Active Vibration Control Using On-line Algebraic Identification and Sliding Modes

Control Activo de Vibraciones Usando Identificación Algebraica en Línea y Modos Deslizantes

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Abstract

In this paper is described the application of an on-line algebraic identification methodology for parameter and signal estimation in vibrating systems. The algebraic identification is employed to estimate the frequency and amplitude of exogenous vibrations affecting the mechanical system using only position measurements. The algebraic identification is combined with an adaptive-like sliding mode control scheme to asymptotically stabilize the system response and, simultaneously, cancel the harmonic vibrations. Numerical and experimental results show the dynamic and robust performance of the algebraic identification and the active vibration control scheme. **Keywords:** Active vibration control, On-line algebraic identification, Sliding mode control.

Resumen

En este artículo se describe la aplicación de una metodología para la identificación algebraica en línea para estimación de parámetros y señales en sistemas vibratorios. La identificación algebraica se utiliza para estimar la frecuencia y la amplitud de vibraciones exógenas que afectan a un sistema mecánico, usando únicamente mediciones de posición. La identificación algebraica se combina con un esquema de control por modos deslizantes del tipo adaptable para estabilizar asintóticamente la respuesta del sistema y, simultáneamente, cancelar las vibraciones armónicas. Resultados numéricos y experimentales muestran el desempeño dinámico y robusto de la identificación algebraica y del esquema de control activo.

Palabras clave: Control activo de vibraciones, Identificación algebraica en línea, Control por modos deslizantes.

1 Introduction

The identification of dynamical systems, involving explicitly the parameter identification, is a well-known process to develop or improve the mathematical description of a physical system by means of a proper use of experimental data. System identification methods provide the analytical tools, algorithms, computational programs and real-time implementation to get good approximations to an adequate model for analysis and control purposes. There exists a vast literature on the area, although most of the identification and estimation methods are essentially asymptotic, recursive or complex that lead to unrealistic implementations. See, e.g., [Ljung, 1987; Soderstrom, 1989; Sagara and Zhao, 1989].

This paper deals with the application of an on-line algebraic identification approach to estimate the physical parameters on mass-spring-damper systems as well as the amplitude and excitation frequency of harmonic perturbations affecting directly the mechanical system. The algebraic identification is combined with an adaptive-like sliding mode control scheme to asymptotically stabilize the system response and, simultaneously, cancel the harmonic vibrations. The main virtue of the proposed identification and adaptive-like sliding mode control scheme for vibrating systems is that only measurements of the transient input/output behavior are used during the identification process, in contrast to the well-known persisting excitation condition and complex algorithms required by most of the traditional identification methods [Ljung, 1987; Soderstrom, 1989]. The proposed results are strongly

based on a theoretical framework on algebraic identification methods reported recently by [Fliess and Sira-Ramírez, 2003].

2 Algebraic parameter identification

To illustrate the basic ideas of the algebraic identification methods proposed by [Fliess and Sira-Ramírez, 2003], it is considered the on-line parameter identification of a simple one degree-of-freedom mass-spring-damper system as well as the parameters associated to an exogenous harmonic perturbation affecting directly its dynamics.

The mathematical model of the mechanical system is described by the ordinary differential equation

$$m\ddot{x} + c\dot{x} + kx = u(t) + f(t) \tag{1}$$

where x denotes the displacement of the mass carriage, u is a control input (force) and $f(t) = F0sin\omega t$ is a harmonic force (perturbation). The system parameters are the mass m, the stiffness constant of the linear spring k and the viscous damping c.

In spite of a priori knowledge of the mathematical model (1), it results evident that this is only an approximation for the physical system, where for large excursions of the mass carriage the mechanical spring has nonlinear stiffness function and close to the rest position there exist nonlinear damping effects (e.g., dry or Coulomb friction). Another inconvenient is that the information used during the identification process contains small measurement errors and noise. It is therefore realistic to assume that the identified parameters will represent approximations to equivalent values into the physical system. As a consequence the algorithms will have to be sufficiently robust against such perturbations. Some of these properties have been already analyzed by [Fliess and Sira-Ramírez, 2003].

On-line algebraic identification

Consider the unperturbed system (1), that is, when $f(t) \equiv 0$, where only measurements of the displacement *x* and the control input *u* are available to be used in the on-line parameter identification scheme. To do this, the differential equation (1) is described in notation of operational calculus [Fliess and Sira-Ramírez, 2003] as follows

$$m(s^{2}x(s) - sx_{0} - \dot{x}_{0}) + c(sx(s) - x_{0}) + kx(s) = u(s)$$
⁽²⁾

where $x_0 = x(t_0)$ and $\dot{x_0} = \dot{x}(t_0)$ are unknown constants denoting the system initial conditions at $t_0 \ge 0$. In order to eliminate the dependence of the constant initial conditions, the equation (2) is differentiated twice with respect to the variable *s*, resulting in

$$m\left(2x+4s\frac{dx}{ds}+s^2\frac{d^2x}{ds^2}\right)+c\left(2\frac{dx}{ds}+s\frac{d^2x}{ds^2}\right)+k\frac{d^2x}{ds^2}=\frac{d^2u}{ds^2}$$
(3)

Now, multiplying (3) by s^{-2} one obtains that

$$m\left(2s^{-2}x + 4s^{-1}\frac{dx}{ds} + \frac{d^2x}{ds^2}\right) + c\left(2s^{-2}\frac{dx}{ds} + s^{-1}\frac{d^2x}{ds^2}\right) + ks^{-2}\frac{d^2x}{ds^2} = s^{-2}\frac{d^2u}{ds^2}$$
(4)

and transforming back to the time domain leads to the integral equation

$$m\left[2\left(\int_{t_0}^{(2)} x\right) - 4\left(\int_{t_0} (\Delta t)x\right) + (\Delta t)^2 x\right] + c\left[-2\left(\int_{t_0}^{(2)} (\Delta t)x\right) + \left(\int_{t_0} (\Delta t)^2 x\right)\right]$$
(5)

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$$+k\left(\int_{t_0}^{(2)} (\Delta t)^2 x\right) = \left(\int_{t_0}^{(2)} (\Delta t)^2 u\right)$$

where $\Delta t = t - t_0$ and $\left(\int_{t_0}^{(n)} \varphi(t)\right)$ are iterated integrals of the form $\int_{t_0}^t \int_{t_0}^{\sigma_1} \cdots \int_{t_0}^{\sigma_{n-1}} \varphi(\sigma_n) d\sigma_n \cdots d\sigma_1$, with $\left(\int_{t_0} \varphi(t)\right) = \int_{t_0}^t \varphi(\sigma) d\sigma$ and *n* a positive integer.

The above integral-type equation (5), after some more integrations, leads to the following *linear* system of equations

$$A(t)\theta = b(t) \tag{6}$$

where $\theta = [m, c, k]^T$ denotes the parameter vector to be identified and A(t), b(t) are 3×3 and 3×1 matrices, respectively, which are described by

$$A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & a_{13}(t) \\ a_{21}(t) & a_{22}(t) & a_{23}(t) \\ a_{31}(t) & a_{32}(t) & a_{33}(t) \end{bmatrix}, b(t) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ b_3(t) \end{bmatrix}$$

whose components are time functions specified as

$$\begin{aligned} a_{11} &= 2 \int_{t_0}^{(2)} x - 4 \int_{t_0} (\Delta t) x + (\Delta t)^2 x & a_{31} = 2 \int_{t_0}^{(4)} x - 4 \int_{t_0}^{(3)} (\Delta t) x + \int_{t_0}^{(2)} (\Delta t)^2 x \\ a_{12} &= -2 \int_{t_0}^{(2)} (\Delta t) x - 4 \int_{t_0} (\Delta t)^2 x & a_{32} = -2 \int_{t_0}^{(4)} (\Delta t) x + \int_{t_0}^{(3)} (\Delta t)^2 x \\ a_{13} &= \int_{t_0}^{(2)} (\Delta t)^2 x & a_{33} = \int_{t_0}^{(4)} (\Delta t)^2 x \\ a_{21} &= 2 \int_{t_0}^{(3)} x - 4 \int_{t_0}^{(2)} (\Delta t) x + \int_{t_0} (\Delta t)^2 x & b_1 = \int_{t_0}^{(2)} (\Delta t)^2 u \\ a_{22} &= -2 \int_{t_0}^{(3)} (\Delta t) x + \int_{t_0}^{(2)} (\Delta t)^2 x & b_2 = \int_{t_0}^{(3)} (\Delta t)^2 u \\ a_{23} &= \int_{t_0}^{(3)} (\Delta t)^2 x & b_3 = \int_{t_0}^{(4)} (\Delta t)^2 u \end{aligned}$$

From the equation (6) can be concluded that the parameter vector θ is *algebraically identifiable* if, and only if, the trajectory of the dynamical system is *persistent* in the sense established by [Fliess and Sira-Ramírez, 2003], that is, the trajectories or dynamic behavior of the system (1) satisfy the condition det $A(t) \neq 0$.

In general, this condition holds at least in a small time interval $(t_0, t_0 + \delta]$, where δ is a positive and sufficiently small value.

By solving the equations (6) it is obtained the following algebraic identifier for the unknown system parameters

$$\widehat{m} = \frac{\Delta_{1}(t)}{detA(t)}
\widehat{c} = \frac{\Delta_{2}(t)}{detA(t)}
\widehat{k} = \frac{\Delta_{3}(t)}{detA(t)}$$

$$\forall t \in (t_{0}, t_{0} + \delta]$$
(7)

where

$$\begin{split} &\Delta_1 = b_1 a_{22} a_{33} - b_1 a_{23} a_{32} - b_2 a_{12} a_{33} + b_2 a_{13} a_{32} + b_3 a_{12} a_{23} - b_3 a_{13} a_{22} \\ &\Delta_2 = -b_1 a_{21} a_{33} + b_1 a_{31} a_{23} + b_2 a_{11} a_{33} - b_2 a_{13} a_{31} - b_3 a_{11} a_{23} + b_3 a_{21} a_{13} \\ &\Delta_3 = b_1 a_{21} a_{32} - b_1 a_{22} a_{31} - b_2 a_{11} a_{32} + b_2 a_{12} a_{31} + b_3 a_{11} a_{22} - b_3 a_{12} a_{21} \\ &det A(t) = a_{11} a_{22} a_{33} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} + a_{12} a_{31} a_{23} + a_{21} a_{13} a_{32} - a_{13} a_{22} a_{31} \end{split}$$

Simulation and experimental results

The performance of the on-line algebraic identifier of the system parameters (7) is now evaluated by means of numerical simulations and experiments on an electromechanical platform (ECP^{TM} rectilinear plant) with a single degree-of-freedom mass-spring-damper system. The physical parameters were previously estimated through several experiments with different excitation inputs (natural and forced vibrations, step and sine sweep inputs, etc.) resulting in the following set of parameters:

$$m = 2.2685[kg], c = 4.1241 \left[\frac{Ns}{m}\right] and k = 356.56[N/m]$$

Nevertheless, it is convenient to remark that the real system clearly exhibits nonlinear effects like nonlinear stiffness and damping functions (hard springs and Coulomb friction on the slides) that were not considered during the synthesis of the algebraic identifier.

Fig. 1 shows the simulation results using the algebraic identifier for a step input u=4 [N]. Here it is clear how the parameter identification is quickly performed (before t=1.02 s) and it is almost exact with respect to the real parameters. It is also evident the presence of singularities in the algebraic identifier, i.e., when the determinant den=det(A) is zero. The first singularity, however, occurs about t=1.02 s, that is too much time (more than 5 times) after the identification has been finished.



Fig. 1. Simulation results of the algebraic identifier. The subscript "e" denotes estimated values and den=detA(t)

Fig. 2 presents the corresponding experimental results using the on-line algebraic identification scheme (7). In this case the actual system response is quite similar to the numerical simulation, resulting in the following (equivalent) parameters:

$$m = 2.25[kg], c = 4.87[Ns/m] and k = 362[N/m]$$

These values represent good approximations for the real parameters. Nevertheless, the identification process starts with some irregular behavior and the estimation takes more time (about $t_s = 0.4 s$), which we have attributed to several factors like neglected nonlinear effects (stiffness and friction), presence of noise on the output measurements and especially the computational algorithms based upon a sampled-time system with fast sampling time $t_s = 0.884 ms$ and numerical integrations based on trapezoidal rules. Some of these problems in the parameter estimation have been already analyzed by [Sagara and Zhao, 1990].



Fig. 2. Experimental results obtained on a ECPTM Rectilinear Plant using algebraic identification methods

Many numerical and experimental results validate the good response of the on-line algebraic identification methods of unknown parameters. In addition, it can be proved the good robustness properties of the algebraic identifiers against stochastic perturbations, noisy measurements, small parameter variations and nonlinearities, which are not included here for space limitations. Moreover, because the algebraic identification process is quickly achieved with a high-speed DSP board, then any possible singularity does not affect significantly the identification results. Otherwise, close to any singularity or variations on the system dynamics, the algebraic identifier can be restarted.

3 Algebraic identification of harmonic vibrations

The algebraic identification methods can be applied to estimate the parameters associated to exogenous perturbations affecting a mechanical vibrating system.

Consider again the mechanical system (1) with known parameters m, k and c and only measurements of the displacement x and the control input u are available for the identification process of the harmonic signal $f(t) = F_0 \sin \omega t$. In this case we proceed to synthesize algebraic identifiers for the excitation frequency ω and amplitude F_0 .

Identification of the excitation frequency ω

System (1) is then expressed in notation of operational calculus as

$$m(s^{2}x(s) - sx_{0} - \dot{x}_{0}) + c(sx(s) - x_{0}) + kx(s) = u(s) + F_{0}\frac{\omega}{s^{2} + \omega^{2}}$$
(8)

where $x_0 = x(0)$ and $\dot{x_0} = \dot{x}(0)$ are the initial conditions. By multiplying (8) by $(s^2 + \omega^2)$ is obtained

$$s^{2}(ms^{2}x + csx + kx - u) + \omega^{2}(ms^{2}x + csx + kx - u) = F_{0}\omega + a_{3}s^{3} + a_{2}s^{2} + a_{1}s + a_{0}$$
(9)

where a_i , $i = 0 \dots 3$, are unknown constants depending on the system initial conditions and the excitation frequency ω , that is,

$$a_0 = \omega^2 (m\dot{x}_0 + cx_0), a_1 = \omega^2 mx_0, a_2 = m\dot{x}_0 + cx_0, a_3 = mx_0$$

In order to eliminate the presence of the amplitude F_0 and the constants a_i , i = 0, ..., 3, we differentiate the equation (9) four times with respect to s and the result is multiplied by s^{-4} , resulting

$$N_1(s) - D_1(s)\omega^2 = 0 \tag{10}$$

where

$$\begin{split} N_1(s) &= 24ms^{-4}x + 96ms^{-3}\frac{dx}{ds} + 72ms^{-2}\frac{d^2x}{ds^2} + 16ms^{-1}\frac{d^3x}{ds^3} + m\frac{d^4x}{ds^4} \\ &+ 24cs^{-4}\frac{dx}{ds} + 36cs^{-3}\frac{d^2x}{ds^2} + 12cs^{-2}\frac{d^3x}{ds^3} + cs^{-1}\frac{d^4x}{ds^4} + 12ks^{-4}\frac{d^2x}{ds^2} \\ &+ 8ks^{-3}\frac{d^3x}{ds^3} + ks^{-2}\frac{d^4x}{ds^4} - 12s^{-4}\frac{d^2u}{ds^2} - 8s^{-3}\frac{d^3u}{ds^3} - s^{-2}\frac{d^4u}{ds^4} \\ D_1(s) &= -12ms^{-4}\frac{d^2x}{ds^2} - 8ms^{-3}\frac{d^3x}{ds^3} - ms^{-2}\frac{d^4x}{ds^4} - 4cs^{-4}\frac{d^3x}{ds^3} - cs^{-3}\frac{d^4x}{ds^4} - ks^{-4}\frac{d^4x}{ds^4} + s^{-4}\frac{d^4u}{ds^4} \end{split}$$

The algebraic equation (10) is now transformed into the time domain, that is,

$$N_1(t) - D_1(t)\omega^2 = 0 \tag{11}$$

where

$$\begin{split} N_{1}(t) &= 24m \left(\int_{t_{0}}^{(4)} x \right) - 96m \left(\int_{t_{0}}^{(3)} (\Delta t) x \right) + 72m \left(\int_{t_{0}}^{(2)} (\Delta t)^{2} x \right) - 16m \left(\int_{t_{0}}^{(\Delta t)^{3} x} \right) \\ &+ m (\Delta t)^{4} x - 24c \left(\int_{t_{0}}^{(4)} (\Delta t) x \right) + 36c \left(\int_{t_{0}}^{(3)} (\Delta t)^{2} x \right) - 12c \left(\int_{t_{0}}^{(2)} (\Delta t)^{3} x \right) \\ &+ c \left(\int_{t_{0}} (\Delta t)^{4} x \right) + 12k \left(\int_{t_{0}}^{(4)} (\Delta t)^{2} x \right) - 8k \left(\int_{t_{0}}^{(3)} (\Delta t)^{3} x \right) + k \left(\int_{t_{0}}^{(2)} (\Delta t)^{4} x \right) \\ &- 12c \left(\int_{t_{0}}^{(4)} (\Delta t)^{2} u \right) + 8 \left(\int_{t_{0}}^{(3)} (\Delta t)^{3} u \right) - \left(\int_{t_{0}}^{(2)} (\Delta t)^{4} u \right) \\ D_{1}(t) &= -12m \left(\int_{t_{0}}^{(4)} (\Delta t)^{2} x \right) + 8m \left(\int_{t_{0}}^{(3)} (\Delta t)^{3} x \right) - m \left(\int_{t_{0}}^{(2)} (\Delta t)^{4} x \right) + 4c \left(\int_{t_{0}}^{(4)} (\Delta t)^{3} x \right) \\ &- c \left(\int_{t_{0}}^{(3)} (\Delta t)^{4} x \right) - k \left(\int_{t_{0}}^{(4)} (\Delta t)^{4} x \right) + \left(\int_{t_{0}}^{(4)} (\Delta t)^{4} u \right) \end{split}$$

Therefore, if the system trajectory or its solution is *persistent* in the sense formulated by [Fliess and Sira-Ramírez, 2003] (i.e., when the condition $D_1(t) \neq 0$ be satisfied at least for a small time interval $(t_0, t_0 + \delta_0]$, where δ_0 is a sufficiently small quantity), we can find from (11) a closed-form expression for the estimated excitation frequency

$$\omega_e = \sqrt{\frac{N_1(t)}{D_1(t)}}, \quad \forall t \in (t_0, t_0 + \delta_0]$$
(12)

which is independent of the amplitude F_0 and the initial conditions.

Identification of the amplitude F_0

To synthesize an algebraic identifier for the amplitude F_0 of the harmonic vibrations acting on the mechanical system (1), we first express the differential equation in notation of operational calculus as follows

$$ms^{2}x(s) + csx(s) + kx(s) = u(s) + f(s) + mx_{0}s + m\dot{x}_{0} + cx_{0}$$
(13)

Taking derivatives, twice, with respect to *s* and multiplying by s^{2} makes possible to remove the dependence on the initial conditions $x_0 = x(0)$ and $\dot{x}_0 = \dot{x}(0)$, resulting

$$m\left(2s^{-2}x + 4s^{-1}\frac{dx}{ds} + \frac{d^2x}{ds^2}\right) + c\left(2s^{-2}\frac{dx}{ds} + s^{-1}\frac{d^2x}{ds^2}\right) + ks^{-2}\frac{d^2x}{ds^2} = s^{-2}\frac{d^2}{ds^2}u(s) + s^{-2}\frac{d^2}{ds^2}f(s)$$
(14)

Now, a time domain representation of (14) leads to

$$m \left[2 \left(\int_{t_0}^{(2)} x \right) - 4 \left(\int_{t_0} (\Delta t) x \right) + (\Delta t)^2 x \right] + c \left[-2 \left(\int_{t_0}^{(2)} (\Delta t) x \right) + \left(\int_{t_0} (\Delta t)^2 x \right) \right]$$

$$+ k \left(\int_{t_0}^{(2)} (\Delta t)^2 x \right) = \left(\int_{t_0}^{(2)} (\Delta t)^2 u \right) + F_0 \left(\int_{t_0}^{(2)} (\Delta t)^2 \sin \omega t \right)$$
(15)

It is important to note that equation (14) still depends on the excitation frequency ω . Therefore, it is required to synchronize both algebraic identifiers for ω and F_0 . This procedure is sequentially executed, first by running the identifier for ω and, after some small time interval with the estimation $\omega_e(t_0 + \delta_0)$ is then started the algebraic identifier for F_0 , which is obtained by solving

$$N_2(t) - D_2(t)F_0 = 0 (16)$$

In this case the system trajectory is persistent if, and only if, the condition $D_2(t) \neq 0$ is satisfied for all $t \in (t_0 + \delta_0, t_0 + \delta_1]$ with $\delta_1 > \delta_0 > 0$ [Fliess and Sira-Ramírez, 2003]. Then the solution F_0 of (16) yields the algebraic identifier

$$F_{0e} = \frac{N_2(t)}{D_2(t)}, \qquad \forall t \in (t_0 + \delta_0, t_0 + \delta_1]$$
(17)

where

$$N_{2}(t) = m \left[2 \left(\int_{t_{0}+\delta_{0}}^{(2)} x \right) - 4 \left(\int_{t_{0}+\delta_{0}} (\Delta t) x \right) + (\Delta t)^{2} x \right] + c \left[-2 \left(\int_{t_{0}+\delta_{0}}^{(2)} (\Delta t) x \right) + \left(\int_{t_{0}+\delta_{0}} (\Delta t)^{2} x \right) \right]$$

$$+k\left(\int_{t_0+\delta_0}^{(2)} (\Delta t)^2 x\right) - \left(\int_{t_0+\delta_0}^{(2)} (\Delta t)^2 u\right)$$
$$D_2(t) = \left(\int_{t_0+\delta_0}^{(2)} (\Delta t)^2 sin[\omega_e(t_0+\delta_0)t]\right)$$

Simulation and experimental results

The performance of algebraic identifier of harmonic vibrations (12) and (17) is now evaluated by means of numerical and experimental results on the ECPTM rectilinear plant, where is configured a single degree-of freedom mechanical system with $u \equiv 0$. The real force applied to the system is given by

$$f(t) = 4sin(5t) [N]$$

where the true amplitude and excitation frequency are $F_0 = 4$ [N] and $\omega = 5$ [rad/s], respectively.

In Fig. 3 are shown some numerical simulations using the algebraic identifiers for the excitation frequency ω and amplitude F_0 . First of all it is started the identifier for ω , which takes about t < 0.2 s to get a good estimation. After the time interval (0,0.7] s, where $t_0 = 0$ s and $\delta_0 = 0.7$ s with an estimated value $\omega_e(t_0 + \delta_0) = 5$ [rad/s], it is activated the identifier for F_0 . In this case the identification process is faster, taking a very small time to obtain a good estimation for F_0 . Both estimations are almost exact with respect to the true values.



Fig. 3. Simulation results using algebraic identification of the excitation frequency ω and amplitude F_0

The experimental results are presented in Fig. 4, where it is easy to note that both identification processes are slower with respect to the numerical simulations.

Again, we assume that such incompatibility is caused by unmodelled dynamics and nonlinearities into the physical system (e.g., nonlinear stiffness, dry and Coulomb friction, backlash), the numerical methods used in the computational algorithms (e.g., sampling-data measurements, numerical integrations) and presence of noise into the input/output measurements. Such obstacles, however, do not affect substantially the algebraic identification process, resulting in fast and good estimations $\omega_e = 4.9$ [rad/s] and $F_{0e} = 3.933$ [N].



Fig. 4. Experimental results using on-line identification for ω and F_0

4 An Active Vibration Control Scheme

Consider the vibrating mechanical system shown in Fig. 5.a, which consists of a passive vibration absorber (secondary system) coupled to the perturbed mechanical system (1). The generalized coordinates are the displacements of both mass carriages, x_1 and x_2 , respectively. In addition, *u* represents the (force) control input and *f* a harmonic perturbation. Here m_1 , k_1 and c_1 denote mass, linear stiffness and linear viscous damping on the primary system; similarly, m_2 , k_2 and c_2 denote mass, stiffness and viscous damping of the passive/active vibration absorber.



Fig. 5. Mechanical system: a) Schematic diagram, and b) ECPTM rectilinear control system

The mathematical model of the two degree-of-freedom system is described by two coupled ordinary differential equations

$$m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) = f(t)$$

$$m_2 \ddot{x}_2 + k_2 (x_2 - x_1) = u(t)$$
(18)

where $f(t) = F_0 \sin \omega t$. In order to simplify the analysis we have assumed that $c_1 \approx 0$ and $c_2 \equiv 0$. Defining the state variables as $z_1 = x_1$, $z_2 = \dot{x}_1$, $z_3 = x_2$ and $z_4 = \dot{x}_2$, one obtains the following state space description

$$\dot{z}_{1} = z_{2}$$

$$\dot{z}_{2} = -\frac{k_{1} + k_{2}}{m_{1}} z_{1} - \frac{c_{1}}{m_{1}} z_{2} + \frac{k_{2}}{m_{1}} z_{3} + \frac{1}{m_{1}} f$$

$$\dot{z}_{3} = z_{4}$$

$$\dot{z}_{4} = -\frac{k_{2}}{m_{2}} z_{1} - \frac{k_{2}}{m_{2}} z_{3} + \frac{1}{m_{2}} u$$

$$y = z_{1}$$
(19)

It is easy to verify that system (19) is completely controllable and observable, as well as marginally stable in case of $c_1=0$, f=0 and u=0 (asymptotically stable when $c_1 > 0$). Note that the disturbance decoupling problem of f(t) using state feedback is not solvable because the output $y = z_1$ has relative degree 4 with respect to u and relative degree 2 with respect to f.

To cancel the exogenous harmonic vibrations on the primary system, the dynamic vibration absorber should apply an equivalent force to the primary system, with the same amplitude but in opposite phase. This means that the vibration energy injected to the primary system is transferred to the absorber through the coupling elements (i.e., spring k_2). Of course, this vibration control method is possible under the assumption of perfect knowledge of the exogenous vibrations f(t) and stable operating conditions. See, e.g., [Korenev and Reznikov, 1993] and references therein.

In what follows we will apply the algebraic identification method to estimate the harmonic force f(t) and design an active vibration controller based on state feedback and feedforward information of f(t).

Differential flatness property

 $\dot{n} = a$

Because the system (19) is completely controllable then, it is differentially flat, with flat output given by $y = z_1$. Then, all the state variables and the control input can be parameterized in terms of the flat output y and a finite number of its time derivatives [Fliess *et al.*, 1993]. As a matter of fact, from y and its time derivatives up to fourth order we can obtain:

$$\begin{aligned} \ddot{y} &= -\frac{k_1 + k_2}{m_1} z_1 + \frac{k_2}{m_1} z_3 \\ y^{(3)} &= -\frac{k_1 + k_2}{m_1} z_2 + \frac{k_2}{m_1} z_4 \\ y^{(4)} &= \left[\frac{(k_1 + k_2)^2}{m_1^2} + \frac{k_2^2}{m_1 m_2} \right] z_1 - \left(\frac{k_1 k_2}{m_1^2} + \frac{k_2^2}{m_1^2} + \frac{k_2^2}{m_1 m_2} \right) z_3 + \frac{k_2}{m_1 m_2} u \end{aligned}$$
(20)

where $c_1=0$ and f=0. Therefore, the parameterization results as follows

$$z_{1} = y$$

$$z_{2} = \dot{y}$$

$$z_{3} = \frac{k_{1} + k_{2}}{k_{2}}y + \frac{m_{1}}{k_{2}}\ddot{y}$$

$$z_{4} = \frac{k_{1} + k_{2}}{k_{2}}\dot{y} + \frac{m_{1}}{k_{2}}y^{(3)}$$

$$u = \frac{m_{1}m_{2}}{k_{2}}y^{(4)} + k_{1}y + \left(m_{1} + m_{2} + \frac{k_{1}}{k_{2}}m_{2}\right)\ddot{y}$$
(21)

From (21) we can obtain a control law to asymptotically track some desired reference trajectory $y^*(t)$ given by

$$u = \frac{m_1 m_2}{k_2} v + k_1 y + \left(m_1 + m_2 + \frac{k_1}{k_2} m_2\right) \ddot{y}$$

$$v = (y^*)^{(4)}(t) - \beta_6 \left[y^{(3)} - (y^*)^{(3)}(t)\right] - \beta_5 \left[\ddot{y} - \ddot{y}^*(t)\right] - \beta_4 \left[\dot{y} - \dot{y}^*(t)\right] - \beta_3 \left[y - y^*(t)\right]$$
(22)

where β_i , i = 3, ..., 6, are positive real constants, which are chosen such that the characteristic polynomial $s^4 + \beta_6 s^3 + \beta_5 s^2 + \beta_4 s + \beta_3$ be *Hurwitz*, i.e., all its roots lying in the open left half complex plane. Nevertheless, this controller is not robust with respect to exogenous signals or parameter uncertainties in the model. In case of $f(t) \neq 0$, the parameterization should explicitly include the effect of f and its time derivatives up to second order.

Next, we will synthesize a sliding mode controller, which combines the property of differential flatness and the integral reconstruction approach, in order to get a robust controller against external vibrations.

Sliding mode control

Consider the perturbed system (19), where only the output $y = z_1$ and the input *u* are available for use on a sliding mode control scheme, under the temporary assumption that the excitation frequency ω is perfectly known.

The state variables and the control input u can be expressed in terms of the flat output y, the perturbation and their time derivatives:

$$z_{1} = y$$

$$z_{2} = \dot{y}$$

$$z_{3} = \frac{k_{1} + k_{2}}{k_{2}}y + \frac{m_{1}}{k_{2}}\ddot{y} - \frac{1}{k_{2}}f(t)$$

$$z_{4} = \frac{k_{1} + k_{2}}{k_{2}}\dot{y} + \frac{m_{1}}{k_{2}}y^{(3)} - \frac{1}{k_{2}}\dot{f}(t)$$

$$u = \frac{m_{1}m_{2}}{k_{2}}y^{(4)} + k_{1}y + \left(m_{1} + m_{2} + \frac{k_{1}}{k_{2}}m_{2}\right)\ddot{y} - f(t) - \frac{m_{2}}{k_{2}}\ddot{f}(t)$$
(23)

Then, when $f(t) = F_0 sin\omega t$, the output y satisfies the following perturbed input-output differential equation

$$y^{(4)} + \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2}\right)\ddot{y} + \frac{k_1k_2}{m_1m_2}y = \frac{k_2}{m_1m_2}u + \frac{1}{m_1}\left(\frac{k_2}{m_2} - \omega^2\right)F_0sin\omega t$$
(24)

For simplicity, we have supposed that $c_1 = c_2 \equiv 0$.

In the equation (24), one can see that when the excitation frequency ω coincides with the uncoupled natural frequency of the dynamic vibration absorber (i.e., $\omega = \omega_2 = \sqrt{k_2/m_2}$) the vibrations affecting the primary system are cancelled.

The main goal is the design of a robust controller that allows to the active dynamic vibration absorber to cancel harmonic vibrations of any excitation frequency affecting the primary system and, simultaneously, the mechanical system follows an off-line pre-specified desired reference trajectory. In addition, we want to preserve the main application of a passive dynamic vibration absorber. This means that the control effort must be zero (u=0) at the tuning frequency of the vibration absorber (ω_2).

To do that, we differentiate (24) twice with respect to time, resulting

$$y^{(6)} + \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2}\right)y^{(4)} + \frac{k_1k_2}{m_1m_2}\ddot{y} = \frac{k_2}{m_1m_2}\ddot{u} - \frac{1}{m_1}\left(\frac{k_2}{m_2} - \omega^2\right)\omega^2 F_0 sin\omega t$$
(25)

Multiplication of (24) by ω^2 and adding it to (25), leads to

$$y^{(6)} + d_1 y^{(4)} + d_2 \ddot{y} + d_3 y = d_4 (\ddot{u} + \omega^2 u)$$
(26)

where

$$d_{1} = \frac{k_{1} + k_{2}}{m_{1}} + \frac{k_{2}}{m_{2}} + \omega^{2} \qquad \qquad d_{3} = \frac{k_{1}k_{2}}{m_{1}m_{2}}\omega^{2}$$
$$d_{2} = \left(\frac{k_{1} + k_{2}}{m_{1}} + \frac{k_{2}}{m_{2}}\right)\omega^{2} + \frac{k_{1}k_{2}}{m_{1}m_{2}} \qquad \qquad d_{4} = \frac{k_{2}}{m_{1}m_{2}}$$

The input-output differential equation (26) allows us to design a controller that does not depend on the amplitude of the vibrations F_0 .

A traditional sliding surface that, under ideal sliding conditions: $\sigma = 0$, $\dot{\sigma} = 0$, gets asymptotic tracking of a desired output reference trajectory $y^*(t)$ is given by

$$\sigma = y^{(5)} - (y^*)^{(5)}(t) + \gamma_4 [y^{(4)}(t) - (y^*)^{(4)}(t)] + \gamma_3 [y^{(3)} - (y^*)^{(3)}(t)] + \gamma_2 [\ddot{y} - \ddot{y}^*(t)] + \gamma_1 [\dot{y} - \dot{y}^*(t)] + \gamma_0 [y - y^*(t)]$$
(27)

where the design gains { γ_0 , γ_1 , γ_2 , γ_3 , γ_4 } are chosen so that the characteristic polynomial $p(s) = s^5 + \gamma_4 s^4 + \gamma_3 s^3 + \gamma_2 s^2 + \gamma_1 s + \gamma_0$ is *Hurwitz*, i.e., all its roots lying in the open left half complex plane.

However, the sliding surface (27) requires the knowledge of some time derivatives of the output variable y.

From (26), the time derivatives of the output variable y up to fifth order can be reconstructed by means of integral reconstruction. That is, they can be expressed in terms of the output y, the input u and iterated integrals of the input and output variables. For more details on this topic see [Fliess *et al.*, 2002].

For zero initial conditions, the integral input-output parameterization of the time derivatives of the output variable is given by

$$\hat{y} = -d_1 \int_{t_0} (y) + \int_{t_0}^{(3)} (d_4 u - d_2 y) + \int_{t_0}^{(5)} (d_4 \omega^2 u - d_3 y)$$

$$\hat{y} = -d_1 y + \int_{t_0}^{(2)} (d_4 u - d_2 y) + \int_{t_0}^{(4)} (d_4 \omega^2 u - d_3 y)$$
(28)

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$$\hat{y}^{(3)} = -d_1\hat{y} + \int_{t_0} (d_4u - d_2y) + \int_{t_0}^{(3)} (d_4\omega^2 u - d_3y)$$
$$\hat{y}^{(4)} = -d_1\hat{y} - d_2y + \int_{t_0}^{(2)} (d_4\omega^2 u - d_3y) + d_4u$$
$$\hat{y}^{(5)} = -d_1\hat{y}^{(3)} - d_2\hat{y} + \int_{t_0} (d_4\omega^2 u - d_3y) + d_4\dot{u}$$

These expressions were obtained by successive integrations of the equation (26). For simplicity, we have denoted the integral $\int_{t_0}^t \varphi(\tau) d\tau$ by $\int_{t_0}^t (\varphi)$ and $\int_{t_0}^t \int_{t_0}^{\tau} \varphi(\rho) d\rho d\tau$ by $\int_{t_0}^{(2)} (\varphi)$ and so on. For non-zero initial conditions, these expressions differ from the actual values by at most a fourth order time

For non-zero initial conditions, these expressions differ from the actual values by at most a fourth order time polynomial of the form: $p_4t^4 + p_3t^3 + p_2t^2 + p_1t + p_0$, where p_i , $i = 0 \dots 4$, are all real constants depending on the unknown initial conditions.

A sliding surface can now be proposed as

$$\hat{\sigma} = \hat{y}^{(5)} - (y^*)^{(5)} + \alpha_9 [\hat{y}^{(4)} - (y^*)^{(4)}(t)] + \alpha_8 [\hat{y}^{(3)} - (y^*)^{(3)}] + \alpha_7 [\hat{y} - \ddot{y}^*] + \alpha_6 [\hat{y} - \dot{y}^*] + \alpha_5 e + \alpha_4 \int_{t_0} e + \alpha_3 \int_{t_0}^{(2)} e + \alpha_2 \int_{t_0}^{(3)} e + \alpha_1 \int_{t_0}^{(4)} e + \alpha_0 \int_{t_0}^{(5)} e$$
(29)

where $y^*(t)$ is a desired output reference trajectory and $e = y - y^*(t)$. Then, the ideal sliding condition $\hat{\sigma} = 0$ results in a tenth order dynamics

$$e^{(10)} + \alpha_9 e^{(9)} + \alpha_8 e^{(8)} + \alpha_7 e^{(7)} + \alpha_6 e^{(6)} + \alpha_5 e^{(5)} + \alpha_4 e^{(4)} + \alpha_3 e^{(3)} + \alpha_2 \ddot{e} + \alpha_1 \dot{e} + \alpha_0 e = 0$$
(30)

which is completely independent of any initial conditions. Therefore, selecting the design parameters α_i , i = 0, ..., 9, such that the associated characteristic polynomial for (30) be *Hurwitz*, one guarantees that the error dynamics on the sliding surface $\hat{\sigma} = 0$ be globally asymptotically stable. In addition, by forcing the sliding surface $\hat{\sigma}$ to satisfy the discontinuous closed loop dynamics:

$$\hat{\sigma} = -W sign(\hat{\sigma}), \quad W > 0$$
 (31)

where sign stands for the *signum* function, one then gets the sliding mode control as a solution of the differential equation

$$\begin{aligned} \ddot{u} + \omega^{2} u &= d_{4}^{-1} v + d_{4}^{-1} \left(d_{1} \hat{y}^{(4)} + d_{2} \hat{y} + d_{3} y \right) \\ v &= y^{*(6)} - \alpha_{9} \left[\hat{y}^{(5)} - y^{*(5)} \right] - \alpha_{8} \left[\hat{y}^{(4)} - y^{*(4)} \right] - \alpha_{7} \left[\hat{y}^{(3)} - y^{*(3)} \right] - \alpha_{6} \left[\hat{y} - \ddot{y}^{*}(t) \right] \\ - \alpha_{5} \left[\hat{y} - \dot{y}^{*}(t) \right] - \alpha_{4} \left[y - y^{*}(t) \right] - \alpha_{3} \xi_{1} - \alpha_{2} \xi_{2} - \alpha_{1} \xi_{3} - \alpha_{0} \xi_{4} - W sign(\hat{\sigma}) \end{aligned}$$
(32)

where $\alpha_5 = \alpha_9(d_2 - d_1^2) + \alpha_7 d_1$. This controller employs only measurements of $y = z_1$ and the excitation frequency ω .

Simulation and experimental results

Fig. 6 shows the numerical and experimental dynamic behavior of the sliding mode control scheme. In this case, the harmonic perturbation $f(t) = 0.515 \sin(17.7t)$ was applied to the mechanical system, which is close to the system's resonance. In the implementation of this controller, an error in the measurement of the excitation frequency was

introduced intentionally, this is ω =19 [rad/s]. In spite of that, we can see how the active vibration absorber dissipates the vibrating energy from the primary system H_1 and allows the asymptotic output regulation about $\bar{y} = \bar{z}_1 = 5$ [mm]. The parameters for the ECPTM rectilinear plant are $m_1 = 3.4316[kg], c_1 = 1.11[Ns/m], k_1 = 356.56[N/m], m_2 = 2.745[kg], c_2 = 3.8122[Ns/m], k_2 = 356.56[N/m].$



Fig. 6. Numerical and experimental responses of the close-loop system

Algebraic identification of harmonic vibrations

The main goal is the algebraic identification of the harmonic force f(t), which will be obtained through similar procedures stated in previous sections using *only* measurements of the output $y = z_1$ and considering that the system parameters are perfectly known.

Consider the input-output differential equation (24) written in operational calculus terms

$$m_{1}s^{4}y(s) + \left(k_{1} + k_{2} + \frac{m_{1}k_{2}}{m_{2}}\right)s^{2}y(s) + \frac{k_{1}k_{2}}{m_{2}}y(s)$$

$$= \frac{k_{2}}{m_{2}}u(s) + \left(\frac{k_{2}}{m_{2}} - \omega^{2}\right)F_{0}\frac{\omega}{s^{2} + \omega^{2}} + a_{3}s^{3} + a_{2}s^{2} + a_{1}s + a_{0}$$
(33)

where a_i , i = 0, ..., 3, denote unknown constants depending on the unknown system initial conditions. Now, equation (33) is multiplied by $(s^2 + \omega^2)$, leading to

$$(s^{2} + \omega^{2}) \left(s^{4}y(s) + \frac{k_{2}}{m_{2}} s^{2}y(s) \right) m_{1} + \left(s^{2}y(s) + \frac{k_{2}}{m_{2}}y(s) \right) k_{1} + k_{2}s^{2}y(s)$$

$$= \frac{k_{2}}{m_{2}} (s^{2} + \omega^{2})u(s) + \left(\frac{k_{2}}{m_{2}} - \omega^{2}\right) F_{0}\omega + (s^{2} + \omega^{2})(a_{3}s^{3} + a_{2}s^{2} + a_{1}s + a_{0})$$
(34)

This equation is differentiated six times with respect to *s* in order to cancel the constants a_i and the unknown amplitude F_0 . The resulting equation is then multiplied by s^{-6} , and next transformed into the time domain, to get

$$[a_{11}(t) + \omega^2 a_{12}(t)]m_1 + [a_{12}(t) + \omega^2 b_{12}(t)]k_1 = c_1(t) + \omega^2 d_1(t)$$
(35)

Where

$$\begin{aligned} a_{11}(t) &= m_2 g_{11}(t) + k_2 g_{12}(t) \\ a_{12}(t) &= m_2 g_{21}(t) + k_2 g_{13}(t) \\ b_{12}(t) &= m_2 g_{13}(t) + k_2 \left(\int_{t_0}^{(6)} t^6 z_1 \right) \\ c_1(t) &= k_2 g_{14}(t) - k_2 m_2 g_{12}(t) \\ d_1(t) &= k_2 \left(\int_{t_0}^{(6)} t^6 u \right) - k_2 m_2 g_{13}(t) \end{aligned}$$

With

$$\begin{split} g_{11}(t) &= 720 \left(\int_{t_0}^{(6)} y \right) - 4320 \left(\int_{t_0}^{(5)} (\Delta t) y \right) + 5400 \left(\int_{t_0}^{(4)} (\Delta t)^2 y \right) - 2400 \left(\int_{t_0}^{(3)} (\Delta t)^3 y \right) \\ &+ 450 \left(\int_{t_0}^{(2)} (\Delta t)^4 y \right) - 36 \left(\int_{t_0} (\Delta t)^5 y \right) + (\Delta t)^6 y \\ g_{12}(t) &= 360 \left(\int_{t_0}^{(6)} (\Delta t)^2 y \right) - 480 \left(\int_{t_0}^{(5)} (\Delta t)^3 y \right) + 180 \left(\int_{t_0}^{(4)} (\Delta t)^4 y \right) - 24 \left(\int_{t_0}^{(3)} (\Delta t)^5 y \right) + \left(\int_{t_0}^{(2)} (\Delta t)^6 y \right) \\ g_{13}(t) &= 30 \left(\int_{t_0}^{(6)} (\Delta t)^4 y \right) - 12 \left(\int_{t_0}^{(5)} (\Delta t)^5 y \right) + \left(\int_{t_0}^{(4)} (\Delta t)^6 y \right) \\ g_{14}(t) &= 30 \left(\int_{t_0}^{(6)} (\Delta t)^4 u \right) - 12 \left(\int_{t_0}^{(5)} (\Delta t)^5 u \right) + \left(\int_{t_0}^{(4)} (\Delta t)^6 u \right) \end{split}$$

Finally, solving for the excitation frequency ω in (35) leads to the following on-line identifier

$$\omega_e^2 = \frac{N_1(t)}{D_1(t)} = \frac{c_1(t) - a_{11}(t)m_1 - a_{12}(t)k_1}{a_{12}(t)m_1 + b_{12}(t)k_1 - d_1(t)}$$
(36)

This estimation is valid if, and only if, the condition $D_1(t) \neq 0$ holds in a sufficiently small time interval $(t_0, t_0 + \delta_0]$ with $\delta_0 > 0$.

By using the same procedure to estimate the amplitude F_0 in the section 2, we obtain the following on-line identifier for the amplitude

$$F_{0e} = \frac{N_2(t)}{D_2(t)}$$
(37)

where

$$\begin{split} N_{2}(t) &= m_{1}P_{1}(t) + \left(k_{1} + k_{2} + \frac{m_{1}k_{2}}{m_{2}}\right)P_{2}(t) + \frac{k_{1}k_{2}}{m_{2}}\left(\int_{t_{0}+\delta_{0}}^{(4)} (\Delta t)^{4}y\right) - \frac{k_{2}}{m_{2}}\left(\int_{t_{0}+\delta_{0}}^{(4)} (\Delta t)^{4}u\right)\\ D_{2}(t) &= \left(\frac{k_{2}}{m_{2}} - \omega_{e}^{2}\right)\left(\int_{t_{0}+\delta_{0}}^{(4)} (\Delta t)^{4}sin[\omega_{e}(t_{0}+\delta_{0})t]\right) \end{split}$$

$$P_{1}(t) = 24 \left(\int_{t_{0}}^{(4)} y \right) - 96 \left(\int_{t_{0}}^{(3)} (\Delta t) y \right) + 72 \left(\int_{t_{0}}^{(2)} (\Delta t)^{2} y \right) - 16 \left(\int_{t_{0}}^{(\Delta t)^{3}} y \right) + (\Delta t)^{4} y$$
$$P_{2}(t) = 12 \left(\int_{t_{0}}^{(4)} (\Delta t)^{2} y \right) - 8 \left(\int_{t_{0}}^{(3)} (\Delta t)^{3} y \right) + \left(\int_{t_{0}}^{(2)} (\Delta t)^{4} y \right)$$

At this point we assume that the excitation frequency has been previously estimated, during a small time interval $(t_0, t_0 + \delta_0]$, using (36). After the time $t = t_0 + \delta_0$ it is started the on-line identifier for the amplitude. Such an estimation is valid as far as the system trajectory be persistent, that is, if the condition $D_2(t) \neq 0$ holds for a sufficiently small time interval $[t_0 + \delta_0, t_0 + \delta_1]$ with $\delta_1 > \delta_0 > 0$.

Simulation results

Fig. 7 shows the identification process of the resonant harmonic vibrations f(t) = 2sin(8.0109t) [N] and the dynamic behavior of the adaptive-like control scheme (32), which starts using the nominal value $\omega = 10$ [rad/s]. We can see that the resonant vibrations are asymptotically and actively cancelled from the displacement of the primary system z_1 . A desired reference trajectory was considered for regulating the evolution of the output variable toward the desired equilibrium $\overline{y} = \overline{z_1} = 0.01$ [m], which is given by a Bezier type polynomial in time. The parameters for the ECPTM rectilinear plant are $m_1 = 10[kg], k_1 = 1000[N/m], m_2 = 2[kg], k_2 = 200[N/m].$



Fig. 7. Controlled system responses and identification of frequency and amplitude of $f(t) = F_0 sin\omega t$

5 Conclusions

In this paper we have described the application of a novel algebraic identification approach for parameter and signal estimation in vibrating systems. This approach is quite promising, in the sense that only input-output information is needed to get precise and fast parameter and signal estimations. This fact was exploited in the formulation of an active vibration control scheme based on sliding modes. Since this active controller requires measurements of the excitation frequency of the harmonic vibrations, the algebraic identification is combined to get an adaptive-like controller. The adaptive-like control scheme results quite precise, fast and robust against parameter uncertainty and variations on the excitation frequency and amplitude of the exogenous perturbations. Further work is being conducted to extend the application to nonlinear vibrating systems.

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