{2}}{\mathrm{d}z^{2}}\tilde{\Theta}(\omega',z) - \sigma_{\mathrm{s}}^{2}(\omega')\tilde{\Theta}(\omega',z) = -\frac{\sqrt{2\pi}I_{0}\delta(z)}{\kappa_{\mathrm{s}}}\sum_{m}C_{m}\delta(\omega'-\omega_{m}) - \kappa_{\mathrm{s}}\left.\frac{\mathrm{d}}{\mathrm{d}z}\tilde{\Theta}(\omega',z)\right|_{z=0} + h\tilde{\Theta}(\omega',0) = 0; \quad \kappa_{\mathrm{s}}\left.\frac{\mathrm{d}}{\mathrm{d}z}\tilde{\Theta}(\omega',z)\right|_{z=l_{\mathrm{s}}} + h\tilde{\Theta}(\omega',l_{\mathrm{s}}) = 0,$$

$$(16)$$

where $\sigma_s^2 \equiv i\omega' \alpha_s^{-1}$ is the square of the complex thermal diffusion coefficient, which is related to the thermal diffusion length. Solving equation (16) by means of the Green's function technique, the following solution is found:

$$\tilde{\Theta}(\omega', z) = \sum_{m} R_m(z)\delta(\omega' - \omega_m), \qquad (17)$$

$$R_m(z) = -\frac{\sqrt{2\pi}I_0 C_m}{\kappa_s} \int_0^{l_s} K(z, z')\delta(z') \,\mathrm{d}z',$$
(18)

where

$$K(z,z') = \begin{cases} \frac{[\cosh \sigma_{s}z + r_{s} \sinh \sigma_{s}z][\cosh \sigma_{s}(z' - l_{s}) - r_{s} \sinh \sigma_{s}(z' - l_{s})]}{\sigma_{s}[(1 + r_{s}^{2}) \sinh \sigma_{s}l_{s} + 2r_{s} \cosh \sigma_{s}l_{s}]}; & z' > z\\ \frac{[\cosh \sigma_{s}(z - l_{s}) - r_{s} \sinh \sigma_{s}(z - l_{s})][\cosh \sigma_{s}z' + r_{s} \sinh \sigma_{s}z']}{\sigma_{s}[(1 + r_{s}^{2}) \sinh \sigma_{s}l_{s} + 2r_{s} \cosh \sigma_{s}l_{s}]}; & z' < z. \end{cases}$$
(19)



Figure 6. Theoretical calculation of the stationary temperature variation profile, at z = 0 (solid line) and z = 8 cm (dashed line).

Here, *K* is the Green's function and $r_s \equiv h(\kappa_s \sigma_s)^{-1}$. Using equations (17) and (19) and the inverse Fourier transform, the temperature variation profile is given by

$$\Theta(t,z) = \frac{I_0}{\kappa_s} \sum_m \frac{C_m}{\sigma_{ms}} \left[\frac{\cosh \sigma_{ms}(z-l_s) - r_{ms} \sinh \sigma_{ms}(z-l_s)}{\left[(1+r_{ms}^2) \sinh \sigma_{ms} l_s + 2r_{ms} \cosh \sigma_{ms} l_s \right]} \right] \exp(i\omega_m t).$$
(20)

Here $\sigma_{ms}^2 \equiv i\omega_m \alpha_s^{-1}$ and $r_{ms} \equiv h(\kappa_s \sigma_{ms})^{-1}$. Note that in this case it was not necessary to introduce any kind of approximation to obtain an analytical solution. In figures 6 and 7 the temperature variations with time for some values of *z* are shown.

As can be seen from figures 6 and 7, there is an amplitude decrease (exponential, as shown in figure 8) and a shift to the right of the maximal values of the temperature oscillations with increasing z. This behaviour of the temperature profile is consistent with experimental results reported in the literature [7].

The small oscillations in the theoretical curves (especially for small values of z) observed in figures 6 and 7 damp out with increasing number of harmonics taken in the calculation of equation (18), but this will produce a notable increment in the computing time.

4. Calorimetric model

Imagine a homogeneous sample with constant thickness l_s and cylindrical symmetry, heated by the power P(t), modulated in amplitude by any periodic function with a modulation frequency f. If the temperature gradient is negligible, the change in its heat energy Q is governed by the following equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}Q(t) + R(t) = P_0 \sum_m C_m \exp\left(\mathrm{i}\omega_m t\right),\tag{21}$$



Figure 7. Theoretical calculation of the stationary temperature variation profile, at positions z = 16 cm (solid line), z = 24 cm (dashed line) and z = 32 cm (dotted line).



Figure 8. Dependence of the temperature variation amplitudes as function of the axial position for the developed temperature profile.

where R(t) is the total heat loss by radiation and/or convection, and P_0 is the amplitude of the absorbed excitation power. Bearing in mind that the rate of increase of the sample's temperature is small enough, we have

$$R(t) = 2hA_{\rm s}\Theta(t), \tag{22}$$



Figure 9. Theoretical calculation of the temperature variations for a solid homogeneous sample by means of the calorimetric model.

$$Q(t) = C_{\rm s} A_{\rm s} l_{\rm s} \Theta(t).$$
⁽²³⁾

In these equations A_s is the area of the illuminated surface, and C_s is the specific volume heat capacity of the sample. Substituting equations (22) and (23) in equation (21) leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}\Theta(t) + 2h\left(C_{\mathrm{s}}l_{\mathrm{s}}\right)^{-1}\Theta(t) = I_0\left(C_{\mathrm{s}}l_{\mathrm{s}}\right)^{-1}\sum_m C_m\exp(\mathrm{i}\omega_m t).$$
(24)

It is important to keep in mind that equation (21) requires $l_s \ll \kappa_s h^{-1}$, that is the sample's thermal resistance to be much smaller than the thermal resistance associated with the heat loss by radiation and/or convection. If the initial condition $\Theta(t = 0) = 0$ is imposed on equation (24), by means of the parameter variations technique [20], we obtain the following general solution:

$$\Theta(t) = \frac{I_0}{C_s I_s} \sum_m \frac{C_m}{i\omega_m + u_s} [\exp(i\omega_m t) - \exp(-u_s t)].$$
(25)

This model is frequently used in the thermal relaxation method [18], where the spatial dependence of the temperature is neglected.

In figure 9, we show the theoretical predictions for the same aluminum sample as before, considering that the C_m coefficients are the same of the periodic rectangular wave defined in equation (1).

Now, the reader can compare equation (25) with equation (15) and conclude that the expressions are exactly the same. Consequently, the calorimetric model is in fact a particular result of the transient heat diffusion model, in the thermally thin sample approximation. This is not surprising, since very small values for the Biot number imply an almost uniform (in space) temperature distribution; which is the initial assumption of the calorimetric model.



Figure 10. (a) Diagram of the measurement system. (b) Detail of the measurement cell, showing the spreading of the thermocouples along the thermometric region.

5. Experimental results

For the general experimental arrangement, described in section 2, the measurement cell consists of an aluminum rod (as a sample) of total thickness L equal to 40 cm, surrounded by a polystyrene isolator. Figure 10(a) shows a schematic representation of the measurement system. The electric resistance, the power supply and the thermocouples were interfaced to a personal computer, through a programmable logic controller (PLC) [21], to control the measurement system and for recording the obtained data.

As figure 10(b) shows, sets of thermocouples were introduced with equidistribution inside the sample to get a thermal profile of the sample. In figure 11, a comparison between the theoretical calculations and the experimental data obtained for z = 0 cm is shown.

It is clear from this figure that as time increases, resulting in a temperature rise, the theoretical and experimental curves fit better and better. This can be attributed to the temperature dependence of the heat exchange coefficient h, whose value increases as the sample gets warmer. In figure 12 the graphs of the experimental data are presented for z = 0, 8 and 16 cm.

The experimental results are qualitatively consistent with the theoretical predictions for the transient response of the sample. However, the separation between the experimental curves for different z values, the temperature rise in the transient state and the amplitude of the temperature oscillations do not correspond with the expected behaviour. This is a consequence of the position of the temperature sensors. Since the sensors almost cross the sample, they act as thermal barriers, dividing the sample in four regions, each one heated by a different power density. Thus, what is really obtained instead of a temperature profile is the result of simultaneous measurements made on four 'different' samples—each one acting as the heating device of the next adjacent portion of the rod. In summary, the experimental setup must be carefully prepared in order to not disturb the continuity and the homogeneity of the heat flux.



Figure 11. Comparison between the experimental results (solid line) and the theoretical transient response (dotted line) for z = 0 cm.



Figure 12. Experimental results obtained for an aluminum sample of 40 cm thickness, for z = 0 cm (solid line), 8 cm (dashed line) and 16 cm (dotted line).

This requires a careful design that allows reading of the sample's surface temperature in order to satisfy the conditions of the model, without major disturbances in experimental results.

6. Conclusions

From the study of the transient state for periodical heating in heat diffusion, it can be possible to obtain information about specific heat capacity and thermal effusivity, just by taking the

mean value of the asymptotic temperature (with respect to the ambient temperature), that is, the temperature close to the end of the transient state. As is demonstrated in section 4, when the sample is such that it can be considered thermally thin (in the context of this paper), the calorimetric model is appropriate to evaluate the specific heat capacity and the heat exchange coefficient.

For samples between the thermally thin and thick regime, and when the transient state is overcome, the analysis of the oscillations around the asymptotic value can serve to evaluate the heat exchange coefficient and the thermal diffusivity, using equation (20). Of course, a nonlinear fitting procedure will be necessary in this case. But, as the comparison between the models and the experimental data shows, one must be very careful while positioning the temperature sensors, and not alter, or even break the continuity of the heat flux.

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