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Análisis y control para sistemas lineales con retardo de tiempo: aplicación a sistemas con reciclo.

T E S I S

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PRESENTA:
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DIRECTOR DE TESIS:
DR. BASILIO DEL MURO CUÉLLAR

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ACTA DE REVISIÓN DE TESIS

En la Ciudad de México siendo las 12:00 horas del día 5 del mes de Enero del 2012 se reunieron los miembros de la Comisión Revisora de la Tesis, designada por el Colegio de Profesores de Estudios de Posgrado e Investigación de SEPI-ESIME-Culhuacan para examinar la tesis titulada:

Análisis y control para sistemas lineales con retardo de tiempo: aplicación a sistemas con reciclo.

Presentada por el alumno:

Marquez	Rubio	Juan Francisco
Apellido paterno	Apellido materno	Nombre(s)

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Juan Francisco Marquez Rubio

Nombre y firma

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Resumen

Éste trabajo presenta nuevos resultados para el diseño de observador-controlador para plantas lineales invariantes en tiempo que pueden contener retardos. El análisis considera principalmente sistemas con reciclo. La principal contribución de éste trabajo es la presentación de las condiciones de existencia de un observador para los sistemas mencionados; se proporcionan resultados relacionados con el seguimiento de referecia y rechazo de perturbaciones, ambas de tipo escalón. Los esquemas de control desarrollados en éste trabajo logran estabilidad y varias especificaciones de comportamiento. Se proporcionan algunos estudios de robustés con respecto a los retardos y parametros. En todos los casos se presentan simulaciones numéricas para mostrar la efectividad de las metodologias propuestas.

Abstract

This work presents new results on the observer-controller design for linear time-invariant plants that may contain times delay. The analysis mainly considers recycling systems. The main contribution described here is the obtained existence conditions of an observer for the systems previously mentioned; results related with step tracking reference as well as step rejecting disturbance are also given. The control schemas developed in this work achieve stability and various performance specifications. Analysis of robustness with respect to time-delay and parameters is provided for some proposed control strategies. In all cases, numerical simulations are presented in order to show the effectiveness of the proposed methodologies.

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⋆ Control basado en un esquema observador para sistemas de primer orden con retardo. J.F. Marquez-Rubio, B. del Muro-Cuéllar, M. Velasco-Villa and J. Alvarez-Ramírez. Revista Mexicana de Ingeniería Química. Vol. 9, No. 1 (2010) pp 43-52. ISSN 1665-2738. (See Appendix B.1)

⋆ Stabilization Strategy for Unstable First Order Linear Systems with large time-delay. J.F. Marquez-Rubio, B. del Muro-Cuéllar, M. Velasco-Villa and J. Alvarez-Ramírez. Asian Journal of Control, Vol. 15, No. 6, pp. 1 9, November 2013. (See Appendix B.2)

⋆ On the Control of Unstable First Order Linear Systems with Large Time lag: Observer Based Approach. B. del Muro-Cuéllar, J.F. Marquez-Rubio, M. Velasco-Villa and J. Alvarez-Ramírez. Accepted in European Journal of Control. (See Appendix B.3)

Submitted Journal Papers (JCR index)

♣ Observer-PID Control for Unstable First Order Linear Systems with Large Time Delay. J.F. Marquez-Rubio, B. del Muro-Cuéllar and M. Velasco-Villa. Submitted to Industrial & Engineering Chemistry Research. (See Appendix B.4)

♣ Control of Delayed Recycling Systems with Unstable First Order Forward Loop. J.F. Marquez-Rubio, B. del Muro-Cuéllar, M. Velasco-Villa, D. Cortés and O. Sename. Submitted to Journal of Process Control. (See appendix B.5)

Journal Paper

⋆ Stabilisation and control of unstable first-order linear delay systems. B. del Muro-Cuéllar, J.F. Marquez-Rubio and M. Velasco-Villa. Int. J. Computer Applications in Technology, Vol. 41, pp 84-90. Nos. 1/2, 2011.

Conference Papers

▷ Stabilization Strategy for Unstable First Order Linear Systems with large time-delay. B. del Muro-Cuéllar, J.F. Marquez-Rubio, M. Velasco-Villa and J. Alvarez-Ramírez. 4th IFAC Symposium on System, Structure and Control, September 15-17, 2010, Ancona, Italy.

▷ Observer scheme for linear recycling systems with time delays. J.F. Marquez-Rubio, B. del Muro-Cuéllar, M. Velasco-Villa and D. Cortes-Rodriguez. 2011 American Control Conference on O'Farrell Street, San Francisco, CA, USA. June 29 - July 01.

Submitted Conference Paper

♣ Control of Delayed Recycling Systems with an Unstable Pole at Forward Path. J.F. Marquez-Rubio, B. del Muro-Cuéllar and O. Sename. Submitted to 2012 American Control Conference.

Article for submitting

▷ Stabilization Strategy for High Order Linear Recycling Systems with Time Delays. J.F. Marquez-Rubio, B. del Muro-Cuéllar and M. Velasco-Villa.

Nomenclature

FOPTD	First Order Plus Time Delay
SPC	Smith Predictor Compensator
A	State matrix of a particular observer
$\mathbf{A}$	State matrix of a particular observer
A_d	State matrix of direct loop
A_r	State matrix of recycle loop
$\mathcal{A}$	State matrix in Laplace domain of the observer-controller
$\mathcal{A}$	State matrix of $H(s)$
$\overline{A}$	State matrix for the open loop recycling system
$\overline{\mathbf{A}}$	State matrix in Laplace domain of the observer-controller
$\overline{A}_\delta$	Matrix uncertainties on $\overline{A}$
A_1	State matrix of a particular observer
$\mathbf{A}_1$	State matrix of a particular observer
$\mathcal{A}_1$	State matrix of $H(s)$
$\mathcal{A}_1$	State matrix in Laplace domain of the observer-controller
$\overline{A}_1$	State matrix for the open loop recycling system
$\overline{\mathbf{A}}_1$	State matrix in Laplace domain of the observer-controller
$\overline{A_{1\delta_1}}$	Matrix uncertainties on $\overline{A}_1$
A_2	State matrix of a particular observer
A_2	State matrix of $G_r(s)$ (Case 3)
$\mathbf{A}_2$	State matrix of a particular observer
$\mathcal{A}_2$	State matrix in Laplace domain of the observer-controller
$\overline{A}_2$	State matrix for the open loop recycling system
$\overline{\mathbf{A}}_2$	State matrix in Laplace domain of the observer-controller
$\overline{A_{2\delta_2}}$	Matrix uncertainties on $\overline{A}_2$

$\mathbf{A}_3$	State matrix of a particular observer
$\overline{\mathbf{A}}_3$	State matrix in Laplace domain of the observer-controller
$\overline{A}_3$	State matrix for the open loop recycling system
$\overline{A}_{3\delta_3}$	Matrix uncertainties on $\overline{A}_3$
a	Location of unstable pole in First Order system
B	Input vector of a particular observer
$\mathbf{B}$	Input vector of a particular observer
B_d	Input vector of direct loop
B_r	Input vector of recycle loop
$\mathcal{B}$	Input vector of $H(s)$
$\mathcal{B}$	Input vector in Laplace domain of the observer-controller
$\overline{B}$	Input vector for the open loop recycling system
$\overline{\mathbf{B}}$	Input vector in Laplace domain of the observer-controller
B_2	Input vector of $G_r(s)$ (Case 3)
b	Gain of a transfer function
b	Location of stable pole
$\overline{b}_i$	Location of stable poles
C	Output vector of a particular observer
$\mathbf{C}$	Output vector of a particular observer
$\mathcal{C}$	Output vector of $H(s)$
$\overline{C}$	Output vector for the open loop recycling system
$C_c(s)$	Static/dynamic controller
C_d	Output vector of direct loop
C_r	Output vector of recycle loop
$C(s)$	Static/dynamic controller
C_1	Output vector of a particular observer
$\mathbf{C}_1$	Output vector of a particular observer
$\mathcal{C}_1$	Output vector of $H(s)$
$\overline{C}_1$	Output vector for the open loop recycling system
C_2	Output vector of a particular observer
C_2	Output vector of $G_r(s)$ (Case 3)
c_i	Location of stable poles
$\overline{D}(s)$	Denominator of the delay free backward model
$D_0(s)$	Step disturbance

D_2	Feedthrough matrix of $G_r(s)$ (Case 3)
G	Observer vector gain
$G(s)$	Unstable first order transfer function
G_c	Block of PI-controller with two degree of freedom
G_d	Direct loop transfer function
G_{ff}	Block of PI-controller with two degree of freedom
G_r	Recycle loop transfer function
$G_s(s)$	Stable part of $G_1(s)$
$\overline{G}_s(s)$	Transfer function of the relocated poles of $G_s(s)$
$G_t(s)$	Open loop transfer function of recycle system
$G_u(s)$	Unstable part of $G_1(s)$
$G_1(s)$	Delay free forward model
$G_{1,2}$	Observer vector gain composed by g_1 and G_2
G_2	Observer vector gain
$G_2(s)$	Delay free backward model
G_3	Observer vector gain
g_1	Static gain
g_2	Static gain
g_3	Static gain designed for disturbance rejection
g_3	Static gain for the observer design
g_4	Static gain for tracking reference
$H(s)$	Step disturbance
$H(s)$	Transfer function
$H_s(s)$	Stable part of $H(s)$
$H_u(s)$	Unstable part of $H(s)$
$J(s)$	Static/dynamic controller
K	Proportional gain for PI-controller with two degree of freedom
$\overline{K}$	Control vector
k	Proportional Gain
$\overline{k}$	Static gain
k_d	Derivative gain
k_i	Integral gain
k_p	Proportional gain

k_1	Static gain
k_2	Static gain
L	Control vector
$\overline{N}(s)$	Numerator of the delay free backward model
$P(s,\theta)$	Time-delay uncertainty acting on the system
Q	Stability quadrilateral for a PID controller
$Q(s)$	Characteristic polynomial of the open loop recycle system
$Q(s,\theta)$	Time-delay uncertainty acting on the system
$R(s)$	Input reference
T_i	Integral time for PI-controller two degree of freedom
$U(s)$	Input system
$U(s)$	Input reference to controlled system
$V(s)$	Input system
w	Internal signal of unstable FOPTD
$\widehat{w}$	Estimated internal signal of unstable FOPTD
x_d	State vector of direct loop
$\widehat{x}_d$	Estimated state vector of direct loop
x_r	State vector of recycle loop
$\widehat{x}_r$	Estimated state vector of recycle loop
$Y(s)$	Output system
α	Gain of a transfer function
α	Lower bound stabilizing of static gain in an output feedback
α_i	Location of stable poles
α_r	Magnitude of step reference
β	Upper bound stabilizing of static gain in an output feedback
β_r	Magnitude of step disturbance
β_d	Static gain for disturbance rejection
β_i	Location of stable zeros
δ_i	Relocated observer poles of $G_s(s)$
ϵ	Constant sufficiently small higher than zero
η	Static gain for tracking reference
$\Theta(s,\theta)$	Plant uncertainty
θ	Time-delay uncertainty
θ_1	Uncertainty for the forward time-delay
θ_2	Uncertainty for the backward time-delay

σ	Constant higher than zero
$\overline{\sigma}$	Constant higher than zero
σ_c	Constant for PI-controller with two degree of freedom
τ	Input/output time-delay
τ_{un}	Unstable time-constant of a First Order system $\tau_{un} = a^{-1}$
τ_0	actual-real time delay system
τ_1	Part of time-delay τ
τ_1	Time delay at forward loop
τ_2	Part of time-delay τ
τ_2	Time delay at backward loop
τ_3	Time delay composed by τ_1 and τ_2
ψ	Observer-controller in nominal case
ω	Frequency
ω	Internal signal in unstable FOPTD
$\widehat{\omega}$	Estimated of ω
$\widehat{\omega}_u$	Estimated internal signal
$\omega_1(s)$	Internal signal of forward loop
$\omega_2(s)$	Internal signal of backward loop

Introduction

Time delay is the property of a physical system by which the response to an applied force (action) is delayed in its effect [1]. Whenever material, information or energy is physically transmitted from one place to another, there is a delay associated with the transmission [2]. Time delays, appearing in the modeling of different classes of systems (chemical processes, manufacturing chains, economy, etc.) are originated by several mechanisms like material transport, recycling loops. In economic systems, delays appear in a natural way since decisions and effects (investment policy, commodity markets evolution: price fluctuations, trade cycles) are separated by some (needed analysis) time interval. In communication, data transmission is always accompanied by a non-zero time interval between the initiation and the delivery-time of a message or signal [3]. In other cases, the delay term comes from the approximation of a high order system by means of a lower dimension one ([4],[5]). The presence of delays (especially, large delays) makes system analysis and control design much more complex [2]. From the control viewpoint, time delays become a challenging situation that should be affronted to yield acceptable closed-loop stability and performance. On the other hand, a particular case of systems with time-delay at states is the recycling system. In recycling systems the output of a process is partially feedback to the input. Recycling processes reuse the energy and the partially processed matter increasing the efficiency of the overall process. They are commonly found in chemical industry, for instance, in a typical plant formed by reactor/separator processes, where reactants are recycled back to the reactor [6], [7], [8]. Recycling processes are systems with positive feedback which can give rise to some undesirable effects even when no time-delay is present in the dynamic system. Moreover, in the case of the continuous stirred tank reactors (CSTR), a difficulty on the recycling is that the model almost always assumes no time delay in the recycle line. While this assumption may make theoretical analysis simpler, it is highly unrealistic [9]. Furthermore, in order to partially overcome the control problem, the control of recycling systems is addressed. Particularly the cases when one unstable pole at direct path, times delay at direct and recycle loops are considered. The main idea used in this work to control recycling systems is derived from the concept of recycle compensation to recuperate inherent process dynamics, i.e. dynamics without recycle (see [10] and [11]). In this way, as a first step an observer is proposed to recuperate the dynamics without recycle. As a second step, a controller for the forward loop is designed. In this work, three control methodologies are proposed in order to deal with recycling systems with times-delay.

In summary, this work presents new results on the observer-controller design. The focus is on linear time-invariant plants that may contain times delay. The analysis mainly considers recycling systems. The main contribution described here is the obtained existence conditions of an observer for the systems previously mentioned; results related with step tracking reference as well as step rejecting disturbance are also given. The control schemas developed in this work achieve stability and various performance specifications. Analysis of robustness with respect to time-delay and other parameters is provided for some proposed control strategies. In all cases, numerical simulations are presented in order to show the effectiveness of the methods. The work is organized as follows, in Chapter 1 the problem formulation is presented. Then as preliminary results, in Chapter 2 some novel control strategies for unstable FOPTD systems as well as some existing results for control of higher order systems with time delay are provided. For the control strategies for unstable FOPTD systems it is used the observer-controller approach. Essentially three control strategies for unstable FOPTD systems are presented. Although the observer-controller approach used in the proposed methodologies is similar it should be pointed out that they allow dealing with different size time-delay. In this way, some results presented in Chapter 2 will be used later in Chapter 3 for the observer design to recycling systems. Tracking step reference and rejecting step disturbance is addressed in Chapter 4. After this, robustness analysis for the developed control methodologies is provided in Chapter 5. The control schemas are evaluated by means numerical simulation in

Chapter 6 and finally some general conclusions of the work are given in Chapter 7.

Chapter 1

Problem Formulation in Control of Recycling Systems

As mentioned before, in recycling systems the output of a process is partially feedback to the input. Recycling processes reuse the energy and the partially processed matter increasing the efficiency of the overall process. However, some effects due to recycle loop could appear, for instance the so called snowball effect is observed in the operation of many chemical plants with recycle streams. Snowball means that a small change in a load variable causes a very large change in the flow rates around the recycle loop. Although snowballing is a steady state phenomenon and has nothing to do with dynamics, it depends on the control structure [12]. Disadvantages of snowball effect has drawn the attention of some researchers; Luyben [13], [14], [15], studied the effects of recycle loops on process dynamics and their implications to plant-wide control; Taiwo [10], discussed the robust control for recycling plants and proposed the concept of recycle compensation to recuperate inherent process dynamics, i.e. dynamics without recycle. Scali and Ferrari [11], analyzed the problem under the same idea. Similar approaches were extended by Lakshminarayanan and Takada [16], and Kwok et. al [17]. Due to the snowball effect, controls of systems with recycle loops are somewhat difficult and interesting in their own. However, when significant transport delay is present in recycled systems the control problem becomes more complex. It is known that in a system with recycle loops and time delays, exponential terms appears in forward and backward paths on its transfer function representation.

In state space representation recycled system with time delay correspond to systems with delays on the input and the state. Model approximation has been proposed to remove the exponential terms from the transfer function denominator of a delayed system, such as the method of moments [18], and Pade-Taylor approximations [19], [20], [21]. Other techniques, such as the seasonal time-series model [17], have been proposed to obtain an approximate model to represent recycle systems. Del Muro et. al. [22] proposed an approximate model to represent recycle systems by using discrete-time approach. In turn, such approximate models can be used for stability analysis or control design [23], [11], [24], [25]. A system with time delay and open-loop unstable poles is notably more difficult to control than a system with only open-loop stable poles. Introducing recycle in such system would lead to a very difficult (although interesting) problem. That is the reason why, to the authors best knowledge, recycle is not used in unstable plants with significant transport delay. To begin to overcome this situation, the problem of recycled system composed of an unstable first order plant in the direct path and a stable system of any order in the recycle loop is addressed in this work. In this way, the following Chapters are devoted to present three proposed control strategies for recycling systems. Before this, in this Chapter it is presented the problem formulation and the class of systems considered in this work is specified. The general idea of the solutions is also outlined in this Chapter. Here the need of an observer-predictor arises. After this, in Chapter 2 some preliminary results concerning to the control of systems with large time delay are presented. Then, in Chapter 3 the observer design for recycling systems is provided. Based on the estimation of necessary internal variables the overall control scheme is presented in Chapter 4 where some results concerning the tracking reference and disturbance rejecting are also given. In Chapter 5 robustness analysis with respect time delay is provided for the proposed observer-controller schemas. Some simulations results are described in Chapter 6.

1.1 Problem formulation

Let us consider the general class of recycling system shown in Figure 1.1, which can be described as,

$$Y(s) = \begin{bmatrix} G_d & G_d G_r \end{bmatrix} \begin{bmatrix} U(s) \\ Y(s) \end{bmatrix} \tag{1.1}$$

with,

$$G_d = G_1(s)e^{-\tau_1 s} = \frac{N(s)}{D(s)}e^{-\tau_1 s} \tag{1.2a}$$

$$G_r = G_2(s)e^{-\tau_2 s} = \frac{\overline{N}(s)}{\overline{D}(s)}e^{-\tau_2 s} \tag{1.2b}$$

where $G_d(s)$, and $G_r(s)$ are transfer functions of the forward (direct) and backward (recycle) paths, respectively; $\tau_1,\ \tau_2 \geq 0$ are the time delays associated to $G_d(s)$, and $G_r(s)$. $N(s)$, $D(s)$, $\overline{N}(s)$ and $\overline{D}(s)$are polynomials on the complex variable s. $U(s)$ is the process input and $Y(s)$ is the process output.

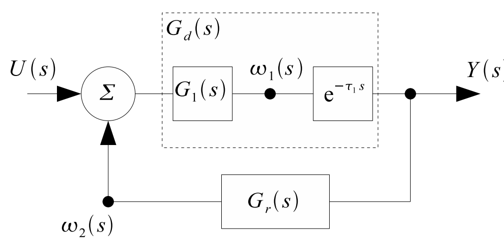

Figure 1.1: A process with recycle.

The closed-loop transfer function of system (1.1) is given by

$$G_t(s) = \frac{Y(s)}{U(s)} = \frac{N(s)\overline{D}(s)e^{-\tau_1 s}}{D(s)\overline{D}(s) - N(s)\overline{N}(s)e^{-(\tau_1+\tau_2)s}} \tag{1.3}$$

Note that exponential terms appear explicitly in numerator and denominator of $G_t(s)$. Stability of (1.3) is determined by the roots of its characteristic equation

$$Q(s) = D(s)\overline{D}(s) - N(s)\overline{N}(s)e^{-(\tau_1+\tau_2)s} = 0 \tag{1.4}$$

More precisely, the overall path $U(s) \rightarrow Y(s)$ is stable if and only if all the roots of $Q(s)$ are contained in the open left-half complex plane. It is well known that the transcendental term in $Q(s)$ induces an infinite number of roots preventing the use of classical control design techniques and stability analysis methods.

Let us to describe some ideas behind the methodology proposed. With reference to Figure 1.1, if signal ω_2 were known, then we could set

$$U(s) = R_1(s) - \omega_2(s) \tag{1.5}$$

obtaining the system shown in Figure 1.2. Then it would be possible to design $R_1(s)$ as $R_1(s) = (R(s) - \omega_1(s))\, J(s)$ like in Figure 1.3. Since ω_1 and ω_2 are internal system signals an observer-predictor is proposed in order to estimate these variables. In fact, Chapter 3 is devoted to this end for different cases in $G_1(s)$. In what follows, the treated cases in this work are defined.

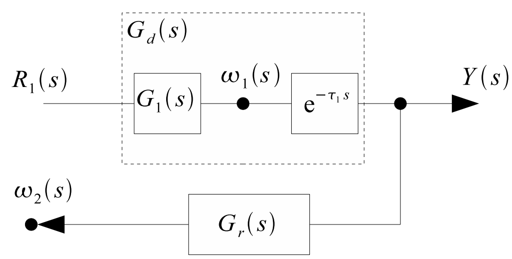

Figure 1.2: System of Figure 1.1 after applying U(s) in (1.5).

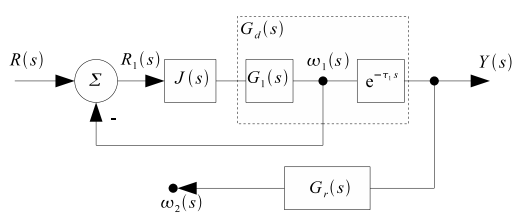

Figure 1.3: A control structure for the system of Figure 1.2.

Case 1.1 *Unstable FOPTD at forward path.*

$$G_d = G_1(s)e^{-\tau_1 s} = \frac{b}{s-a}e^{-\tau_1 s} \tag{1.6a}$$

$$G_r = G_2(s)e^{-\tau_2 s} = \frac{\overline{N}(s)}{\overline{D}(s)}e^{-\tau_2 s} \tag{1.6b}$$

where $a, b \in \mathbb{R}$, *with* $a > 0$, *that is* G_d *is unstable;* $\overline{N}(s)$ *and* $\overline{D}(s)$ *are polynomials on the complex variable* s.

Case 1.2 *One unstable pole and stable poles at forward path.*

$$G_d = G_1(s)e^{-\tau_1 s} = \frac{\alpha}{(s-a)(s+\overline{b_1})(s+\overline{b_2})...(s+\overline{b_m})}e^{-\tau_1 s}, \tag{1.7a}$$

$$G_r = G_2(s)e^{-\tau_2 s} = \frac{\overline{N}(s)}{\overline{D}(s)}e^{-\tau_2 s}, \tag{1.7b}$$

where $a, \overline{b}_i, \alpha \in \mathbb{R}$, *with* $a, \overline{b}_i > 0$, *that is* $G_d(s)$ *is unstable;* $\overline{N}(s)$ *and* $\overline{D}(s)$ *are polynomials on the complex variable* s.

Case 1.3 *Generalized forward path.*

$$G_d = G_1(s)e^{-\tau_1 s} = \frac{N_1(s)}{D_1(s)}\frac{b}{s-a}e^{-\tau_1 s} \tag{1.8a}$$

$$G_r = G_2(s)e^{-\tau_2 s} = \frac{\overline{N}(s)}{\overline{D}(s)}e^{-\tau_2 s} \tag{1.8b}$$

where $a, b \in \mathbb{R}$, *with* $a > 0$, *that is* G_d *is unstable;* $N_1(s)$, $\overline{N}(s)$, $D_1(s)$ *and* $\overline{D}(s)$ *are polynomials on the complex variable* s.

Remark 1.1 *In the three cases it is assumed that the backward loop of recycle system,* $G_2(s)$ *is stable, i.e., the roots of* $\overline{D}(s)$ *are in the left half of the complex plane.*

In Chapter 3 an observer-predictor is designed in order to estimate the interest variables for each case depicted in this Chapter.

Chapter 2

Preliminary results: Control for Systems with Time Delay

Several control strategies have been developed to deal with time delays. The simplest one consists in ignoring the effects of the time delay, designing a compensator for the delay-free process and applying the obtained controller to the actual delayed system. It is clear that this method works only in the case of stable processes with sufficiently small time delay. When the continuous case is considered, the delay operator can be approximated by means of a Taylor or Padé series expansions which could leads to a non-minimum-phase process with rational transfer function representation [26]. With the same stability purpose analysis, some works have applied the Rekasius substitution; see for instance [27].

A different class of compensation strategies consist of counteracting the time delay effects by means of schemes intended to predict the effects of current inputs in future outputs. The Smith Prediction Compensator (SPC) ([28],[29]) is the most common prediction strategy considered in the literature that provides a future output estimation by means of a type of open-loop observer scheme. The main limitation of the original SPC is the fact that the prediction scheme does not have a stabilization step, which restricts its application to open-loop stable plants. To alleviate this problem, some modifications to the original structure, and the ability to handle processes with an integrator and large time delay, have been reported ([24], [30], [31]). Some extensions to the non-stable case have been also reported, see for instance [32],

[33] and the references included there in.

Also as an example, Seshagiri *et al.*, [34] present an efficient modification to the SPC in order to control unstable FOPTD systems. Their methodology is restricted to systems satisfying $\tau < 1.5\tau_{un}$ where τ is the process time-delay and τ_{un} the unstable time-constant. With a different perspective, in [35] upper bounds on the delay size ($\tau < 2\tau_{un}$) are provided when using linear time invariant controllers on the stabilization of strictly proper delayed real rational plants. It is important to note that in general, the provided bounds are not tight and the authors prevent that the developed controllers are not intended as practical solutions and are only used as a tool to compute the achievable delay margin for some particular cases. In [36], based in a numerical method, it is considered the stabilization of linear time-delay systems of order n. However, stability conditions with respect to time-delay and time constant of the process are not provided. The proposed method consists in shifting the unstable eigenvalues to the left half plane by static state feedback by applying small changes to the feedback gain, the same approach is implemented by considering an observer-based strategy. Furthermore, special attention is devoted in [36] to the unstable FOPTD case, where the problem of stabilization of systems satisfying $\tau < 2\tau_{un}$ is solved by the observer-based approach. It is important to point out that for the proposed observer-based scheme in [36] it is not evident how to implement a Proportional-Integral (PI) or Proportional-Integral-Derivative (PID) controller to get step reference tracking and step disturbance rejection.

The case of unstable FOPTD processes has been analyzed in [37], where a stability analysis is done in order to calculate all the stabilizing values of proportional controllers for such systems. In [38], the well-known D-partition technique is used to estimate the stabilization limits of PID controllers. For unstable processes with dominant unstable time-constant and under the action of a PID control, the analysis leads to a constraint of the type $\tau < \tau_{un}$. Based on an extension of the Hermite-Bielhler Theorem, a complete set of PID-controllers for time-delay systems has been analyzed in [39, 40]. Different bounds for the stabilization of first order dead-time unstable systems are provided as well as a complete parameterization of the stabilizing P and PI controller in the case $\tau < \tau_{un}$ and the stabilizing PID controllers for the case $\tau < 2\tau_{un}$. Under a different perspective, in [41] it is presented a complete analysis that includes also the case of neutral systems.

In [42] a modification to the original SPC is proposed, in order to deal with unstable FOPTD systems. Using a similar structure, the result is extended to delayed high order systems ([43]). In both works, a robustness analysis is done concluding that for unstable dead time dominant systems, the resulting closed-loop can be destabilized with an infinitesimal value of the modeling error, i.e., that robustness is strongly dependent on the relationship τ/τ_{un}. For the control scheme proposed in ([43], [42]), it can be easily proven that in the case of unstable plants, the internal stability is not guaranteed. In fact, it is obtained an unstable dynamics for the estimation error and, as a result, a minimal initial condition error between the original plant and the model produces an internal unbounded signal.

This Chapter presents some preliminary results that will be used later in the observer design for recycling systems. The presentation of such results is given in two parts: existing stability results and new control strategies for FOPTD systems. In the first part, some stability results with respect to unstable FOPTD systems as well as higher order systems with time delay are provided. In particular, when a static output feedback is used. Also, the tuning of controllers P, PI and PID for unstable FOPTD systems is presented. It should be pointed out that the results presented in this part are results existing in the literature and are presented only for the sake of the completeness.

The results presented in the second part of this chapter focuses on the stabilization problem of linear unstable first-order systems with time delay in the input channel. The main motivation to deal with this class of systems is based on the fact that, in some cases, high order systems can be approximated with first or second order system with time delays that could be considered as a first step toward studying the stability properties of high-order unstable delayed plants ([4], [34]).

The presentation of the results concerning unstable FOPTD systems is organized as follows. In Section 2.3 it is presented the first stabilizing control strategy for systems satisfying $\tau < \tau_{un}$. Then Section 2.4 considers the stabilization problem for first order unstable processes with significant large time delay at the direct path ($\tau > \tau_{un}$), stating in particular, a stabilization problem as the one posed in [42]. It is shown that our control strategy produces a stable closed loop system able to solve the regulation problem without the initial condition problem of ([43] [42]) described above and without the use of a SPC strategy. In addition it is shown that our method allows a stability condition $\tau < 2\tau_{un}$ instead of the one obtained in ([34]) restricted to $\tau \leqslant 1.5\tau_{un}$. The closed loop stability of the proposed scheme is analyzed by means of a discrete time representation of the systems when the sampling period T approaches zero.

On the other hand, in some works cited above as well as the proposed one in Section 2.4, the stabilization problem of a first order dead-time system is restricted to the condition $\tau < 2\tau_{un}$. This stabilization upper bound is precisely the main topic of Section 2.5, i.e., the consideration of continuous first order linear unstable processes subject to large input time-delays, with special interest in the case $\tau > 2\tau_{un}$. The proposed strategy is based on an observer-controller design inspired in the methodology reported in [36], using the PID stabilizing parameterization in [39, 40]. As main results, this part of the work presents an observer-based stabilization structure with a P or PI controller that provides a new larger stabilization bound $\tau < 3\tau_{un}$. With the same stabilization scheme and considering as a controller a PID action it is proved that the closed-loop system can be stabilized for the improved condition $\tau < 4\tau_{un}$. Up to our best knowledge, until now, it has not been reported in the literature a control structure stabilizing a system under the condition $\tau > 2\tau_{un}$. To complete our stabilization strategy it is shown that a particular modification of our control scheme, based on an additional static internal loop, allows to reject input step disturbances when a PI or PID control is used.

This Chapter is organized as follows. Before providing the control strategies for the cases depicted previously, in Section 2.1 the considered class of systems is defined as well as the problem formulation. Then in Section 2.2 some results existing in the literature are presented. In Section 2.3 a control structure for unstable FOPTD systems satisfying $\tau < \tau_{un}$ is presented. After this, Section 2.4 is devoted to analyze two proposed control strategies for systems satisfying $\tau < 2\tau_{un}$. Then, Section 2.5 presents a control strategy that allows to deal with systems satisfying $\tau < 3\tau_{un}$ and $\tau < 4\tau_{un}$. Finally, some conclusions are given in Section 2.6.

2.1 Class of Systems

Consider the linear, unstable, input-output delay system,

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s}, \tag{2.1}$$

where $U(s)$ and $Y(s)$ are the input and output signals respectively, $\tau \geq 0$ is the input time-delay and,

$$G(s) = \frac{b}{s-a} = \frac{ba^{-1}}{a^{-1}s-1},$$

is the delay-free transfer function with $a, b > 0$. Notice that $\tau_{un} = a^{-1}$ can be seen as the unstable time-constant of the process. With respect to the class of systems (2.1) a traditional control strategy based on an output feedback of the form,

$$U(s) = C(s)[R(s) - Y(s)], \tag{2.2}$$

produces a closed loop system given by,

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)e^{-\tau s}}{1 + C(s)G(s)e^{-\tau s}}, \tag{2.3}$$

where the term $e^{-\tau s}$ located at the denominator of the transfer function (2.3) leads to a system with an infinite number of poles and where the closed loop stability properties should be carefully stated.

In this Chapter, some control structures for systems with time delay are studied. First, the static output feedback for some time delayed systems is analyzed in Section 2.2. Also in Section 2.2 it is presented the existence conditions of P, PI and PID controller for unstable FOPTD systems. Then in Section 2.3 it is presented a control strategy for unstable FOPTD systems satisfying $\tau < \tau_{un}$. In Section 2.4 we will propose two control schemas for unstable FOPTD systems that yields stable closed-loop operation, satisfying $\tau < 2\tau_{un}$. Then in Section 2.5 a control strategy for unstable FOPTD systems is also given, where it is proposed an observer-based scheme for the class of unstable systems (2.1) that yields stable closed-loop operation, based on the traditional observer theory. It is show how two static gains are enough in order to get an adequate estimation of a specific internal signal. The main idea is to propose a prediction scheme with a simpler structure and stronger stability margin face to large time-delays, when compared to controllers proposed in recent literature [34–36, 42–44], as well as the proposed control strategies in Section 2.4. The strategy is completed by incorporating P, PI, PID controllers.

2.2 Existing stability results

In this section some preliminary results on the stability of systems with one unstable pole plus time delay are presented. These results will be used later in the proofs of the observer-predictor convergence.

Lemma 2.1.

Consider the unstable input-output delay system

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} = \frac{b}{s-a}e^{-\tau s}, \quad a > 0 \tag{2.4}$$

with a proportional output feedback

$$U(s) = R(s) - kY(s), \tag{2.5}$$

where $R(s)$ is the new reference input. There exist a proportional gain k such that the closed loop system

$$\frac{Y(s)}{R(s)} = \frac{be^{-\tau s}}{s - a + kbe^{-\tau s}} \tag{2.6}$$

is stable if and only if $\tau < \tau_{un}$.

Stability of (2.6) has been previously studied in the literature. Lemma 2.1 can be proved using classical frequency domain, D-decomposition or even by the classical Pontryagin Method [5], [45], [39], [46], [3]. The proof of Lemma 2.1 is provided in Section A.1 (Appendix A) by using an alternative method: discrete-time approach.

A useful practical result in order to compute the parameter k involved on the control scheme is the following.

Corollary 2.1.

Consider system given by (2.4) with $\tau < \tau_{un}$. Then, there exists $k \in R^+$ that stabilizes the closed loop system (2.6), satisfying $\alpha < k < \beta$, with $\alpha = a/b$ and some constant $\beta > a/b$.

Proof. The proof of Corollary 2.1 is presented in Appendix A. ■

Remark 2.1 *From an analysis on the frequency domain, it is not difficult to determinate accurately the value β given in Corollary 2.1. In fact, such value is given by $\beta = \frac{a}{b}\sqrt{1 + (\frac{\omega}{a})^2}$, where ω satisfy $\frac{\omega}{a} = \tan(\omega\tau)$ for $0 < \omega < \frac{\pi}{2\tau}$. The usefulness of Corollary 2.1 comes from that any $k = \frac{a}{b} + \epsilon$, with $\epsilon > 0$ stabilizes the closed loop system (2.6) for ϵ sufficiently small.*

The following result is similar to Lemma 2.1, but for a class of unstable delayed system more complex than the previous one.

Lemma 2.2.

[47] Consider the system,

$$\frac{Y(s)}{U(s)} = \frac{\alpha}{(s-a)(s+b)} e^{-\tau s} \tag{2.7}$$

with $a, b > 0$, $\frac{1}{a} - \frac{1}{b} > 0$ and a proportional output feedback

$$U(s) = R(s) - kY(s), \tag{2.8}$$

where $R(s)$ is the new reference input. Then, there exist a gain k such that the closed loop system,

$$\frac{Y(s)}{R(s)} = \frac{\alpha e^{-\tau s}}{(s-a)(s+b) + k\alpha e^{-\tau s}}, \tag{2.9}$$

is BIBO stable if and only if

$$\tau < \frac{1}{a} - \frac{1}{b}. \tag{2.10}$$

Proof. The proof of this Lemma is provided in Appendix A. ■

Note that the necessary condition $\frac{1}{a} - \frac{1}{b} > 0$ in Lemma 2.2 is derived from a stability analysis of the transfer function (2.7) when $\tau = 0$.

The following result is a generalization of the Lemma 2.2, i.e. the stabilization condition for a system with one unstable pole and more than one stable pole by considering an output static feedback.

Lemma 2.3.

[47] Consider the system,

$$\frac{Y(s)}{U(s)} = \frac{b}{(s-a)(s+c_1)(s+c_2)...(s+c_n)} e^{-\tau s} \tag{2.11}$$

with $n \in \mathbb{R}$, $a, c_i > 0$ $\forall i = 1, 2, ..., n$, $\frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i} > 0$ and a proportional output feedback

$$U(s) = R(s) - kY(s), \tag{2.12}$$

where $R(s)$ is the new reference input. There exist a gain k such that the closed loop system,

$$\frac{Y(s)}{R(s)} = \frac{be^{-\tau s}}{(s-a)(s+c_1)(s+c_2)...(s+c_n) + kbe^{-\tau s}}, \tag{2.13}$$

is stable if and only if

$$\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}. \tag{2.14}$$

Proof. The proof of this result is given in Appendix A. ■

In what follows, taking into account the results presented in [39], the existence conditions for the stabilizing feedback given by (2.2) are stated, for the following three types of $C(s)$ compensators.

i) P controller, $C(s) = k_p$

ii) PI controller, $C(s) = k_p + k_i/s$

iii) PID controller, $C(s) = k_p + k_i/s + k_d s$.

The following results are recalled, for the sake of completeness, and will be used later to obtain the main results of this part of the work.

Theorem 2.1.

[39] Consider the transfer function (2.1) and the control feedback (2.2), a necessary condition for a proportional controller P to simultaneously stabilize the delay-free plant and the plant with delay is $\tau < \tau_{un}$. If this necessary condition is satisfied, then the set of all stabilizing gains k_p for a given open-loop unstable plant with transfer function as in (2.1) is given by,

$$\frac{a}{b} < k_p < \frac{1}{b\tau}\sqrt{z_1^2 + a^2\tau^2}$$

where z_1 is the solution of the equation, $\tan(z) = \frac{1}{a\tau}z$, in the interval $(0, \frac{\pi}{2})$.

Theorem 2.2.

[39] Consider the transfer function (2.1) and the control feedback (2.2), a necessary condition for a PI controller to simultaneously stabilize the delay-free plant and the plant with delay is $\tau < \tau_{un}$. If this necessary condition is satisfied, then the range of k_p values for which a solution exists to the PI stabilization problem of a given open-loop unstable plant with transfer function as in (2.1) is given by,

$$\frac{a}{b} < k_p < \frac{1}{b\tau}\sqrt{\alpha_1^2 + a^2\tau^2}$$

where α_1 is the solution of the equation, $\tan(\alpha) = \frac{1}{a\tau}\alpha$, in the interval $(0, \frac{\pi}{2})$.

Remark 2.2 *It is important to note that Theorem 2.1 (Theorem 2.2) is stated (as in [39]) as a necessary condition. However, it is easy to see that the condition $\tau < \tau_{un}$ is necessary and sufficient for the existence of an stabilizing proportional (proportional-integral) feedback.*

Remark 2.3 *Once the range of k_p has been obtained from Theorem 2.2, one should choose and fix a value of the parameter k_p inside of such range. For this value of k_p,* *the range of k_i is given by,*

$$0 < k_i < -\frac{az_1}{b\tau}[\sin(z_1) - \frac{1}{a\tau}z_1\cos(z_1)],$$

where z_1 is the first positive real root of,

$$-\frac{b}{a}k_p + \cos(z) + \frac{z}{a\tau}\sin(z) = 0.$$

Theorem 2.3.

[39] A necessary and sufficient condition for the existence of a stabilizing PID controller for the open-loop unstable plant (2.1) is $\tau < 2\tau_{un}$. If this condition is satisfied, then the range of k_p values for which a given open-loop unstable plant, with transfer function as in (2.1), can be stabilized using a PID controller is given by,

$$\frac{a}{b} < k_p < \frac{a}{b}[\frac{\alpha_1}{a\tau}\sin(\alpha_1) + \cos(\alpha_1)]$$

where α_1 is the solution of the equation

$$\tan(\alpha) = \frac{1}{a\tau - 1}\alpha$$

in the interval $(0, \pi)$. In the special case of $\tau = \frac{1}{a}$, we have $\alpha_1 = \frac{\pi}{2}$. For k_p values outside this range, there are no stabilizing PID controllers. Moreover, the complete stabilizing region is given by Figure 2.1. For each $k_p \in (k_l := \frac{a}{b}, \frac{a}{b}[\frac{\alpha_1}{a\tau}\sin(\alpha_1) + \cos(\alpha_1)])$, the cross-section of the stabilizing region in the (k_i, k_d) space is the quadrilateral Q.

Remark 2.4 *The parameters involved in the stability quadrilateral Q depicted in*

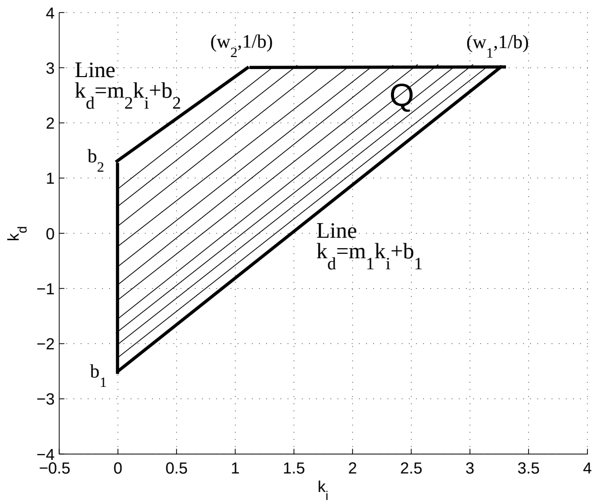

Figure 2.1: Stability region (k_i, k_d)

Figure 2.1 are given by,

$$
\begin{aligned}
m_j &= \frac{\tau^2}{z_j^2}, \\
b_j &= \frac{a\tau}{bz_j}[\sin(z_j) - \frac{1}{a\tau}z_j\cos(z_j)], \\
w_j &= -\frac{az_j}{b\tau}[\sin(z_j) - \frac{1}{a\tau}z_j(\cos(z_j)+1)],
\end{aligned}
$$

for $j = 1, 2$, where z_1 and z_2 are the first and second positive real roots of,

$$-\frac{b}{a}k_p + \cos(z) + \frac{z}{a\tau}\sin(z) = 0,$$

respectively.

2.3 Stabilization Strategy for systems $\tau < \tau_{un}$

The control structure presented in this Section gives as result the publication of the article "Control basado en un esquema observador para sistemas de primer orden con retardo" at Revista Mexicana de Ingeniería Química. Also a version of the work in english was published at Int. J. Computer Applications in Technology. This article can be consulted in Appendix B.1.

In this Section it is presented a control strategy that allows to deal with unstable FOPTD systems satisfying $\tau < \tau_{un}$. The control strategy presented here is based on the observer design together with a PI controller. The following result provides the convergence conditions of the observer schema.

Theorem 2.4.

Consider the observer schema shown in Figure 2.2. There exists a static gain $k \in R$ such that $\lim_{t\to\infty}[\widehat{w}(t) - w(t)] = 0$ if and only if $\tau < \tau_{un}$.

Proof. Consider the observer schema shown in Figure 2.2, where its dynamic can be written as,

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\widehat{x}}(t) \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} x(t) \\ \widehat{x}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ bk & -bk \end{bmatrix} \begin{bmatrix} y(t) \\ \widehat{y}(t) \end{bmatrix} + \begin{bmatrix} b \\ b \end{bmatrix} u(t)$$

$$\begin{bmatrix} y(t+\tau) \\ \widehat{y}(t+\tau) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ \widehat{x}(t) \end{bmatrix}.$$

By defining the error prediction as $e_x(t) = \hat{x}(t) - x(t)$, it is easy to obtain,

$$\dot{e}_x(t) = ae_x(t) - kbe_x(t-\tau).$$

Therefore, from Lemma A.1 (Appendix A) the result of the theorem follows. ■

As in the case of the SPC, the proposed control strategy has a module of prediction (observer) and a controller for the delay free model (PI controller). In this case it is considered a PI-controller with two degree of freedom proposed by [48] (see also Section A.5), which can be expressed as,

$$U(s) = R(s)G_{ff}(s) - G_c(s)\hat{W}(s), \tag{2.15}$$

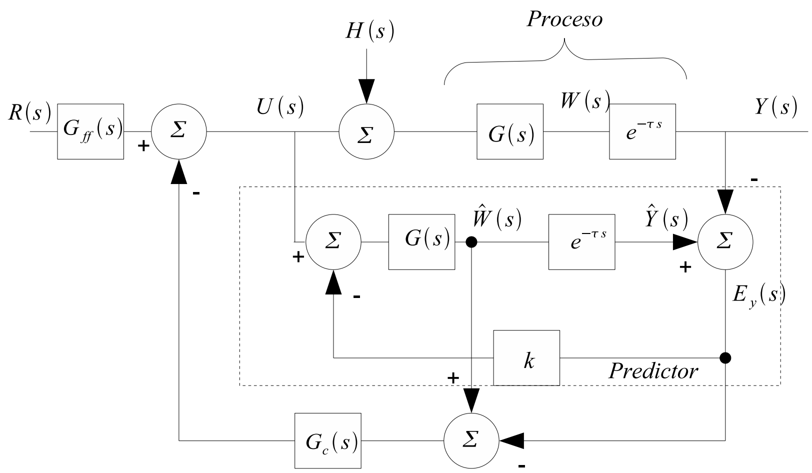

Figure 2.2: Proposed control schema for $\tau < \tau_{un}$

where,

$$G_{ff}(s) = K(\sigma_c + \frac{1}{sT_i}) \tag{2.16a}$$

$$G_c(s) = K(1 + \frac{1}{sT_i}). \tag{2.16b}$$

However, if rejecting step disturbance is taken into account the control law is modified as,

$$U(s) = R(s)G_{ff}(s) - G_c(s)(\hat{W}(s) - E_y(s)). \tag{2.17}$$

In this way, the observer-control strategy depicted in this Section is able to track step references $R(s)$, and rejecting step disturbances $H(s)$. Additional details can be found in Appendix B.1. In the following sections, two control strategies are presented in order to tackle a more difficult problem i.e., the case of unstable FOPTD systems satisfying $\tau > \tau_{un}$.

2.4 Stabilization Strategy for systems $\tau < 2\tau_{un}$

In this Section two control strategies are proposed in order to deal with unstable FOPTD satisfying $\tau < 2\tau_{un}$. The first control strategy presented in Section 2.4.1 is based on an output feedback that assumes the knowledge of the time delay size and not the complete unstable FOPTD model. The second control strategy is analyzed in Section 2.4.2 by using the observer-controller approach (similar to the presented in Section 2.3). This latest strategy considers the knowledge of the unstable FOPTD model. Although nominal process or a part of the model is used for the design, in both control strategies an analysis about robustness with respect to time delay is given.

2.4.1 Output feedback approach

The control structure provided in this Section gives as result the publication of the article "Stabilization Strategy for Unstable First Order Linear Systems with Large Time Delay" in Asian Journal of Control. Also a simpler version of this work was presented at 4th IFAC Symposium on System, Structure and Control. This work can be also consulted in Appendix B.2.

In order to improve the stability properties of system (2.1) with respect to a proportional feedback of the form (2.2), i.e., C(s) being a proportional gain, in what follows it is proposed a stabilization scheme based on two proportional gains together with an induced time delay on the feedback loop. This particular array is depicted in Figure 2.3.

Notice that, from Figure 2.3, the feedback function $f(t)$ satisfies the difference equation,

$$f(t) = -k_1 k_2 f(t-\tau) + k_2 y(t). \tag{2.18}$$

At a first glance, it is evident that the implicit discrete nature of feedback (2.18) imposes the restriction,

$$|k_1 k_2| < 1, \tag{2.19}$$

in order to satisfy the stability conditions of the difference equation (2.18).

The closed loop transfer function of the system depicted in Figure 2.3, with $G(s)$

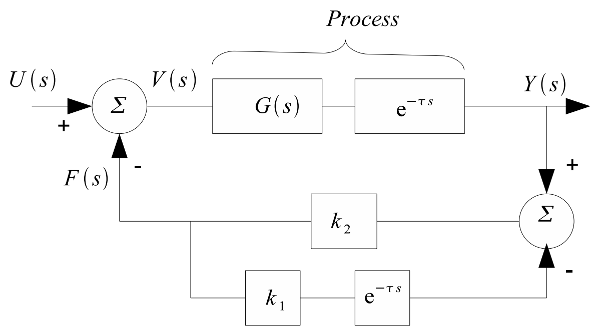

Figure 2.3: Proposed stabilization scheme.

as in (2.1), can be expressed as,

$$\begin{aligned}\frac{Y(s)}{U(s)} &= \frac{G(s)e^{-\tau s}}{1+G(s)e^{-\tau s}\frac{k_2}{1+k_1k_2e^{-\tau s}}}\\ &= \frac{be^{-\tau s}(1+k_1k_2e^{-\tau s})}{(s-a)(1+k_1k_2e^{-\tau s})+k_2be^{-\tau s}}.\end{aligned} \tag{2.20}$$

The complete stability condition for the overall control scheme given in Figure 2.3 is stated in the following result.

Theorem 2.5.

Consider the delayed system (2.1) and the feedback scheme shown in Figure 2.3. There exist constants k_1 and k_2 such that the corresponding closed loop system given in equation (2.20), is stable if and only if $\tau < 2\tau_{un}$.

Proof. The result of the theorem will be proven in an indirect way by considering the alternate system depicted in Figure 2.4 where its corresponding closed loop system is obtained as,

$$\frac{Y(s)}{U(s)} = \frac{be^{-\tau s}}{(s-a)(1+g_1e^{-\tau s})+g_2be^{-\tau s}}. \tag{2.21}$$

Note that in this equation, its corresponding characteristic equation is equivalent to the one derived from equation (2.20) when considering $g_1 = k_1k_2$ and $g_2 = k_2$. In

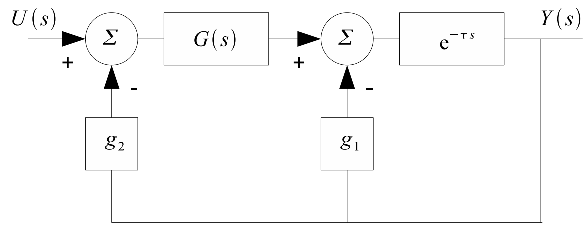

Figure 2.4: Alternative stabilization structure.

this way, the proof concludes by stating the stability conditions of the system given by (2.21). Such result is presented in Appendix A (Lemma A.2). ■

A useful practical result in order to compute the parameters involved on the control scheme is the following.

Corollary 2.2.

Consider the control scheme described in Figure 2.3. If $\tau < 2\tau_{un}$, then the parameters k_1 and k_2 that stabilize the closed loop system (2.20) satisfy,

$$a\tau - 1 < k_1 k_2 \leq a\tau - 1 + \sigma,$$

for some constant $\sigma > 0$, and

$$\frac{a}{b}(k_1 k_2 + 1) < k_2 \leq \frac{a}{b}(k_1 k_2 + 1) + \bar{\sigma},$$

for some constant $\bar{\sigma} > 0$.

Proof. Taking into account that $g_1 = k_1 k_2$ and $g_2 = k_2$, the Corollary A.1 provided in Appendix A can be applied ■

Remark 2.5 *It should be pointed out that the closed loop stability of equation (2.20) can be stated by considering the continuous time approach, presented in ([49]) where the conditions are stated by considering the set of finite poles of the equivalent transfer function of the free delay system ($\tau = 0$) and theirs behavior when the input delay* *is not null. However, the results are significantly different, in ([49]) the stability property of the system is obtained for fixed g_1 and g_2 (or k_1 and k_2). In our case, the conditions are given explicitly for the system parameters τ and $1/a$ and a practical and easy way is proposed to obtain the stabilizing controller parameters k_1 and k_2 as stated in Corollary 2.2.*

From Corollary 2.2 it is now possible to state a recursive algorithm in order to obtain stabilizing parameters k_1 and k_2. This procedure can be given as follows.

Algorithm 2.1
Step 0:

1. *Define* $\sigma_0 = 0.6(2/a - \tau)$ *and* $\bar{\sigma}_0 = 0.02(2/a - \tau)$.

Step i:

1. *Define* $\sigma_i = \sigma_{i-1}/2$ *and* $\bar{\sigma}_i = \bar{\sigma}_{i-1}/2$.

2. *Obtain* k_2 *and* k_1 *as,*

$$\begin{aligned} k_2 &= \frac{a}{b}(a\tau + \sigma_i) + \bar{\sigma}_i, \\ k_1 &= \frac{a\tau - 1 + \sigma_i}{k_2}. \end{aligned}$$

If at the step i it is obtained an unstable closed loop system for the obtained k_1 and k_2, proceed to step $i+1$. The algorithm ends when the obtained closed loop system is stable.

Notice that even when the resulting closed loop dynamic for the obtained k_1 and k_2 is stable, it is possible to continue the algorithm without breaking the stability properties of the system in order to improve the general closed loop response.

Simulation Results

The proposed methodology will be now illustrated by means of an academic example.

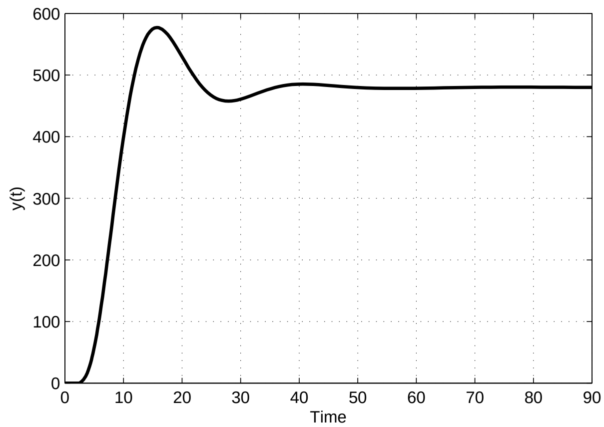

Figure 2.5: Output signal in Example 2.1.

Example 2.1 *Consider the unstable input delay system given by:*

$$\frac{Y(s)}{V(s)} = \frac{6}{s-1}e^{-\tau s}, \tag{2.22}$$

with $\tau = 1.5$. From Theorem 2.5, it is clear that there exist gains k_1 and k_2 that stabilize the closed loop system depicted in Figure 2.3 since the time delay satisfy $\tau < 2\tau_{un}$.

For the simulation experiments it is considered $\sigma = 0.1$ and $\bar{\sigma} = 0.0033$ and therefore it is obtained $k_2 = 0.27$ and $k_1 = 2.22$. In Figure 2.5 it is depicted the behavior of the stabilized system by means of the evolution of the output signal $y(t)$. To carry out this experiment it was assumed the exact knowledge of the plant parameters.

Let us consider now the particular control strategy presented by Seshagiri ([34]) and implemented here by considering the parameters design $\lambda = 0.7$, $\theta_m = 1.5$, $k_c = 0.4841$, $\tau_i = 3.2021$, $\epsilon = 0.35$ and $k_d = 0.1701$. To evaluate the output signal evolution on both schemes it is considered uncertainties acting on the time delay. Figure 2.6 and 2.7 compare the output response of the strategies by considering -13% and $+3\%$, respectively. It can be seen that the method proposed in this work gives a better performance under time delay uncertainty.

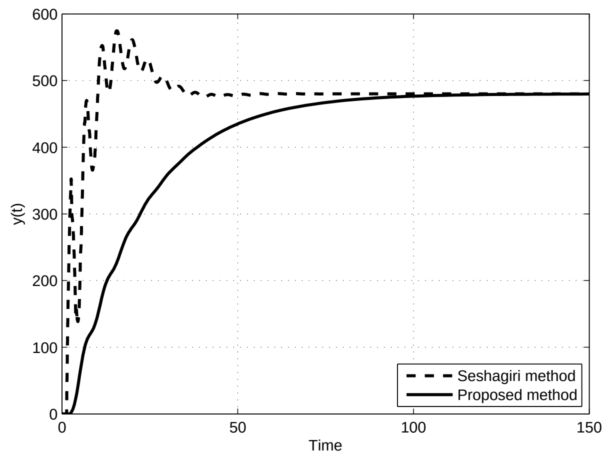

Figure 2.6: Time delay uncertainty in the process of −13%.

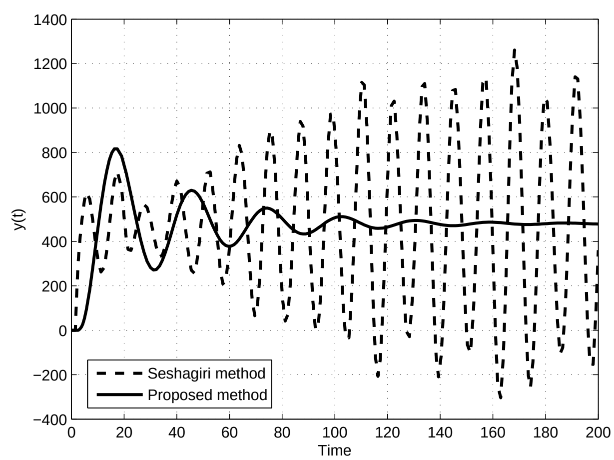

Figure 2.7: Time delay uncertainty in the process of +3%.

Now, consider the control strategy proposed by Normey-Rico ([43]) applied to system (2.22). The tuning parameters are set as recommended, this is, $T_r = L_n = 1.5$. *Assuming also a dead-time estimation error of 5%,* $T_0 = 1.575$ *is chosen. The above parameters give as a result the following controllers (see ([43]) for details on the control structure),*

$$C(s) = \frac{0.3889(5.25s+1)}{5.25s}, \;\; F(s) = \frac{1.5s+1}{5.25s+1}$$

and

$$F_r(s) = \frac{(1.5s+1)^2(28.71s+1)}{(5.25s+1)(1.575s+1)^2}.$$

In Figure 2.8, it is shown the output response of the process by considering an initial condition in the plant with a magnitude of 0.01. *As mentioned in the Introduction, it is verified that the control strategy proposed in ([43]) and ([42]) it is not able to handle the specified problem due to the minimal initial condition error between the plant and the compensator. On the other hand, it is important to note that when the time delay is large enough (i.e. near of the limit $\frac{2}{a}$), the stability region of the closed loop system becomes more limited. In this case, the computation of the controller parameters is more involved. In order to illustrate this fact, let us consider again the system given by (2.22) together with different values of τ. The parameters of the controller for each τ are provided in Table (2.1).*

From the closed loop characteristic equation,

$$(s-1) + \beta(s+\alpha)6e^{-\tau s} = 0, \tag{2.23}$$

with, $\beta = \frac{k_1 k2}{6}$ and $\alpha = -a + \frac{6}{k_1}$, the corresponding Nyquist diagram for different values of τ is shown in Figure 2.9 where from the Nyquist criterion the conditions that assure stability are satisfied. In Table (2.1) is also presented the gain margin M_g of the closed loop system for each case, making evident the fact that when the time delay increases its value, the stability region of the system decrease in size.

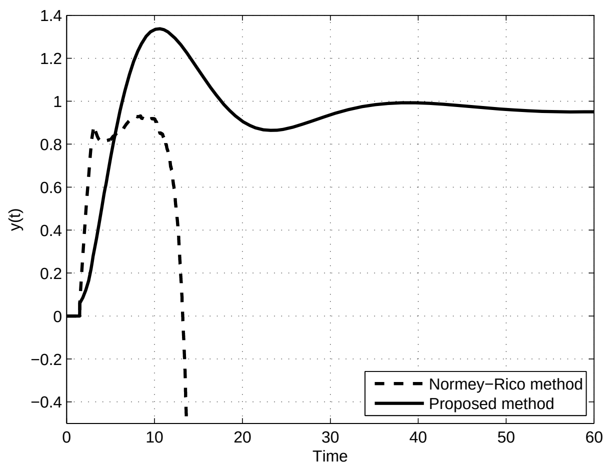

Figure 2.8: Output response under initial condition different to zero.

Table 2.1: Parameter values.

τ	k_1k_2	k_2	M_g
1.1	0.2	0.21	1.06
1.5	0.6	0.27	1.02
1.8	0.86	0.311	1.006

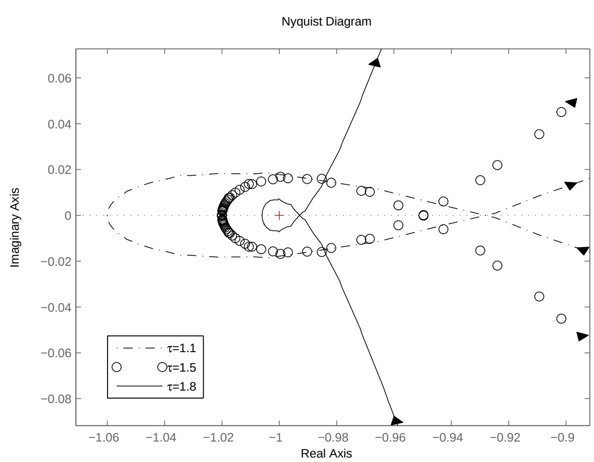

Figure 2.9: Nyquist diagram for different values of τ.

2.4.2 Observer based approach

The control structure presented in this Section gives as result the publication of the article "On the Control of Unstable First Order Linear Systems with Large Time lag: Observer Based Approach" in European Journal of Control. This article can be also consulted in Appendix B.3.

An observer based scheme is proposed in order to consider the case when a significant large time delay $\tau > \tau_{un}$ is present at the direct path. This scheme is depicted in Figure 2.10. In what follows, necessary and sufficient conditions are obtained for the existence of a future output estimator for unstable plants that improves the conditions given in the Section 2.3. In addition, it is proposed a simple and effective methodology in order to explicitly obtain the mentioned estimator.

Theorem 2.6.

Consider the observer based scheme shown in Figure 2.10. Then there exist constants g_1 and g_2 such that $\lim_{t\to\infty} [\hat{w}(t) - w(t)] = 0$ if and only if $\tau < 2\tau_{un}$.

Proof. The proof can be easily done by taking into account the stability conditions given in Lemma A.2 (Appendix A). With this aim, consider the dynamic of the prediction scheme shown in Figure 2.10 that can be written in state space form as,

$$\begin{bmatrix} \dot{w}(t) \\ \dot{\widehat{w}}(t) \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} w(t) \\ \widehat{w}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ bg_2 & -bg_2 \end{bmatrix} \begin{bmatrix} y(t) \\ \widehat{y}(t) \end{bmatrix} + \begin{bmatrix} b \\ b \end{bmatrix} u(t) \tag{2.24}$$

$$\begin{bmatrix} y(t+\tau) \\ \widehat{y}(t+\tau) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w(t) \\ \widehat{w}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ g_1 & -g_1 \end{bmatrix} \begin{bmatrix} y(t) \\ \widehat{y}(t) \end{bmatrix} \tag{2.25}$$

with $\widehat{w}(t)$ the estimation of $w(t)$. Defining first the state prediction error $e_w(t) = \widehat{w}(t) - w(t)$ and the output estimation error $e_y(t) = \widehat{y}(t) - y(t)$ it is possible to describe the behavior of the error signal as:

$$\begin{bmatrix} \dot{e}_w(t) \\ e_y(t+\tau) \end{bmatrix} = \begin{bmatrix} a & -bg_2 \\ 1 & -g_1 \end{bmatrix} \begin{bmatrix} e_w(t) \\ e_y(t) \end{bmatrix}. \tag{2.26}$$

Consider now a state space realization of system (A.15) (described in Figure A.4) that can be written as,

$$\begin{bmatrix} \dot{x}(t) \\ y(t+\tau) \end{bmatrix} = \begin{bmatrix} a & -bg_2 \\ 1 & -g_1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} u(t). \tag{2.27}$$

It is clear now that the stability conditions of system (2.27), given in Lemma A.2 (Appendix A), are equivalent to the ones of system (2.26), from where, the result of the theorem follows. ■

The parameters involved in the observer design (g_1 and g_2) can be calculated by using Corollary A.1 and Algorithm A.1, provided in Appendix A.

Once the prediction scheme has been established, the proposed control structure will be complemented with a proportional-integral action and a simple and effective step disturbance rejection strategy. It should be noticed that the control strategy can be implemented independent of the estimation strategy and therefore we are not forced to use a PI control structure. The proposed control schema suggests the use of a two degree of freedom controller given as,

$$U(s) = R(s)G_{ff}(s) - \hat{W}(s)G_c(s) \tag{2.28}$$

where,

$$G_{ff}(s) = K(\sigma_c + \frac{1}{sT_i}) \tag{2.29a}$$

$$G_c(s) = K(1 + \frac{1}{sT_i}). \tag{2.29b}$$

which can be designed as if the delay free output signal $w(t)$ were available. Obviously, at the implementation, the estimated signal $\hat{w}(t)$ is used according to Figure 2.10 (See also Section A.5, Appendix A).

In order to achieve step disturbance rejection an additional gain g_3 is required, and the control input becomes,

$$U(s) = R(s)G_{ff}(s) - \hat{W}(s)G_c(s) + G_c(s)g_3E_y(t) \tag{2.30}$$

The following result shows the conditions in the parameter g_3 to achieve step disturbance rejection in the proposed schema shown in Figure 2.10.

Lemma 2.4.

Consider the proposed observer scheme shown in Figure 2.10. Then, there exist a PI controller with two degree of freedom given by (2.30) able to reject the step disturbance $H(s)$ if $g_3 = g_1 + 1$.

Proof. Consider the transfer function $Y(s)/H(s)$ of the control structure given in Figure 2.10,

$$\frac{Y(s)}{H(s)} = \frac{G(s)e^{-\tau s}[T(s) + G_c(s)G(s)(g_1e^{-\tau s} - g_3e^{-\tau s})]}{T(s) + G_c(s)G(s)[g_1e^{-\tau s} - G(s)g_2e^{-\tau s}]}, \tag{2.31}$$

where, $T(s) = 1 + g_1e^{-\tau s} + G(s)g_2e^{-\tau s} + G_c(s)G(s)$ and $G(s)$, $G_c(s)$ are defined previously in equations (2.1) and (2.29) respectively. Consider the application of the final value theorem to equation (2.31) with $H(s) = \frac{1}{s}$ and $g_3 = g_1 + 1$, then it is an easy task to verify that under the assumption of the lemma, it is obtained,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 0.$$

Hence, the control strategy is able to reject step disturbance. ■

The proposed methodology, intended to stabilize and at the same time improve the overall response of the system, can be summarized as follows:

1. Fulfillment of the conditions of Theorem 2.6 ($\tau < 2\tau_{un}$). This fact states the existence of a prediction scheme.

2. Predictor stabilization. This can be achieved by tuning the parameters g_1 and g_2, using the results in Corollary A.1 and Algorithm A.1.

3. Compute g_3, in order to reject step disturbance.

4. Design of a PI-controller with a "set point weighting" strategy (refer to Figure 2.10), or any other desired controller stabilizing the delay free plant $G(s)$.

Robustness with respect to time delay uncertainty.

In the preceding developments, a control strategy has been presented under the assumption of a complete knowledge of the actual process. In practice, it is desired to get a control strategy that provides stability conditions with respect to model uncertainties, in particular, due to the observer based strategy considered in this work, the observer time-delay may be different from one associated with the plant. In what follows, it will be shown that the results presented in [50], can be used in the case presented in this work in order to analyze the robustness properties of the control strategy addressed in this work with respect to the time-delay. With this aim, consider a characteristic quasipolynomial of the form,

$$p(s) = p_0(s) + p_1(s)e^{-\tau s} + p_2(s)e^{-\tau_0 s} = 0 \tag{2.32}$$

where its stability properties will be established as a function of the time-delays τ and τ_0.

Following [50] it is possible to give a general framework for our particular case. Let $\mathcal{T}$ denote the set of all points $(\tau, \tau_0) \in \mathbb{R}_+^2$ such that $p(s)$ has at least one zero on the imaginary axis. Any $(\tau, \tau_0) \in \mathcal{T}$ is known as a crossing point and $\mathcal{T}$ is the collection of all stability crossing curves. Consider now system (2.1) and the predictor scheme shown in Figure 2.10 with $G_{ff} = 1$, $g_3 = 0$ and $G_c = \overline{k}$ which leads to the feedback law $U(s) = \overline{k}\widehat{W}(s)$. After straightforward computations, considering τ as the delay in the observer and τ_0 as the one in the process, the closed-loop characteristic equation is given by,

$$p_A(s) = p_a(s) + p_b(s)e^{-\tau s} + p_c(s)e^{-\tau_0 s} = 0, \tag{2.33}$$

with,

$$\begin{aligned}
p_a(s) &= s^2 + (b\overline{k} - 2a)s + a(a - b\overline{k}) \\
p_b(s) &= g_1 s^2 + \left[b(g_2 + \overline{k}g_1) - 2ag_1\right] s + \left[a^2 g_1 - ab(g_2 + \overline{k}g_1)\right] \\
p_c(s) &= b^2\overline{k}g_2.
\end{aligned}$$

It is clear that the characteristic equation (2.33) has the form of (2.32), therefore it is possible to identify the regions of (τ, τ_0) in $\mathbb{R}^2_+$ such that $p_A(s)$ is stable.

Following [50], Figure 2.11 shows the region (τ, τ_0) for the characteristic equation (2.33). This figure illustrates the range of values $[\tau_{0\,\min}, \tau_{0\,\max}]$ such that the proposed observer-based controller with a nominal delay τ is able to stabilize the closed-loop system, i.e., such that the characteristic equation (2.33) remains stable. Additional details (taken from [50]) of the applied method can be found in Section A.7 (Appendix A).

Simulation Results

The effectiveness of the proposed methodology will be now evaluated by means of three academic examples. The results will be compared with alternative strategies taken from the recent related literature.

Example 2.2 *Consider the control concentration of the unstable reactor addressed in [43]. The open-loop system is given by,*

$$\frac{Y(s)}{U(s)} = \frac{3.433}{103.1s - 1} e^{-20s}. \tag{2.34}$$

The control structure proposed in this work (depicted in Figure 2.10) is implemented by considering,

$$g_1 = a\tau - 1 + \sigma,\ g_2 = \frac{a}{b}(1 + g_1) + \bar{\sigma} \text{ and } g_3 = 1 + g_1$$

with $\sigma = 0.8060$ *and* $\bar{\sigma} = 0.4087$*, obtaining* $g_1 = 0$*,* $g_2 = 0.7$ *and* $g_3 = 1$*. The PI controller parameters, given in equation (2.29), are set to* $K = 22.6$*,* $T_i = 1$ *and* $\sigma_c = 0.5$*.*

For process (2.34), Normey-Rico et. al [43] proposed the following controllers (for the considered control structure see [43]),

$$C(s) = \frac{3.29(43.87 + 1)}{43.87s},\ F(s) = \frac{20s + 1}{43.87s + 1}$$

and

$$F_r(s) = \frac{(20s + 1)^2(93.16s + 1)}{(43.87s + 1)(26s + 1)^2}.$$

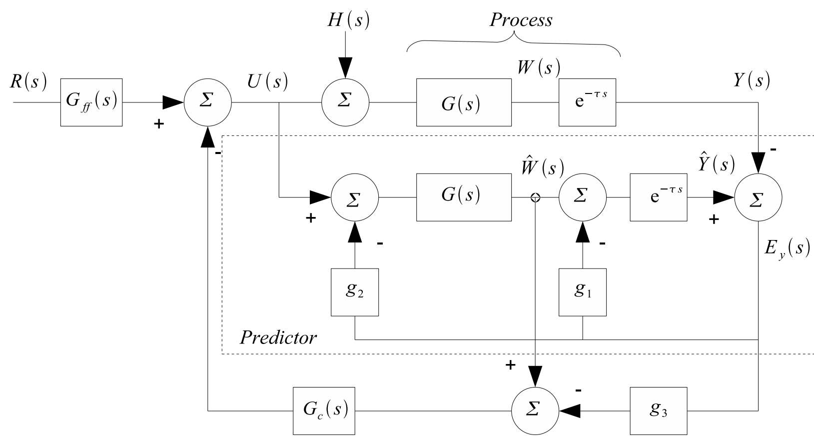

Figure 2.10: Proposed control scheme for $\tau < 2\tau_{un}$.

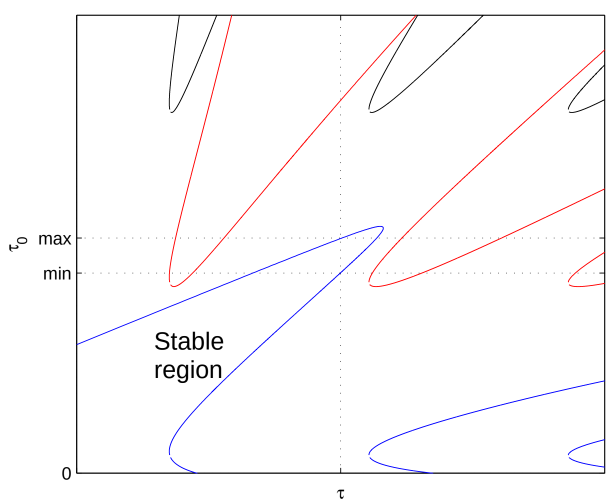

Figure 2.11: $\mathcal{T}^1$ for the characteristic equation (2.33)

The performance of the two schemes is compared by considering a positive unit step input and a step disturbance $H(s)$ acting at $t = 300$. Figure 2.12, shows the closed-loop responses when considering an exact knowledge of the model parameters. Notice that the proposed methodology produces a better disturbance rejection result than the one obtained by the method addressed in [43]. The initial conditions problems mentioned in the introduction for the methodology proposed in [43] are evident in Figure 2.13 where a minimal initial conditions error $y(t) - \widehat{y}(t) = 0.01$ shows the unstable error dynamics.

Example 2.3 *Consider the unstable delay system,*

$$\frac{Y(s)}{U(s)} = \frac{6}{s-1}e^{-1.5s}. \tag{2.35}$$

Let us consider the particular control strategy presented by Seshagiri et al., [34] and implemented here by considering the parameters design: $\lambda = 0.7$, $\theta_m = 1.05$, $k_c = 0.4841$, $\tau_i = 3.2021$, $\epsilon = 0.35$ and $k_d = 0.1701$. For the methodology proposed in the present work and depicted in Figure 2.10, it was considered the controller parameters $\sigma = 0.1$, $\bar{\sigma} = 0.0033$ producing as a consequence $g_1 = 0.6$, $g_2 = 0.27$ and $g_3 = 1.6$. The PI compensator, given in equation (2.29), was tuned by considering $K = 0.4841$, $T_i = 3.2021$ and $\sigma_c = 0.35$ in order to achieve a similar set-point tracking speed as that one of [34]. To evaluate the output signals evolution on both schemes it was considered a positive unit step input and a positive step disturbance $h(t) = 0.003$ acting at $t = 20$. Figure 2.14 shows the obtained responses of both cases when it is considered the exact knowledge of the model parameters. The method proposed in this work is observed to give a better response. In Figure 2.15 it is shown the responses obtained for the two control structure when the input time-delay is increased by 5%. In this case, the structure proposed in [34] becomes unstable while the method proposed here remains stable. It can be shown from Section of Robustness that for a 5% disturbance on the time-delay, our strategy remains on the stability region described by τ and τ_0. Figure 2.16 presents the estimation error $e_w(t) = \widehat{w}(t) - w(t)$ of the scheme given in Figure 2.10.

The next example, that cannot be treated with the strategy reported in [43] or in [34] is presented in order to show the effectiveness of our control scheme.

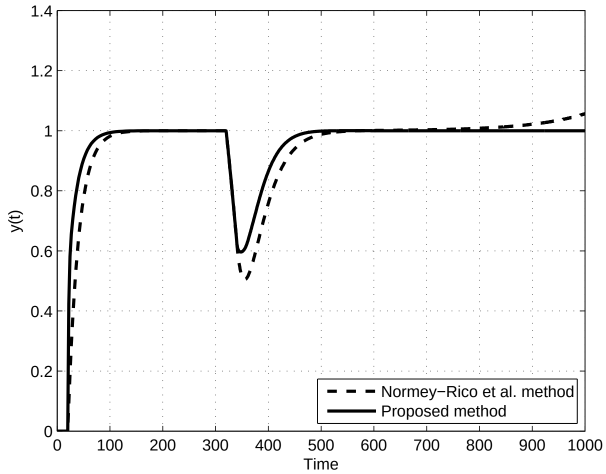

Figure 2.12: Output evolution when considering exact knowledge of parameters.

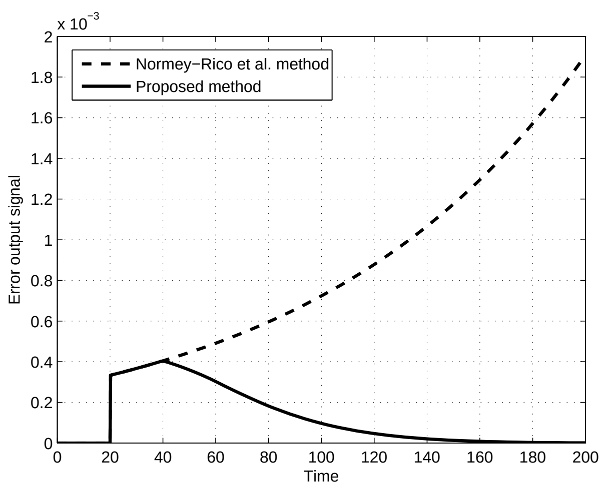

Figure 2.13: Error output signals under initial state conditions.

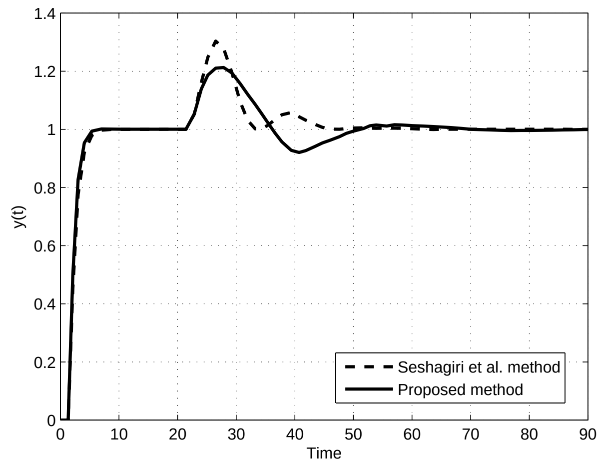

Figure 2.14: Output evolution when considering exact knowledge of parameters.

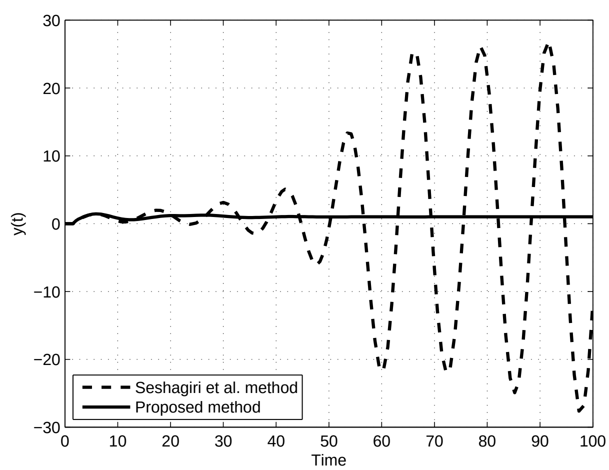

Figure 2.15: Output time evolution for a parametric variation of +20%.

Example 2.4 *Consider the unstable delayed system,*

$$\frac{Y(s)}{U(s)} = \frac{2}{s-1}e^{-1.8s}. \tag{2.36}$$

It is clear that condition $\tau < 2\tau_{un}$ of Theorem 2.6 is satisfied. Notice that the delay term is almost equal to the relation $2\tau_{un}$. The control strategy presented in this work and depicted in Figure 2.10 is computed by using Corollary A.1 producing the gains $g_1 = 0.86$ and $g_2 = 0.934$ for $\sigma = 0.06$ and $\bar{\sigma} = 0.001$. The PI controller parameters (equation 2.29) are $K = 3$, $T_i = 3$ and $\sigma_c = 0.3$. The observer based structure proposed in [36] is considered with $k = 0.53$ and $l = 0.51$, see [36] for details about its implementation. Consider also the PID controller proposed in [39] with a stabilizing proportional gain region given by $-0.5078 < k_p < -0.5$ and the set of PID parameters picked as $k_p = -0.503$, $k_i = -0.0002$ and $k_d = -0.46$. Figure 2.17 shows the stability region (k_i, k_d) for different values of k_p, as we can see, the stable region (inside the quadrilateral areas) becomes difficult to find due to the size of the time delay. It is important to note that the transfer function in [39] is defined with a negative gain when compared with (2.36), then the parameters (k_p, k_i and k_d) in the implementation must be inverted.

Considering an exact knowledge of the plant parameters and a small error between the initial conditions of the plant and its model, Figure 2.18 shows the comparison of the output signal evolution with respect to the control strategy proposed by [36], note how our strategy provide a slightly better performance that the one proposed by [36]. Notice also that for the strategy proposed by [36], due to its numerical nature, it is not evident to include a PI or PID controller in order to get step tracking reference since the closed loop stability is compromised. Figure 2.19 shows the robustness of the strategy by considering a process time-delay variation of $\tau = 1.8043$ and from where the advantages of our strategy are evident. To end the comparison with [36], Figure 2.20 shows the rejection of a step disturbance $h(t)$ acting at 100 sec.

Under ideal conditions, the PID controller performance, designed in [39] only for the stabilization problem, is presented in Figure 2.21, where the output signal response is depicted. It is clear the excessive overshoot in the output response as well as the large setting time, even when a unitary step reference is considered. This result is a consequence of the challenging delay consideration $\tau = 1.8 < 2/a$ that

Table 2.2: Range of time delay uncertainties.

a	b	τ	g_1	g_2	$\bar{k}$	$\tau_{0\,\min}$	$\tau_{0\,\max}$
1	6	1.5	0.6	0.27	1.1	1.313	1.5367
1	6	1.5	0.75	0.292	1.1	1.3904	1.6198
1	2	1.8	0.86	0.934	1.1	1.7834	1.8097
1	2	1.8	0.87	0.94	1.1	1.7875	1.8105

restrict the stabilization conditions of the closed-loop system as is described in the following remark.

Remark 2.6 *To make emphasis on the problems when dealing with large time delays as in the Example 2.4 ($\tau \to 2\tau_{un}$), the analysis developed in Section of Robustness can be used to show the range of time delay that guarantee closed-loop stability for several values of parameters a, b, τ, g_1, g_2 and $\bar{k}$. This case is shown in Table 2.2 from where it is possible to see how the robustness can be improved by changing the observer parameters g_1 and g_2, leading to a compromise between performance and robustness.*

2.5 Stabilization Strategy for Systems with $\tau < 3\tau_{un}$ and $\tau < 4\tau_{un}$

Based on the control strategy presented in this section the article entitled "Observer-PID Control for Unstable First Order Linear Systems with Large Time Delay" has been submitted to Industrial & Engineering Chemistry Research.

This Section presents a control strategy that allows to deal with unstable FOPTD systems with time delay satisfying $\tau < 3\tau_{un}$ and $\tau < 4\tau_{un}$. As a first step the observer design is considered, then the control schema is complemented by using a PI or PID controller. The proposed control strategy is developed as follows. In Section 2.5.1 the observer design is provided. The main results of this strategy are given in Section 2.5.2. After this, Section 2.5.3 presents a result that concerns with step disturbance rejection and finally, some numerical examples are shown in Section 2.5.4

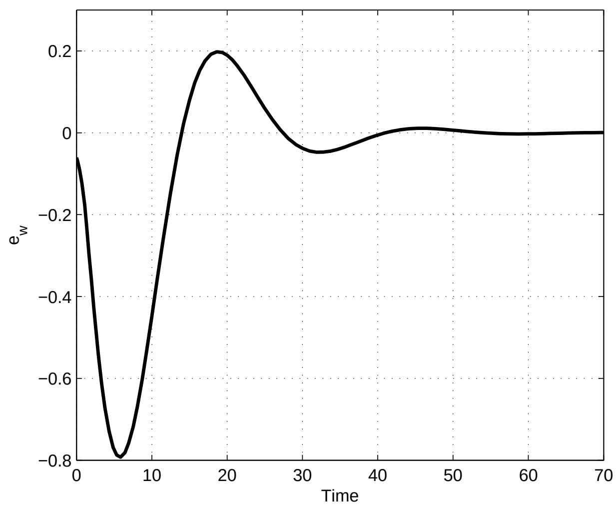

Figure 2.16: Estimation error $e_w(t) = \widehat{w}(t) - w(t)$ on Example 2.3.

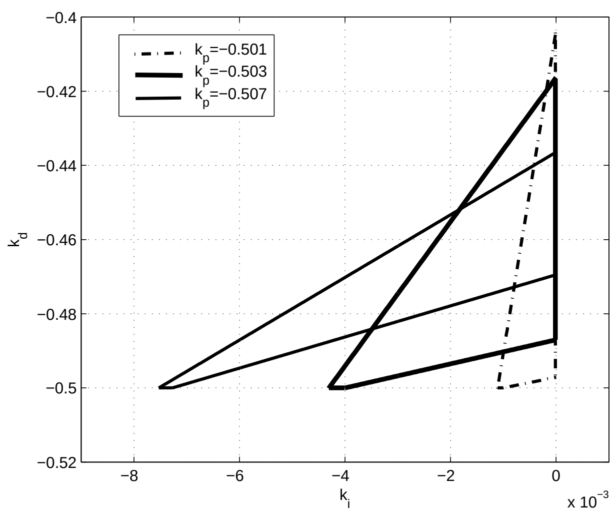

Figure 2.17: Stable regions (k_i, k_d), for different values of k_p

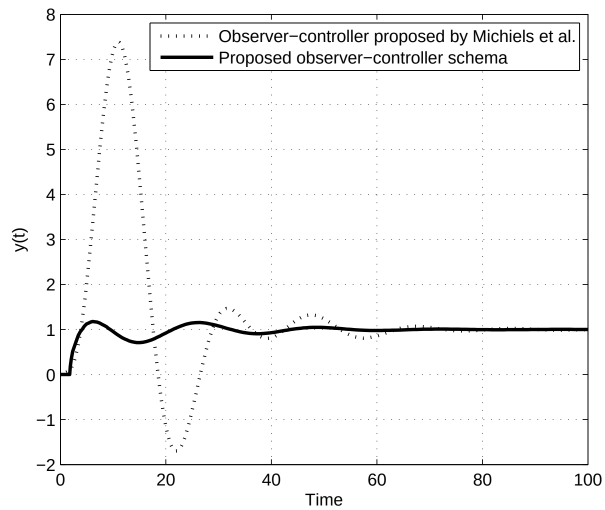

Figure 2.18: Output signal in Example 2.4.

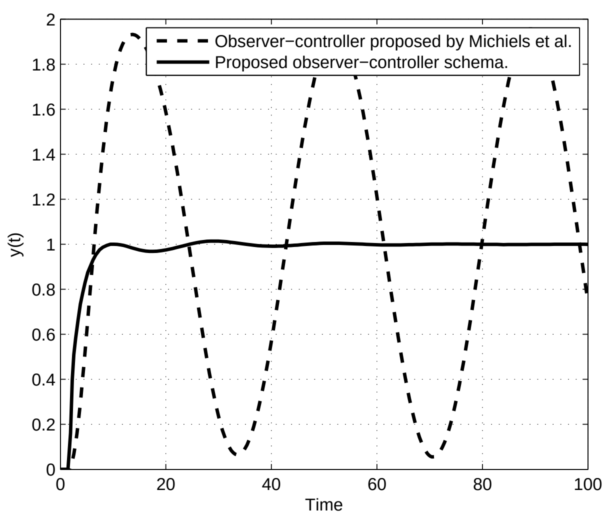

Figure 2.19: Output signal under parametric variations.

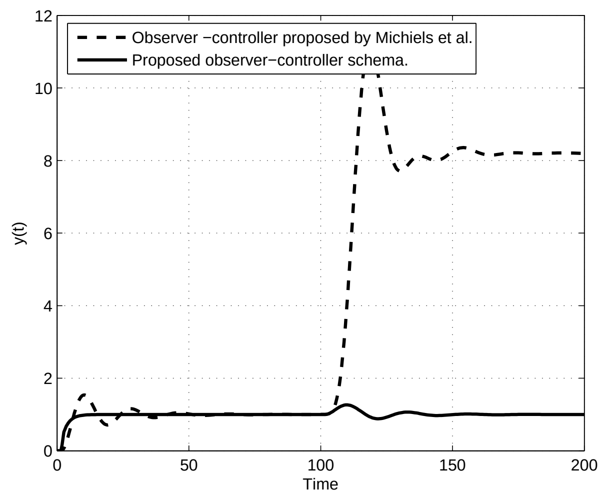

Figure 2.20: Output signal under step disturbance.

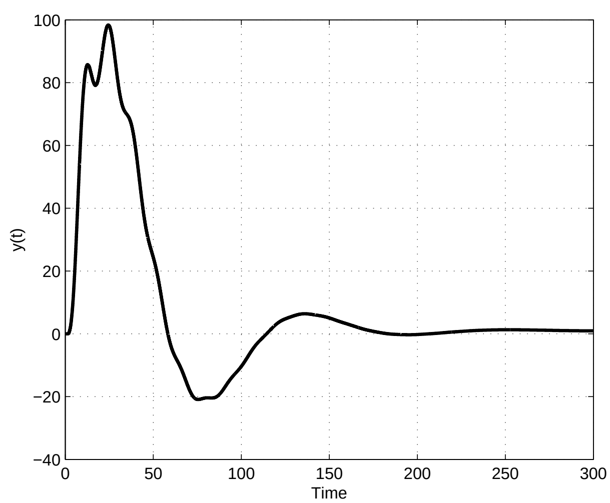

Figure 2.21: Output signal controlled by a PID structure [39].

2.5.1 Observer Strategy

Consider now the unstable, input-output delayed system (2.1), rewritten by splitting the input time-delay in the form,

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} = e^{-\tau_2 s}\frac{b}{s-a}e^{-\tau_1 s} \tag{2.37}$$

where $a, b > 0$, and $\tau = \tau_1 + \tau_2$.

In what follows, taking into account, the new delay-splited representation (2.37), a novel control structure will be presented in order to stabilize the original system (2.1) and at the same time solve the regulation and step disturbance problems. This new strategy considers an observer-based scheme together with a P or PI compensator defined by observed states that as a consequence allows to get the new stabilization bound $\tau < 3\tau_{un}$. Also, it can be shown that when it is considered a PID compensator, the stabilization bound is improved to $\tau < 4\tau_{un}$. In particular, the general strategy presented here, considers the notation and parameterization for the P, PI, PID compensators analyzed in [39, 40]. In order to present the main result of the proposed observer, as a preliminary step, it will be presented the stabilizing conditions for the static output injection scheme shown in Figure 2.22.

Lemma 2.5.

Consider the stabilizing scheme shown in Figure 2.22. Then there exist constants g_1 and g_2 such that the closed-loop system,

$$\frac{Y(s)}{U(s)} = \frac{be^{-\tau s}}{(s-a)(1+g_1 e^{-\tau_2 s}) + g_2 b e^{-\tau_2 s}} \tag{2.38}$$

is stable, if and only if, $\tau_2 < 2\tau_{un}$.

Proof. Based on the splitting strategy of τ, system (2.38) can be rewritten as,

$$\frac{Y(s)}{U(s)} = \frac{be^{-\tau_2 s}}{(s-a)(1+g_1 e^{-\tau_2 s}) + g_2 b e^{-\tau_2 s}}e^{-\tau_1 s}$$

Note from Figure 2.22 that the delay term $e^{-\tau_1 s}$ does not affect the stability of the

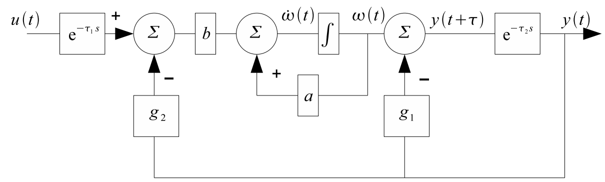

Figure 2.22: Static output injection.

closed-loop system. Therefore, it is considered the expression,

$$\frac{Y(s)}{U(s)} = \frac{be^{-\tau_2 s}}{(s-a)(1+g_1 e^{-\tau_2 s}) + g_2 b e^{-\tau_2 s}}. \tag{2.39}$$

In this way, the proof ends when it is stated that the system (2.39) is stable if and only if $\tau_2 < 2\tau_{un}$. This can be concluded by considering Lemma A.2 in Appendix A, which presents the stability conditions for the system (2.39). ■

As a consequence of Lemma 2.5 it is possible to state the following result.

Theorem 2.7.

Consider the observer scheme shown in Figure 2.23. Then, there exist constants g_1 and g_2 such that $\lim_{t\to\infty} [\omega(t) - \widehat{\omega}(t)] = 0$ if and only if $\tau_2 < 2\tau_{un}$.

Proof. Consider the original system (2.1) where the input delay has been splitted as given in (2.37). Under this new representation, it is possible to give a state space representation for the internal signal $\omega(t)$ as,

$$\begin{aligned} \dot{\omega}(t) &= a\omega(t) + bu(t-\tau_1) \\ y_w(t+\tau_2) &= \omega(t) \end{aligned} \tag{2.40}$$

It is now possible to consider a Luenverger-type observer for system (2.40) in the form,

$$\dot{\hat{\omega}}(t) = a\hat{\omega}(t) + bg_2(y(t) - \hat{y}(t)) + bu(t-\tau_1) \tag{2.41a}$$

$$\hat{y}_w(t+\tau_2) = \hat{\omega}(t) + g_1(y(t) - \hat{y}(t)) \tag{2.41b}$$

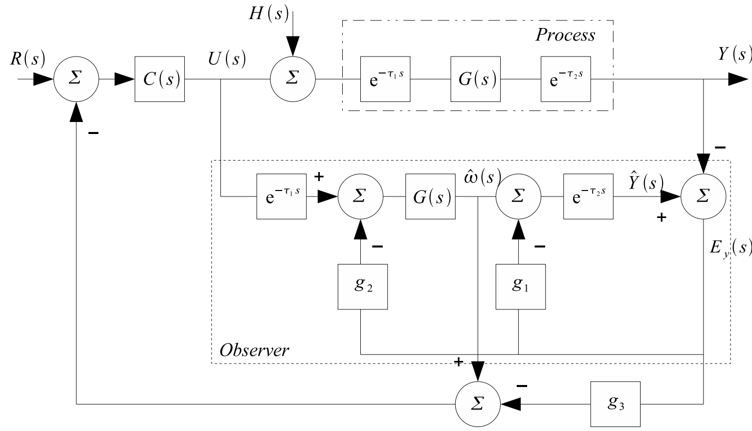

Figure 2.23: Proposed control strategy.

Systems (2.40)-(2.41) can be rewritten as,

$$\begin{aligned}
\begin{bmatrix} \dot{\omega}(t) \\ \dot{\widehat{\omega}}(t) \end{bmatrix} &= \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} \omega(t) \\ \widehat{\omega}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ bg_2 & -bg_2 \end{bmatrix} \begin{bmatrix} y(t) \\ \widehat{y}(t) \end{bmatrix} + \begin{bmatrix} b \\ b \end{bmatrix} u\left(t-\tau_1\right) \\
\begin{bmatrix} y(t+\tau_2) \\ \widehat{y}(t+\tau_2) \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \omega(t) \\ \widehat{\omega}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ g_1 & -g_1 \end{bmatrix} \begin{bmatrix} y(t) \\ \widehat{y}(t) \end{bmatrix}
\end{aligned} \tag{2.42}$$

with $\widehat{\omega}(t)$ the estimation of $\omega(t)$. Defining first the state prediction error $e_\omega(t) = \omega(t) - \widehat{\omega}(t)$ and the output estimation error $e_y(t) = y(t) - \widehat{y}(t)$ it is possible to describe the behavior of the error signal as:

$$\begin{bmatrix} \dot{e}_\omega(t) \\ e_y(t+\tau_2) \end{bmatrix} = \begin{bmatrix} a & -bg_2 \\ 1 & -g_1 \end{bmatrix} \begin{bmatrix} e_\omega(t) \\ e_y(t) \end{bmatrix}. \tag{2.43}$$

Consider now a state space realization of system (2.38) (described in Figure 2.22) written as,

$$\begin{bmatrix} \dot{x}(t) \\ y(t+\tau_2) \end{bmatrix} = \begin{bmatrix} a & -bg_2 \\ 1 & -g_1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} u(t-\tau_1). \tag{2.44}$$

It is clear now that the stability conditions of systems (2.44), given in Lemma 2.5, are equivalent to the ones of system (2.43), from where, the result of the theorem follows. ■

Notice that with the observer structure presented above, it is possible to estimate the internal signal $\omega(t)$ that correspond to the prediction of the output signal $y(t)$, τ_2 units of time ahead.

It should be pointed out that the discrete dynamics in (2.43),

$$e_y(t+\tau_2) = -g_1 e_y(t) + e_\omega(t)$$

imposes the initial restriction, $|g_1| < 1$. It is not an easy task to get the stability region associated with gains g_1 and g_2. However, note that a practical tuning of parameters g_1 and g_2 can also be done by considering the stability properties of the closed-loop system shown in Figure 2.22. In this way, a useful and practical result in order to compute the parameters involved on the predictor scheme is the Corollary A.1 and Algorithm A.1 when it is considered $\tau = \tau_2$ in Corollary A.1.

In [36] it is shown by means of an observer-based strategy that the delayed system (2.1) is stabilizable if and only if $\tau < 2\tau_{un}$. The proposed methodology allows to stabilize the system but not to implement a PI controller to achieve step tracking or step disturbance rejection. In what follows, the conditions in order to improve this bound are stated.

Notice that from the conditions of Theorem 2.7, for the estimation of $\omega(t)$ it is required $\tau_2 < 2\tau_{un}$. If $\tau_1 \neq 0$ on the delays distributions of Figure 2.23, then the control strategy should try to compensate the effect of a total time-delay $\tau > 2\tau_{un}$. In what follows, the admissible value of τ_1 will be analyzed in order to get an overall stable closed-loop system depending on $\tau = \tau_1 + \tau_2$.

2.5.2 Stabilization, regulation and disturbance rejection problem

Once the prediction scheme has been established, the proposed control structure will be complemented with a proportional P, proportional-integral PI or proportional-integral-derivative PID action. For the PI and PID parameterization the stabilization results given in [39, 40] are considered. The main result provides an observer-based structure that when complemented with a P (PI) controller produces the necessary and sufficient stabilization condition $\tau < 3\tau_{un}$. This necessary and sufficient stabilization condition is improved to $\tau < 4\tau_{un}$ when a PID controller is alternatively considered.

Proportional Action

Theorem 2.8.

Consider the observer-based control scheme depicted in Figure 2.23, with a control law,

$$U(s) = k_p[R(s) - \hat{\omega}(s)] \tag{2.45}$$

i.e., $C(s) = k_p$ and $g_3 = 0$. Then, there exists a constant k_p such that the closed-loop system (2.1)-(2.41)-(2.45) is stable if, and only if, $\tau < 3\tau_{un}$.

Proof. Consider the observer scheme shown in Figure 2.23. From Theorem 2.7, an adequate estimation $\widehat{\omega}(s)$ of the signal $\omega(s)$ is assured if and only if $\tau_2 < 2\tau_{un}$. Therefore, by Theorem 2.1 (see also Remark 2.2), it is possible to find a proportional controller of the form (2.45), such that the closed loop system is stable if and only if $\tau_1 < \tau_{un}$. Then we can conclude that the closed loop system is stable if and only if $\tau < 3\tau_{un}$. ■

Remark 2.7 *If $\tau < 3\tau_{un}$, then the proportional gain k_p can be computed by using Theorem 2.1.*

Proportional-Integral Action

Theorem 2.9.

Consider the observer-based control scheme depicted in Figure 2.23, with the control law,

$$U(s) = (k_p + k_i/s)[R(s) - \hat{\omega}(s)] \tag{2.46}$$

i.e., $C(s) = k_p + k_i/s$ and $g_3 = 0$. Then, there exists constants k_p and k_i such that the closed-loop system (2.1)-(2.41)-(2.46) is stable if, $\tau < 3\tau_{un}$.

Proof. The proof is similar to the one of Theorem 2.8, by using Theorem 2.2 instead of Theorem 2.1. ■

Remark 2.8 *If $\tau < 3\tau_{un}$, then set of stabilizing (k_p, k_i) can be determined by Theorem 2.2 and Remark 2.3.*

Proportional-Integral-Derivative Action

Theorem 2.10.

Consider the observer-based control scheme depicted in Figure 2.23, with the control law,

$$U(s) = (k_p + k_i/s + k_d s)[R(s) - \widehat{\omega}(s)], \tag{2.47}$$

i.e., $C(s) = k_p + k_i/s + k_d s$ and $g_3 = 0$. Then, there exists constants k_p, k_i and k_d such that the closed-loop system (2.1)-(2.41)-(2.47) is stable if, and only if, $\tau < 4\tau_{un}$.

Proof. Consider the observer scheme shown in Figure 2.23. From Theorem 2.7, an adequate estimation $\widehat{\omega}(s)$ of the signal $\omega(s)$ is assured if and only if $\tau_2 < 2\tau_{un}$. Therefore, by Theorem 2.3, it is possible to find a PID controller of the form (2.47), such that the closed loop system is stable if and only if $\tau_1 < 2\tau_{un}$. Then we can conclude that the closed loop system is stable if and only if $\tau < 4\tau_{un}$. ■

Remark 2.9 *If $\tau < 4\tau_{un}$ then the gains k_p, k_i, k_d can be computed from Theorem 2.3 and Remark 2.4.*

2.5.3 Step disturbance rejection

The observer-based control scheme presented previously can be improved by adding a step disturbance rejection property in the cases that an integral action is present in the compensator $C(s)$, i.e., for the cases given in equations (2.46)-(2.47). This result is stated under the following conditions.

Lemma 2.6.

Consider system (2.1) together with the observer-based control scheme depicted in Figure 2.23. Under these conditions, there exists a PI or PID controller able to reject input step disturbance ($H(s)$) if $g_3 = g_1 + 1$.

Proof. Consider the transfer function $Y(s)/H(s)$ of the control structure given in Figure 2.23 with $G(s)$ defined in equation (2.1), and $C(s)$ being a PI or PID controller defined by equations (2.46) and (2.47), respectively.

To verify the assertion of the lemma, the classical "Final value theorem" can be applied to $Y(s)$ when the disturbance signal is given as $H(s) = \frac{1}{s}$. It is an easy task to verify that under the condition $g_3 = g_1 + 1$, it is obtained,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 0$$

proving the result. ■

2.5.4 Simulation Results

The effectiveness of the proposed methodology will be now evaluated by means of two numerical examples.

Example 2.5 *An isothermal chemical reactor exhibiting multiple steady state solutions is considered. The mathematical model of the reactor is given as,*

$$\frac{dC}{dt} = \frac{Q}{V}(C_f - C) - \frac{k_1 C}{(k_2 C + 1)^2}$$

where Q is the inlet flow rate and C_f is the inlet concentration. The values of the operating parameters are given as $Q = 0.03333$ L/s, $V = 1$ L, $k_1 = 10$ L/s, and $k_2 = 10$ L/mol. For the nominal value of $C_f = 3.288$ mol/L, the steady-state solution of the model equation gives the following two stable steady states at $C = 1.7673$ and 0.01424 mol/L. There is one unstable steady state at $C = 1.316$ mol/L. Feed concentration is considered as the manipulated variable. Linearization of the manipulated variable around this operating condition $C = 1.316$ gives the unstable *transfer function model as* $3.433/(103.1s-1)$. *In [34] and [43] a measurement time delay of* 20 *s is considered. For our particular case, time-delay is considered to be two times the time constant of the system, i.e.,* 206.2 *s. In this way the unstable transfer function model is obtained as,*

$$\frac{Y(s)}{U(s)} = \frac{3.433}{103.1s-1}e^{-206.2s}$$

Since the time-delay satisfy $\tau < 3\tau_{un}$, *from Theorem 2.9 there exist gains* g_1 *and* g_2 *that stabilize the closed-loop system depicted in Figure 2.23. For the observer design,* $\tau = \tau_1 + \tau_2$ *is considered, with* $\tau_1 = 56.2$ *and* $\tau_2 = 150$. *For the simulation experiments* $g_1 = 0.6$ *and* $g_2 = 0.47$. *Then, since* $\tau_1 < \tau_{un}$, *a PI controller is used as controller C(s), see Figure 2.23. From Theorem 2.2, the range of proportional gain is* $0.29 < k_p < 0.66$; *and* $k_p = 0.46$ *is chosen (see Remark 2.3). The* k_i *range is obtained as* $0 < k_i < 0.0012$. *The set of PI parameters is picked as* $k_p = 0.46$ *and* $k_i = 0.0008$. *In order to improve the performance of the response, a PI with two degree of freedom proposed by [48] is used (see also Section A.5). In this way, the control law is modified as,*

$$U(s) = R(s)G_{ff}(s) - G_c[\hat{\omega}(s) + g_3E_y(s)] \tag{2.48}$$

where $g_3 = 1.6$, $G_{ff}(s) = k_p\sigma_c + \frac{k_i}{s}$ *and* $G_c(s) = k_p + \frac{k_i}{s}$. σ_c *can be chosen from* $0 < \sigma_c < 1$, *in this case* $\sigma_c = 0.001$. *In Figure 2.24 it is depicted the behavior of the stabilized system by means of the evolution of the output signal* $y(t)$. *In this experiment the exact knowledge of the plant parameters is assumed and an initial condition in the plant with a magnitude of* 0.1. *Also a small step disturbance (of magnitude* -0.005*) is considered acting at* 3500 *s. Figure 2.25 presents the estimation error* $e_\omega(t) = \omega(t) - \hat{\omega}(t)$ *of the scheme given in Figure 2.23.*

Example 2.6 *Consider the unstable first order system given by,*

$$\frac{Y(s)}{U(s)} = \frac{2}{s-1}e^{-\tau s}$$

with $\tau = 3.2$. *From Theorem 2.10, it is clear that there exist gains* g_1 *and* g_2 *that stabilize the closed-loop system depicted in Figure 2.23 since the time-delay satisfies*

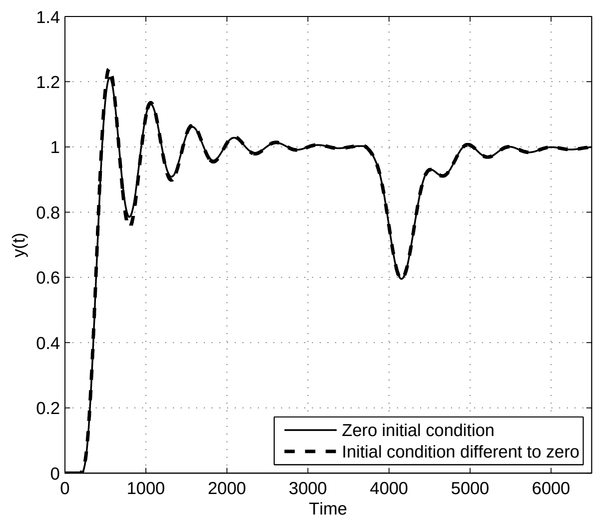

Figure 2.24: Output response $y(t)$, Example 2.5.

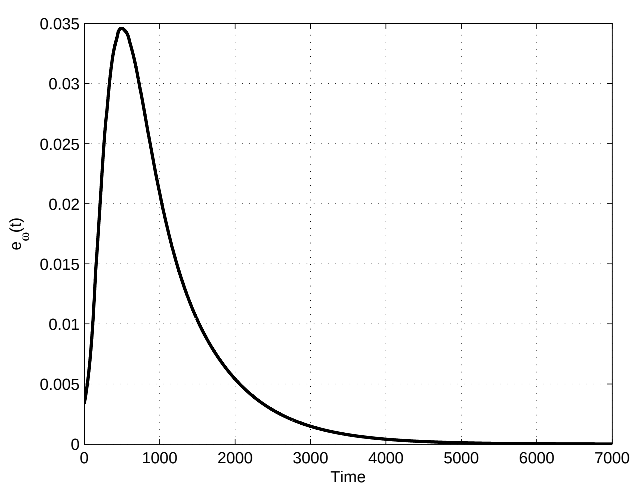

Figure 2.25: Estimation error $e_\omega(t) = \omega(t) - \hat{\omega}(t)$ on Example 2.5.

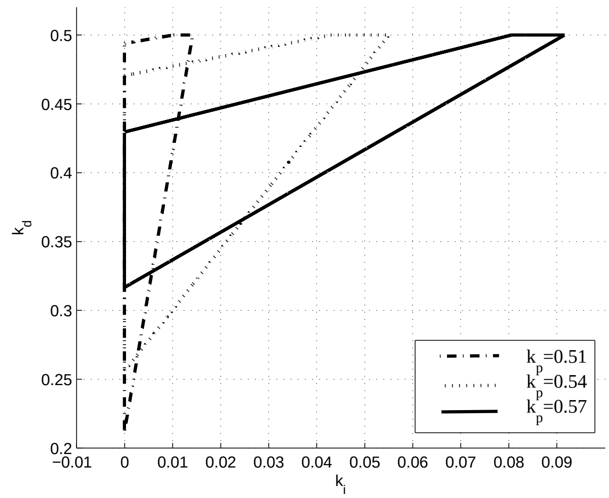

Figure 2.26: Region (k_i, k_d) for different values of k_p, Example 2.6.

$\tau < 4\tau_{un}$. For the observer design it is considered $\tau = \tau_1 + \tau_2$, where $\tau_1 = 1.4$ and $\tau_2 = 1.8$. For the simulation experiments it is considered $\epsilon = 0.06$ and $\bar{\epsilon} = 0.004$ and therefore it is obtained $g_1 = 0.86$ and $g_2 = 0.934$. Then, since $\tau_1 < 2\tau_{un}$, a PID controller is used as controller $C(s)$, see Figure 2.23. From Theorem 2.3, the range of proportional gain is $0.5 < k_p < 0.5813$; the stability region (k_i, k_d) for different values of k_p is shown in Figure 2.26, and the set of PID parameters is picked as $k_p = 0.503$, $k_i = 0.0002$ and $k_d = 0.46$. Also a PID with two degree of freedom as the one in (2.48) is used instead of a simple controller PID, with $g_3 = 1.86$, $G_{ff}(s) = k_p\sigma_c + \frac{k_i}{s} + k_d s$ and $G_c(s) = k_p + \frac{k_i}{s} + k_d s$. σ_c can be chosen from $0 < \sigma_c < 1$, in this case $\sigma_c = 0.001$ is used. In Figure 2.27 it is depicted the behavior of the stabilized system by means of the evolution of the output signal $y(t)$. To carry out this experiment it was assumed the exact knowledge of the plant parameters and an initial condition in the plant with a magnitude of 0.001. Figure 2.28 presents the estimation error $e_\omega(t) = \omega(t) - \hat{\omega}(t)$ of the scheme given in Figure 2.23.

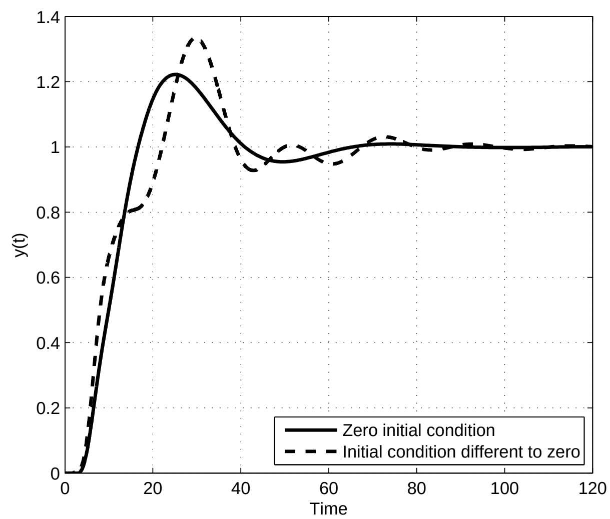

Figure 2.27: Output response $y(t)$, Example 2.6.

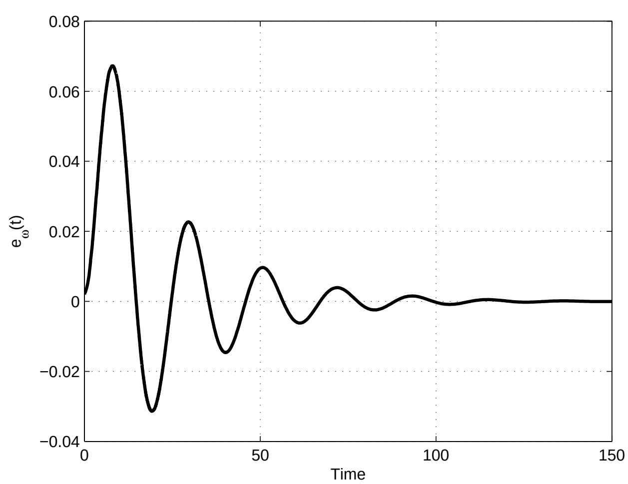

Figure 2.28: Estimation error $e_\omega(t) = \omega(t) - \hat{\omega}(t)$ on Example 2.6.

2.6 Conclusions

In this Chapter some preliminary results toward to deal with systems with recycle are presented. Some existing results in the literature are provided (Section 2.2). It seems that such results are simple, however it has been shown that more complicated control structures can be obtained in order to improve the performance of the system. As examples, four control strategies have been developed in order to deal with unstable FOPTD systems. In particular, unstable processes with significant time delays. This class of systems are commonly a challenging control problem. In fact, the existence of large input delay represents the worst case scenario of the regulation problem due to the instability problems associated with this phenomenon.

As a first step in Section 2.3 a control structure for systems satisfying $\tau < \tau_{un}$ is presented. Then two control strategies are presented in Section 2.4 in order to deal with FOPTD systems satisfying $\tau < 2\tau_{un}$. In first control strategy necessary and sufficient conditions for the stabilization of unstable first order systems with large time delays at the input-output path are given. In fact, the stabilization conditions presented here allows to stabilize First Order Unstable systems with a large time delay in a simpler manner (which is the main advantage) that the tuning methods presented in recent works in the literature. The problem is solved by proposing a two degree of freedom feedback that considers two simple static gains and adds a time delay effect. The stability conditions are obtained by considering an alternative discrete time approach that allows us to derive the continuous time case by taking the sampling time T tending to zero. The effectiveness of the proposed strategy is evaluated by simulations on an academic example. The second proposed control strategy presents the conditions to ensure the stability of the system in closed loop with an output injection strategy. In a second step, the stability conditions are used to design an observer-based scheme that provides a forward output estimation together with a feedback compensation to guarantee prediction convergence. The robustness of the overall observer- based strategy is analyzed when considering uncertainties on the magnitude of the time delay associated with the plant and the one considered on the design of the observer. A stability region as a function of these two time delays is obtained. The proposed prediction scheme is complemented by the use of a PI compensator to track step reference signals and to reject step disturbances. Also some numerical simulations are provided to show the performance of the proposed schema. As it is seen the second proposed control has abilities that the first proposed strategy does not have. However it should be pointed out that the second proposed strategy is more complicated since implies to design an observer module and a controller for the delay free model.

In the fourth proposed control schema, provided in Section 2.5, it is presented a new stabilization bound for a class of unstable first order linear system with large time-delay at the input-output path. Considering an splitting strategy for the original time delay τ, i.e. $\tau = \tau_1 + \tau_2$, it is presented a double action methodology that is based on the estimation of the internal signal $\omega(t)$ of the original plant that represent the future value of the output, τ_2 units of time ahead. This estimation structure is used at the same time to counteract the effect of this τ_2 output delay. The predicted internal signal $\omega(t)$ is used also in an outer P, PI, PID loop that takes into account the effects of the remaining input delay τ_1. The novel strategy presented in this work allows stabilizing open loop plants up to the limit $\tau < 4\tau_{un}$ that up to our best knowledge has not been reported in the literature. The strategy is complemented with an input step disturbance rejection property for the overall closed-loop system. Numerical simulations show the effectiveness of the proposed control structure. Some results presented in this Chapter will be used in Chapter 3 in order to get observer schemas for recycling systems. Also, it should be noticed that the presented observer-controller schemas in this Chapter serve as reference for the developed strategies to recycling systems. This is, some ideas applied in this Chapter are taken in order to propose control schemas for systems with recycle. In fact, the reader can see that some ideas formulated in Chapter 1 are similar to the ones used in this Chapter.

Chapter 3

Observer design for Recycling systems

In Chapter 1 some ideas have been developed in order to overcome the control problem of recycling systems. As a first step, the observer-predictor design is required. Therefore, in this Chapter it is proposed an observer-predictor as well as a tuning method for each case defined in problem formulation given in Chapter 1. This Chapter is organized as follows, the observer design for the recycling systems in cases 1.1, 1.2 and 1.3 is addressed in Sections 3.1, 3.2 and 3.3, respectively.

3.1 Observer design for unstable FOPTD system at forward path

This section presents the observer design for unstable FOPTD system at forward path, i.e., when Case 1.1 is considered. Two observer strategies are developed in order to tackle the estimation of the variables ω_1 and ω_2 as depicted in Section 1.1. It should be noticed that the first observer strategy (presented in Section 3.1.1) can be applied for systems satisfying $\tau_1 < \tau_{un}$, while the second estimation strategy (provided in Section 3.1.2) considers larger time-delay in the systems, i.e., $\tau_1 < 2\tau_{un}$. Note that τ_{un} is defined as the unstable time-constant of the system i.e., $\tau_{un} = 1/a$.

3.1.1 Delayed forward loop $\tau_1 < \tau_{un}$.

To estimate ω_1 and ω_2 shown in Figure 1.1, the observer-predictor depicted in Figure 3.1 is proposed. Its convergence is established in the following result.

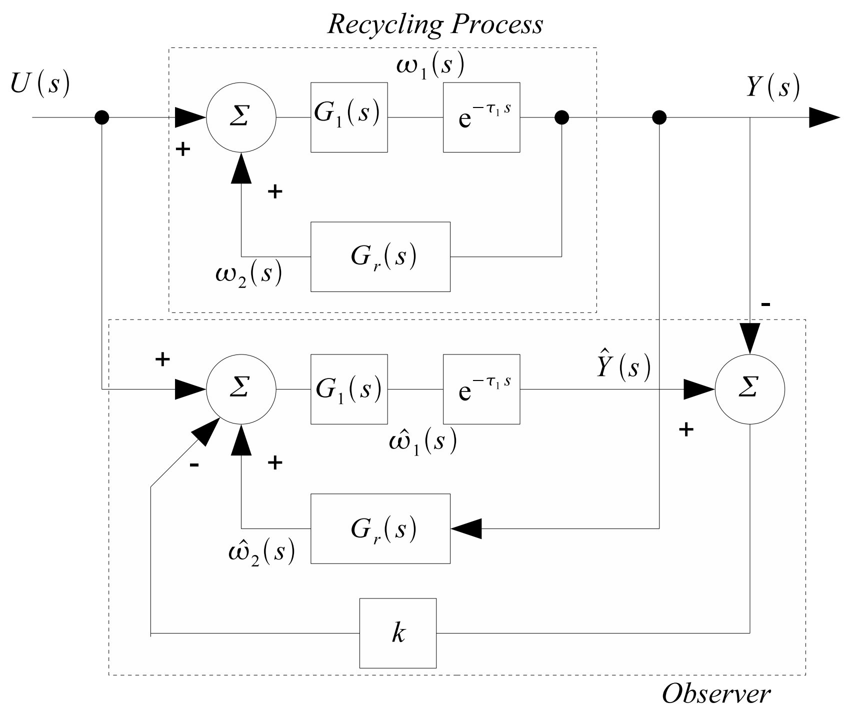

Figure 3.1: Observer predictor proposed

Theorem 3.1.

Consider the observer-predictor scheme shown in Figure 3.1, with G_r a stable transfer function. There exists constant k such that

$$\lim_{t\to\infty} [\omega_i - \widehat{\omega}_i] = 0, \text{ for } i = 1, 2, \tag{3.1}$$

if and only if $\tau_1 < \tau_{un}$.

Proof. A state space representation of the observer-predictor scheme shown in Figure 3.1 is

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + A_1\mathbf{x}(t-\tau_1) + A_2\mathbf{x}(t-\tau_2) + Bu(t) \tag{3.2a}$$

$$\mathbf{y}(t) = C\mathbf{x}(t-\tau_1) \tag{3.2b}$$

with,

$$\mathbf{x}(t) = \begin{bmatrix} x_d(t) & x_r(t) & \widehat{x}_d(t) & \widehat{x}_r(t) \end{bmatrix}^T,$$

$$\mathbf{y}(t) = \begin{bmatrix} y(t) & \widehat{y}(t) \end{bmatrix}^T, B = \begin{bmatrix} B_d & 0 & B_d & 0 \end{bmatrix}^T,$$

$$A = \begin{bmatrix} A_d & 0 & 0 & 0 \\ 0 & A_r & 0 & 0 \\ 0 & 0 & A_d & 0 \\ 0 & 0 & 0 & A_r \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ B_rC_d & 0 & 0 & 0 \\ B_rkC_d & 0 & -B_dkC_d & 0 \\ B_rC_d & 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & B_dC_r & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_dC_r \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} C_d & 0 & 0 & 0 \\ 0 & 0 & C_d & 0 \end{bmatrix},$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input, $\mathbf{y} \in \mathbb{R}^2$ is the output, $\tau_1 \geq 0$ and $\tau_2 \geq 0$ are the time delays present in the system. $A_d \in \mathbb{R}^{n\times n}$, $B_d \in \mathbb{R}^{n\times 1}$, and $C_d \in \mathbb{R}^{1\times n}$ are matrices and vectors parameters that corresponds to the forward loop in the process, and $A_r \in \mathbb{R}^{n\times n}$, $B_r \in \mathbb{R}^{n\times 1}$, and $C_r \in \mathbb{R}^{1\times n}$ are matrices and vectors parameters that correspond to backward path in the process, $\widehat{x}(t)$ is the estimation of $x(t)$.

Defining the state prediction errors

$$e_{x_d}(t) = \widehat{x}_d(t) - x_d(t), \quad e_{x_r}(t) = \widehat{x}_r(t) - x_r(t), \tag{3.4}$$

and the output estimation

$$e_y(t) = \widehat{y}(t) - y(t), \tag{3.5}$$

it is possible to describe the behavior of the error signals as,

$$\begin{bmatrix} \dot{e}_{x_d}(t) \\ \dot{e}_{x_r}(t) \\ e_y(t+\tau_1) \\ e_{\omega_2}(t+\tau_2) \end{bmatrix} = \begin{bmatrix} A_d & 0 & -B_d k & B_d \\ 0 & A_r & 0 & 0 \\ C_d & 0 & 0 & 0 \\ 0 & C_r & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{x_d}(t) \\ e_{x_r}(t) \\ e_y(t) \\ e_{\omega_2}(t) \end{bmatrix} \tag{3.6}$$

Note that $e_y(t) = C_d e_{x_d}(t-\tau_1)$ and that $e_{\omega_2}(t) = C_r e_{x_r}(t-\tau_2)$. Then, system (3.6) can be rewritten as

$$\dot{e}_{x_d}(t) = A_d e_{x_d}(t) - B_d k C_d e_{x_d}(t-\tau_1) + B_d C_r e_{x_r}(t-\tau_2) \tag{3.7a}$$
$$\dot{e}_{x_r}(t) = A_r e_{x_r}(t)$$

Since A_r is a Hurwitz matrix, the stability of system (3.7) can be analyzed by considering the partial dynamics

$$\dot{e}_{x_d}(t) = A_d e_{x_d}(t) - B_d k C_d e_{x_d}(t-\tau_1) \tag{3.8}$$

or equivalently,

$$\begin{bmatrix} \dot{e}_{x_d}(t) \\ e_y(t+\tau_1) \end{bmatrix} = \begin{bmatrix} A_d & -B_d k \\ C_d & 0 \end{bmatrix} \begin{bmatrix} e_{x_d}(t) \\ e_y(t) \end{bmatrix} \tag{3.9}$$

Consider now a state space realization of system (2.6). It is easy to see that this dynamics can be written in state space form as,

$$\begin{bmatrix} \dot{x}(t) \\ y(t+\tau_1) \end{bmatrix} = \begin{bmatrix} a & -bk \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_d(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} B_d \\ 0 \end{bmatrix} u(t) \tag{3.10}$$

Comparing (3.10) and (3.9) it is clear that Lemma 2.1 can be applied to system (3.9). Hence the result of the theorem follows. ■

Remark 3.1 *Since the observer dynamic is equivalent to system (2.6), the observer design parameter k can be calculated by means of the Corollary 2.1.*

3.1.2 Delayed forward loop $\tau_1 < 2\tau_{un}$.

Here is presented also an observer-predictor design for unstable FOPTD system at forward path. In comparison with the observer given in Section 3.1.1, the observer presented in this Section allows to deal with a larger time delay i.e., $\tau < 2\tau_{un}$. Furthermore, to estimate ω_1 and ω_1 shown in Figure 1.1, the observer-predictor depicted in Figure 3.2 is proposed. The corresponding convergence is established in the following result.

Theorem 3.2.

Consider the observer-predictor scheme shown in Figure 3.2, with G_2 a stable transfer function and $g_3 = 1$. Then, there exists constants g_1 and g_2 such that

$$\lim_{t\to\infty} [\omega_i - \widehat{\omega}_i] = 0, \text{ for } i = 1, 2,$$

if and only if $\tau_1 < 2\tau_{un}$.

Proof. The dynamics of the observer-prediction scheme shown in Figure 3.2 can be written in state space form as,

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + A_1\mathbf{x}(t-\tau_2) + A_2\mathbf{y}(t) + Bu(t), \tag{3.11}$$

$$\mathbf{y}(t+\tau_1) = C_1\mathbf{x}(t) + C_2\mathbf{y}(t) \tag{3.12}$$

with, $\mathbf{x}(t) = \begin{bmatrix} x_d(t) & x_r(t) & \widehat{x}_d(t) & \widehat{x}_r(t) \end{bmatrix}^T$, $\mathbf{y}(t) = \begin{bmatrix} y(t) & \widehat{y}(t) \end{bmatrix}^T$,
$B = \begin{bmatrix} B_d & 0 & B_d & 0 \end{bmatrix}^T$,

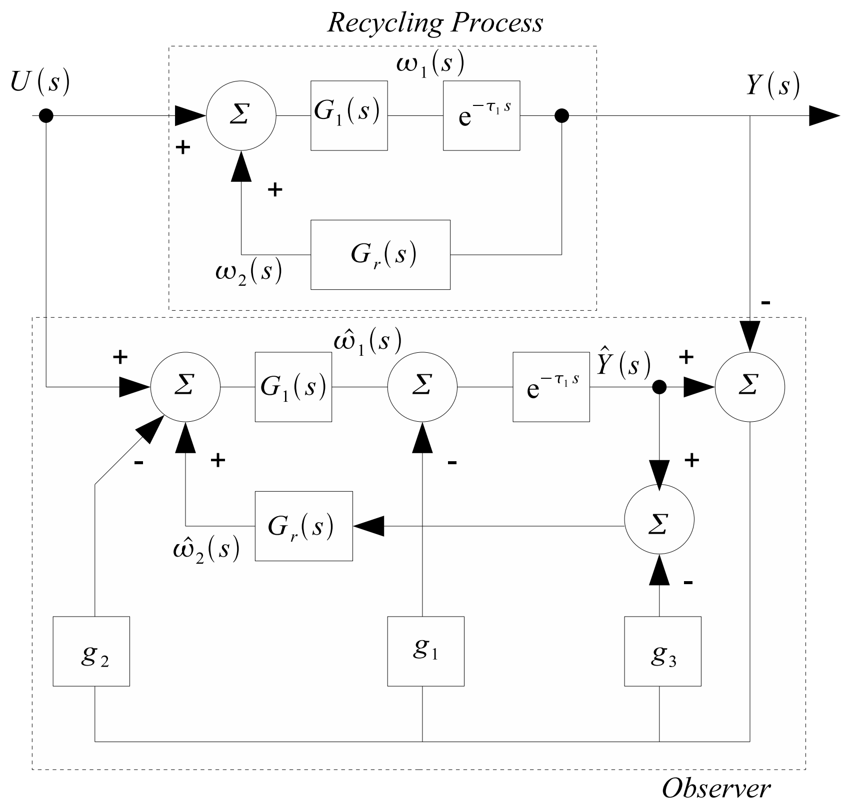

Figure 3.2: Proposed predictor.

$$A = \begin{bmatrix} A_d & 0 & 0 & 0 \\ 0 & A_r & 0 & 0 \\ 0 & 0 & A_d & 0 \\ 0 & 0 & 0 & A_r \end{bmatrix}, A_1 = \begin{bmatrix} 0 & B_dC_r & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_dC_r \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 \\ B_r & 0 \\ B_dg_2 & -B_dg_2 \\ g_3 & B_r - g_3 \end{bmatrix}, C_1 = \begin{bmatrix} C_d & 0 & 0 & 0 \\ 0 & 0 & C_d & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 0 \\ g_1 & -g1 \end{bmatrix}.$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input, $\mathbf{y} \in \mathbb{R}^2$ is the output, $\tau_1 \geq 0$ and $\tau_2 \geq 0$ are the time delays associated with the states, $A_d \in \mathbb{R}^{1\times1}$, $B_d \in \mathbb{R}^{1\times1}$, and $C_d \in \mathbb{R}^{1\times1}$ are matrices and vectors parameters that correspond to the forward loop in the process, and $A_r \in \mathbb{R}^{n-1\times n-1}$, $B_r \in \mathbb{R}^{n-1\times1}$, and $C_r \in \mathbb{R}^{1\times n-1}$ are matrices and vectors parameters that corresponds to backward path in the process, $\widehat{x}(t)$ is the estimation of $x(t)$.

Remark 3.2 *In the state space representation given by equations (3.11)-(3.12), times delay at the direct and recycle loops of the corresponding transfer functions in the model of the process are considered in the output.*

Defining the state prediction errors $e_{x_d}(t) = \widehat{x}_d(t) - x_d(t)$, $e_{x_r}(t) = \widehat{x}_r(t) - x_r(t)$, the output estimation $e_y(t) = \widehat{y}(t) - y(t)$ and the function of the delayed state $e_{\omega_2}(t) = C_r e_{x_r}(t-\tau_2)$, it is possible to describe the behavior of the error signals as,

$$\begin{bmatrix} \dot{e}_{x_d}(t) \\ \dot{e}_{x_r}(t) \\ e_y(t+\tau_1) \\ e_{\omega_2}(t+\tau_2) \end{bmatrix} = \begin{bmatrix} A_d & 0 & -B_d g_2 & B_d \\ 0 & A_r & 0 & 0 \\ C_d & 0 & -g_1 & 0 \\ 0 & C_r & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{x_d}(t) \\ e_{x_r}(t) \\ e_y(t) \\ e_{\omega_2}(t) \end{bmatrix}.$$

Note that $e_y(t) + g_1 e_y(t-\tau_1) = C_d e_{x_d}(t-\tau_1)$ and that $e_{\omega_2}(t) = C_r e_{x_r}(t-\tau_2)$. Then, this later system can be rewritten as,

$$\dot{e}_{x_d}(t) = A_d e_{x_d}(t) - B_d g_2 e_y(t) + B_d C_r e_{x_r}(t-\tau_2) \tag{3.13a}$$

$$\dot{e}_{x_r}(t) = A_r e_{x_r}(t) \tag{3.13b}$$

Since A_r is a Hurwitz matrix, the stability of system (3.13) can be analyzed by considering the partial dynamics,

$$\dot{e}_{x_d}(t) = A_d e_{x_d}(t) - B_d g_2 e_y(t)$$

or equivalently,

$$\begin{bmatrix} \dot{e}_{x_d}(t) \\ e_y(t+\tau_1) \end{bmatrix} = \begin{bmatrix} A_d & -B_d g_2 \\ C_d & -g_1 \end{bmatrix} \begin{bmatrix} e_{x_d}(t) \\ e_y(t) \end{bmatrix}. \tag{3.14}$$

On the other hand, consider now a state space realization of the system (A.15) or equivalently the output injection shown in Figure A.4 given in Appendix A. Then, it is easy to see that this dynamic can be written as,

$$\begin{bmatrix} \dot{x}(t) \\ y(t+\tau) \end{bmatrix} = \begin{bmatrix} a & -bg_2 \\ 1 & -g_1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t). \tag{3.15}$$

From the above developments, it is clear that the stability conditions for system (3.15), given in Lemma A.2, are valid for the obtained system (3.14), and the result of the theorem follows. ■

Remark 3.3 *Since the observer dynamic is equivalent to system (A.15), the observer parameters g_1 and g_2 can be calculated by means of the Corollary A.1 and Algorithm A.1 given in Appendix A.*

3.2 Observer design for one unstable pole and stable poles at forward path

In order to estimate ω_1 and ω_2 in Figure 1.1 when Case 1.2 is considered, we propose the observer-predictor depicted in Fig. 3.3. As a first step the convergence of such observer is shown by considering a system with one unstable pole, one stable pole plus time delay at forward loop. In this way, we have the following result.

Theorem 3.3.

Consider the observer-predictor scheme shown in Fig. 3.3, with G_r a stable transfer function and $\frac{1}{a} - \frac{1}{b} > 0$. There exists constant k such that

$$\lim_{t\to\infty} [\omega_i - \widehat{\omega}_i] = 0, \text{ for } i = 1, 2, \tag{3.16}$$

if and only if

$$\tau_1 < \frac{1}{a} - \frac{1}{b}. \tag{3.17}$$

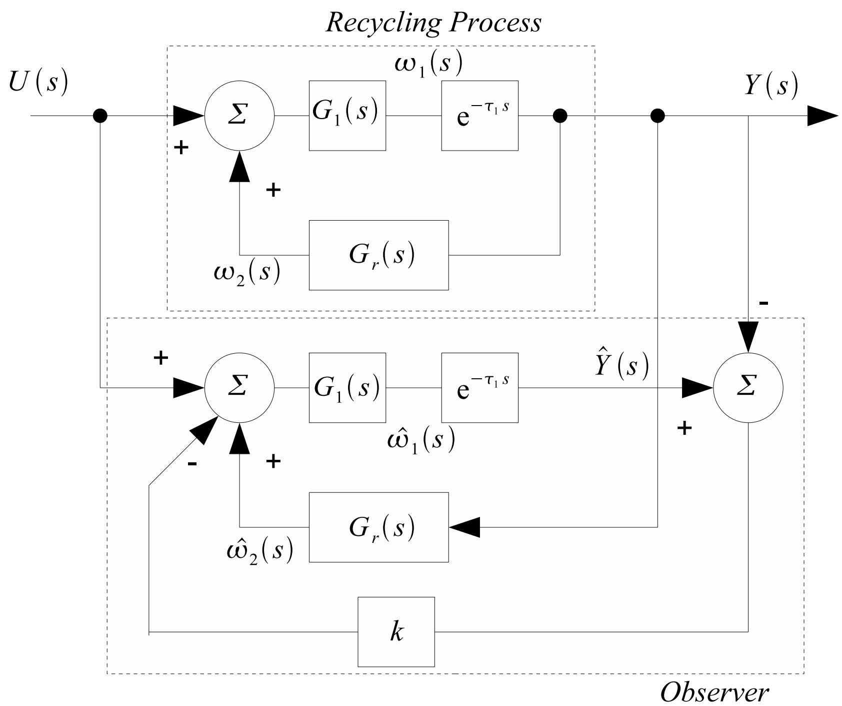

Figure 3.3: Proposed observer schema

Proof. A state space representation of the observer-predictor scheme shown in Figure 3.3 is,

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + A_1\mathbf{x}(t-\tau_1) + A_2\mathbf{x}(t-\tau_2) + Bu(t) \tag{3.18}$$

$$\mathbf{y}(t) = C\mathbf{x}(t-\tau_1) \tag{3.19}$$

with, $\mathbf{x}(t) = \begin{bmatrix} x_d(t) & x_r(t) & \widehat{x}_d(t) & \widehat{x}_r(t) \end{bmatrix}^T$, $\mathbf{y}(t) = \begin{bmatrix} y(t) & \widehat{y}(t) \end{bmatrix}^T$, $B = \begin{bmatrix} B_d & 0 & B_d & 0 \end{bmatrix}^T$,

$$A = \begin{bmatrix} A_d & 0 & 0 & 0 \\ 0 & A_r & 0 & 0 \\ 0 & 0 & A_d & 0 \\ 0 & 0 & 0 & A_r \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ B_rC_d & 0 & 0 & 0 \\ B_rkC_d & 0 & -B_dkC_d & 0 \\ B_rC_d & 0 & 0 & 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & B_dC_r & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_dC_r \\ 0 & 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} C_d & 0 & 0 & 0 \\ 0 & 0 & C_d & 0 \end{bmatrix},$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input, $\mathbf{y} \in \mathbb{R}^2$ is the output, $\tau_1 \geq 0$ and $\tau_2 \geq 0$ are the time delays present in the system. $A_d \in \mathbb{R}^{n\times n}$, $B_d \in \mathbb{R}^{n\times 1}$, and $C_d \in \mathbb{R}^{1\times n}$ are matrices and vectors parameters that corresponds to the forward loop in the process, and $A_r \in \mathbb{R}^{n\times n}$, $B_r \in \mathbb{R}^{n\times 1}$, and $C_r \in \mathbb{R}^{1\times n}$ are matrices and vectors parameters that correspond to recycling (or backward) path in the process, $\widehat{x}(t)$ is the estimation of $x(t)$. Defining the state prediction errors $e_{x_d}(t) = \widehat{x}_d(t) - x_d(t)$, $e_{x_r}(t) = \widehat{x}_r(t) - x_r(t)$, and the output estimation $e_y(t) = \widehat{y}(t) - y(t)$, it is possible to describe the behavior of the error signals as,

$$\begin{bmatrix} \dot{e}_{x_d}(t) \\ \dot{e}_{x_r}(t) \\ e_y(t+\tau_1) \\ e_{\omega_2}(t+\tau_2) \end{bmatrix} = \begin{bmatrix} A_d & 0 & -B_dk & B_d \\ 0 & A_r & 0 & 0 \\ C_d & 0 & 0 & 0 \\ 0 & C_r & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{x_d}(t) \\ e_{x_r}(t) \\ e_y(t) \\ e_{\omega_2}(t) \end{bmatrix}, \tag{3.20}$$

Note that $e_y(t) = C_d e_{x_d}(t-\tau_1)$ and that $e_{\omega_2}(t) = C_r e_{x_r}(t-\tau_2)$. Then, system (3.6) can be rewritten as,

$$\dot{e}_{x_d}(t) = A_d e_{x_d}(t) - B_dkC_d e_{x_d}(t-\tau_1) + B_dC_r e_{x_r}(t-\tau_2), \tag{3.21a}$$
$$\dot{e}_{x_r}(t) = A_r e_{x_r}(t). \tag{3.21b}$$

Since A_r is a Hurwitz matrix, the stability of system (3.21) can be analyzed by considering the partial dynamics

$$\dot{e}_{x_d}(t) = A_d e_{x_d}(t) - B_d k C_d e_{x_d}(t - \tau_1), \tag{3.22}$$

or equivalently,

$$\begin{bmatrix} \dot{e}_{x_d}(t) \\ e_y(t+\tau_1) \end{bmatrix} = \begin{bmatrix} A_d & -B_d k \\ C_d & 0 \end{bmatrix} \begin{bmatrix} e_{x_d}(t) \\ e_y(t) \end{bmatrix}. \tag{3.23}$$

Consider now a state space realization of system (2.7) together a static output feedback . It is easy to see that this dynamics can be written in state space form as,

$$\begin{bmatrix} \dot{x}(t) \\ y(t+\tau_1) \end{bmatrix} = \begin{bmatrix} A_d & -B_d k \\ C_d & 0 \end{bmatrix} \begin{bmatrix} x_d(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} B_d \\ 0 \end{bmatrix} u(t), \tag{3.24}$$

Comparing (3.24) and (3.23) it is clear that Lemma 2.2 can be applied to system (3.23). Hence the result of the theorem follows. ■

Note that the proposed prediction strategy can be generalized to recycling systems with an unstable pole and m stable poles at forward loop i.e., Case 1.2, defined in Chapter 1. Let rewrite the forward loop considered in Case 1.2,

$$G_d = \frac{\alpha}{(s-a)(s+\overline{b_1})(s+\overline{b_2})...(s+\overline{b_m})} e^{-\tau_1 s}. \tag{3.25}$$

In such case, the convergence of the observer is assured with the following result.

Theorem 3.4.

Consider the observer-predictor scheme shown in Fig. 3.3, with G_r a stable transfer function, G_d defined as in (3.25) and $\frac{1}{a} - \sum_{i=1}^{m} \frac{1}{b_i} > 0$. There exists constant k such that

$$\lim_{t \to \infty} [\omega_i - \widehat{\omega}_i] = 0, \text{ for } i = 1, 2, \tag{3.26}$$

if and only if $\tau < \frac{1}{a} - \sum_{i=1}^{m} \frac{1}{b_i}$.

Proof. The proof of this Theorem can be stated in a similar way that the proof of Theorem 3.3, by using Lemma 2.3 instead of Lemma 2.2 ■

Remark 3.4 *It should be pointed out that the observer design presented here has a disadvantage with respect to the allowable size of time delay at direct loop. For illustrate this fact, let us analyze the the convergence condition (3.17) given in Theorem 3.3. When the necessary condition $b > a$ is satisfied, the allowable size of time delay at direct loop is decreased from $1/a$. Moreover if b is near of the the limit $b > a$ the allowable size of time delay at direct loop is very small. In this way, if b is incremented the condition (3.17) is less restrictive on the allowable time delay. Hence, the convergence condition depends on the position of the open root loop at forward loop of recycling systems, since the parameters a and b are imposed by the system dynamic. Also, similar conclusion can be obtained from an analysis of the Theorem 3.4. In order to partially overcome this problem, the observer design developed in the following section is proposed.*

3.3 Observer design for generalized forward path

Consider the results presented in Section 2.2, which will be useful in the proof of the following result. In this way, let us consider a static output injection for the class of unstable delayed systems given by,

$$H(s) = \frac{b(s+\beta_1)(s+\beta_2)...(s+\beta_m)e^{-\tau s}}{(s-a)(s+\alpha_1)(s+\alpha_2)...(s+\alpha_n)} \tag{3.27}$$

with

$$H_u(s) = \frac{b}{s-a}e^{-\tau s}, \tag{3.28}$$

$$H_s(s) = \frac{(s+\beta_1)(s+\beta_2)...(s+\beta_m)}{(s+\alpha_1)(s+\alpha_2)...(s+\alpha_n)}, \tag{3.29}$$

$n, m \in \mathbb{R}$, $n \geqslant m$, $a, \alpha_i, \beta_j > 0$ $\forall i = 1, 2, ..., n$ and $\forall j = 1, 2, ..., m$. A state space representation for the unstable part of $H(s)$ given by (3.28) can be obtained as,

$$\begin{aligned} \dot{x}(t) &= ax(t) + bu(t) \\ y_1(t) &= x(t-\tau) \end{aligned}$$

Then, a state space representation for the stable part $H_s(s)$ given in equation (3.29) can be expressed as,

$$\dot{x}_1(t) = A_1 x_1(t) + B_1 u_1(t) \tag{3.30a}$$

$$y(t) = C_1 x_1 + D_1 u_1(t) \tag{3.30b}$$

In order to get a general state space representation for the transfer function given by (3.27) it is considered $u_1(t) = y_1(t)$. Thus, we have,

$$\dot{x}(t) = \mathcal{A}x(t) + \mathcal{A}_1 x(t-\tau) + \mathcal{B}u(t) \tag{3.31a}$$

$$y(t) = \mathcal{C}x(t) + \mathcal{C}_1 x(t-\tau) \tag{3.31b}$$

where $x(t) = \begin{bmatrix} x(t) & x_1(t) \end{bmatrix}^T$,

$$\mathcal{A} = \begin{bmatrix} a & 0 \\ 0 & A_1 \end{bmatrix}, \mathcal{A}_1 = \begin{bmatrix} 0 & 0 \\ B_1 & 0 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} b \\ 0 \end{bmatrix} \tag{3.32}$$

$$\mathcal{C} = \begin{bmatrix} 0 & C_1 \end{bmatrix}, \mathcal{C}_1 = \begin{bmatrix} D_1 & 0 \end{bmatrix} \tag{3.33}$$

Notice that due to the realization (3.31) comes from a transfer function, it is possible to set the delay term at the input, output or even between dynamics. In this case, time delay term is mixed with the unstable part of $H(s)$.

A state space representation for the system (3.31) together a static output injection as shown in Figure 3.4 can be obtained as,

$$\dot{x}(t) = (\mathcal{A} - G\mathcal{C})x(t) + (\mathcal{A}_1 - G\mathcal{C}_1)x(t-\tau) + \mathcal{B}u(t) \tag{3.34a}$$

$$y(t) = \mathcal{C}x(t) + \mathcal{C}_1 x(t-\tau) \tag{3.34b}$$

and its transfer function can be expressed as,

$$\frac{Y(s)}{U(s)} = (\mathcal{C} + \mathcal{C}_1 e^{-\tau s})(sI - (\mathcal{A} - G\mathcal{C}) - (\mathcal{A}_1 - G\mathcal{C}_1)e^{-\tau s})^{-1}\mathcal{B} \tag{3.35}$$

where $G = \begin{bmatrix} g_1 & G_2 \end{bmatrix}^T$. In what follows stability conditions for the system (3.27) when a static output injection is considered, or equivalently the transfer function (3.35) are presented.

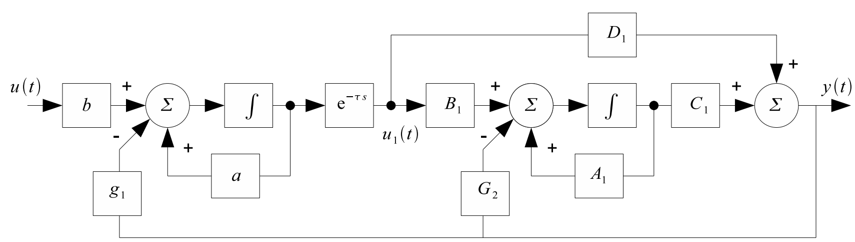

Figure 3.4: Output injection schema

Lemma 3.1.

Consider the delayed system given by (3.27), its state space representation given by (3.31) and the static output feedback shown in Figure 3.4. There exist $G = \begin{bmatrix} g_1 & G_2 \end{bmatrix}^T$, such that the closed loop transfer function given by (3.35) is stable if and only if $\tau < \frac{1}{a}$ for $n = m$; or $\tau < \frac{1}{a} - \sum_{i=1}^{n-m} \frac{1}{\alpha_i}$ for $n < m$.

Proof. Consider the output injection for the system given by (3.27) shown in Figure 3.4. Notice that since there is not cancellation zero-pole, the state space representation (3.30) is observable. Therefore, it is possible to locate all poles of $H_s(s)$ in any desirable position by means G_2. Moreover, if it is proposed to locate the poles at the same position of the zeros of $H_s(s)$, stable cancellation pole-zero is then allowed. For the case $n = m$, this leads to a system of the form (2.4), and then it is possible to get a static gain g_1 (by Lemma 2.1) such that the closed loop transfer function (3.35) is stable if and only if,

$$\tau < \frac{1}{a}. \tag{3.36}$$

In the case $n < m$, this lies to a system of the form (2.11). Then, there exist a static gain g_1 (by Lemma 2.3) such that the system (3.35) is stable if and only if

$$\tau < \frac{1}{a} - \sum_{i=1}^{n-m} \frac{1}{\alpha_i}. \tag{3.37}$$

■

Now, to estimate ω_1 and ω_2 in Figure 1.1 when Case 1.3 is considered, the observer-predictor depicted in Figure 3.5 is proposed. Its convergence is established in the following result.

Theorem 3.5.

Consider the observer-predictor scheme shown in Figure 3.5, with G_r a stable transfer function. There exists a vector $G = \begin{bmatrix} g_1 & G_2 & G_3 \end{bmatrix}^T$ such that

$$\lim_{t\to\infty} [\omega_i - \widehat{\omega}_i] = 0, \text{ for } i = 1, 2, \tag{3.38}$$

if and only if $\tau_1 < \frac{1}{a}$.

Proof. A state space representation for the forward loop of recycling system G_d is

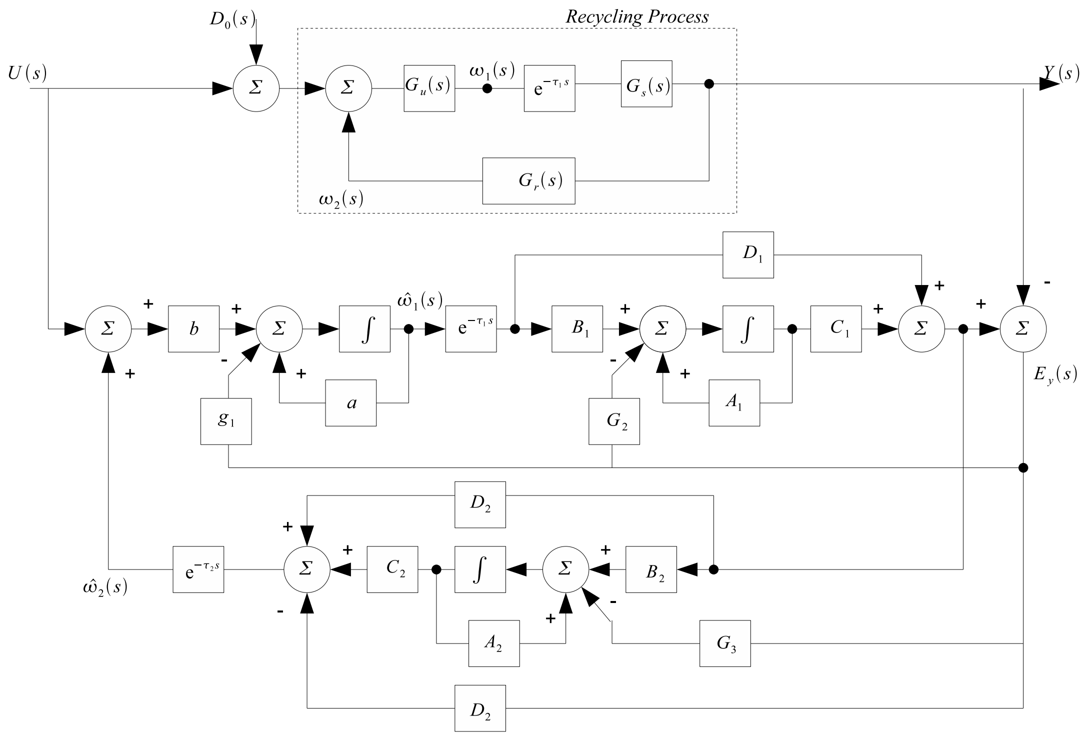

Figure 3.5: Observer predictor proposed

of the form (3.31) and can be rewritten as,

$$\dot{x}_d(t) = \mathcal{A}x_d(t) + \mathcal{A}_1 x_d(t-\tau_1) + \mathcal{B}u(t) \tag{3.39a}$$

$$y(t) = \mathcal{C}x_d(t) + \mathcal{C}_1 x_d(t-\tau_1) \tag{3.39b}$$

Then a state space representation for the backward path of recycling system, G_r is defined as,

$$\dot{x}_r(t) = A_2 x_r(t) + B_2 y(t) \tag{3.40a}$$

$$\omega_2(t+\tau_2) = C_2 x_r(t) + D_2 y(t) \tag{3.40b}$$

In this way, a state space representation of the observer-predictor scheme shown in Figure 3.5 is,

$$
\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{A}_1\mathbf{x}(t-\tau_1) + \mathbf{A}_2\mathbf{x}(t-\tau_2) \\
&\quad + \mathbf{A}_3\mathbf{x}(t-\tau_3) + \mathbf{B}u(t) \qquad (3.41a) \\
\mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{C}_1\mathbf{x}(t-\tau_1) \qquad (3.41b)
\end{aligned}
$$

with $\tau_3 = \tau_1 + \tau_2$, $\mathbf{x}(t) = \begin{bmatrix} x_d(t) & x_r(t) & \widehat{x}_d(t) & \widehat{x}_r(t) \end{bmatrix}^T$, $\mathbf{y}(t) = \begin{bmatrix} y(t) & \widehat{y}(t) \end{bmatrix}^T$

$$
\mathbf{B} = \begin{bmatrix} \mathcal{B} \\ 0 \\ \mathcal{B} \\ 0 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} \mathcal{A} & 0 & 0 & 0 \\ B_2\mathcal{C} & A_2 & 0 & 0 \\ G_{1,2}\mathcal{C} & 0 & \mathcal{A} - G_{1,2}\mathcal{C} & 0 \\ G_3\mathcal{C} & 0 & B_2\mathcal{C} - G_3\mathcal{C} & A_2 \end{bmatrix}
$$

$$
\mathbf{A}_1 = \begin{bmatrix} \mathcal{A}_1 & 0 & 0 & 0 \\ B_2\mathcal{C}_1 & 0 & 0 & 0 \\ G_{1,2}\mathcal{C}_1 & 0 & \mathcal{A}_1 - G_{1,2}\mathcal{C}_1 & 0 \\ G_3\mathcal{C}_1 & 0 & B_2\mathcal{C}_1 - G_3\mathcal{C}_1 & 0 \end{bmatrix} \mathbf{A}_2 = \begin{bmatrix} \mathcal{B}D_2\mathcal{C} & \mathcal{B}C_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \mathcal{B}D_2\mathcal{C} & 0 & 0 & \mathcal{B}C_2 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
\mathbf{A}_3 = \begin{bmatrix} \mathcal{B}D_2\mathcal{C}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \mathcal{B}D_2\mathcal{C}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{C} = \begin{bmatrix} \mathcal{C} & 0 & 0 & 0 \\ 0 & 0 & \mathcal{C} & 0 \end{bmatrix} \mathbf{C}_1 = \begin{bmatrix} \mathcal{C}_1 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{C}_1 & 0 \end{bmatrix}
$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input, $\mathbf{y} \in \mathbb{R}^2$ is the output, $\tau_1 \geq 0$ and $\tau_2 \geq 0$ are the time delays present in the system. $\mathcal{A}, \mathcal{A}_1 \in \mathbb{R}^{n\times n}$, $\mathcal{B} \in \mathbb{R}^{n\times 1}$, $\mathcal{C} \in \mathbb{R}^{1\times n}$, $\mathcal{C}_1 \in \mathbb{R}^{1\times n}$ are matrices and vectors parameters that corresponds to the forward loop in the process, and $A_2 \in \mathbb{R}^{n\times n}$, $B_2 \in \mathbb{R}^{n\times 1}$, $C_2 \in \mathbb{R}^{1\times n}$ and D_2 are matrices and vectors parameters that corresponds to backward path in the process, $\widehat{x}(t)$ is the estimation of $x(t)$. Defining the state prediction errors $e_{x_d}(t) = \widehat{x}_d(t) - x_d(t)$, $e_{x_r}(t) = \widehat{x}_r(t) - x_r(t)$, the output estimation $e_y(t) = \widehat{y}(t) - y(t)$ and with $G_3 = B_2$, it is possible to describe the behavior of the error signals as,

$$
\dot{e}_{x_d}(t) = \mathcal{A}e_{x_d}(t) + \mathcal{A}_1 e_{x_d}(t-\tau_1) - G_{1,2}e_y(t) + \mathcal{B}C_2 e_{x_r}(t-\tau_2) \qquad (3.43a)
$$

$$
\dot{e}_{x_r}(t) = A_r e_{x_r}(t) \qquad (3.43b)
$$

Note that,

$$e_y(t) = \mathcal{C}_1 e_{x_d}(t - \tau_1) + \mathcal{C} e_{x_d} \tag{3.44}$$

Since A_r is a Hurwitz matrix, the stability of system (3.43) can be analyzed by considering the partial dynamics

$$\dot{e}_{x_d}(t) = \mathcal{A} e_{x_d}(t) + \mathcal{A}_1 e_{x_d}(t - \tau_1) - G_{1,2} e_y(t) \tag{3.45}$$

or equivalently,

$$\dot{e}_{x_d}(t) = (\mathcal{A} - G_{1,2}\mathcal{C}) e_{x_d}(t) + (\mathcal{A}_1 - G_{1,2}\mathcal{C}_1) e_{x_d}(t - \tau_1) \tag{3.46}$$

Consider now the dynamic of system (3.34) and rewritten as,

$$\dot{x}(t) = (\mathcal{A} - G\mathcal{C}) x(t) + (\mathcal{A}_1 - G\mathcal{C}_1) x(t - \tau) \tag{3.47}$$

Comparing (3.47) and (3.46) it is clear that Lemma 3.1 can be applied to system (3.46). Hence the result of the theorem follows. ■

3.4 Conclusions

In this Chapter some observer schemas for recycling systems have been presented. For the Case 1.1 (Section 3.1), two observers have been proposed. First observer scheme is based on a static output feedback. With a more straightforward structure, a second observer is proposed where the internal variables of the system can be estimated with a larger time delay at forward loop. Then, derived from results on frequency domain approach, an observer for the Case 1.2 (Section 3.2) is proposed. This proposed schema allows dealing with systems composed by one unstable pole, several stable poles and time delay at direct path. The allowable time delay in the observer convergence condition depends directly on the location of the poles of the system as it is illustrated by Remark 3.4. In order to improve the mentioned disadvantage of the observer strategy given in Section 3.2, Section 3.3 provides an observer strategy that overcome such disadvantage. Additionally the observer proposed in Section 3.3 can be used for plants with zeros in the real half left complex plane (Case 1.3) at forward path. Note that the observer schema proposed in Section 3.3 can be seen as a generalized schema with respect to the class of system at forward loop. However, the observer structures are somehow different. In all proposed observers in this Chapter convergence conditions with respect to time delay and constant time of the forward loop have been obtained. Also, results that allow calculating the parameters involved in the observer design have been provided. The observer design studied in this Chapter will be used in Chapter 4 to obtain observer-controller structures for dealing with delayed recycling systems.

Chapter 4

Tracking reference and disturbance rejecting

In Chapter 3, observer design for the Cases 1.1, 1.2 and 1.3 have been developed. Therefore, in this Chapter it is assumed that estimated internal signals $\widehat{\omega}_1$ and $\widehat{\omega}_2$ converge to its real value. In this way, the ideas depicted in Section 1.1 can be implemented. In what follows, a control strategy (based on an observer) for each case defined in Section 1.1 is presented. Also, results related to step tracking reference and rejecting step disturbance are provided.

4.1 Control for unstable FOPTD system at forward path

4.1.1 Delayed forward loop, $\tau_1 < \tau_{un}$.

The observer-controller methodology given in this section was presented in American Control Conference 2011. Also, an improved version of the work has been submitted to Journal of Process Control, which can be also seen in Appendix B.5.

Consider the observer presented in Section 3.1.1 shown in Figure 3.1 and the control proposed given as,

$$U(s) = J(s)(R(s) - \hat{\omega}_1 + E_y(s)) - \hat{\omega}_2(s) \tag{4.1}$$

Hence, the complete control scheme is depicted in Figure 4.1. Then, in order to improve the output response performance, consider a PI controller with two degree of freedom provided in [48] (see also Section A.5), instead of a simple controller $J(s)$. In such case, the control law can be implemented as,

$$U(s) = R(s)G_{ff}(s) - G_c(s)(\widehat{\omega}_1(s) - E_y(s)) - \widehat{\omega}_2(s) \tag{4.2}$$

where,

$$G_{ff}(s) = K(\sigma_c + \frac{1}{T_i s}) \tag{4.3a}$$

$$G_c(s) = K(1 + \frac{1}{T_i s}) \tag{4.3b}$$

The following results show that such scheme achieves step input tracking and a particular disturbance rejection action by using a PI controller with two degree of freedom depicted above.

Lemma 4.1.

Consider the proposed observer scheme shown in Figure 3.1. Then, there exist a PI controller with two degree of freedom given by (4.2) such that $\lim_{t\to\infty} y(t) = \alpha_r$, where $R(s) = \alpha_r/s$ is the step input reference and $D_0(s) = 0$.

Proof. Consider the observer scheme shown in Figure 3.1 and the control strategy given by (4.2). Then,

$$\frac{Y(s)}{R(s)} = \frac{G_d G_{ff}(1 + kG_1 e^{-s\tau_1})}{G_1 G_c + G_c G_d - G_1 G_c e^{-s\tau_1} + kG_1 e^{-s\tau_1} + kG_1 G_c G_d + 1} \tag{4.4}$$

Applying the Final Value Theorem with $R(s) = \alpha_r/s$, as input reference. Now, from equation (4.4), we have,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\frac{N_d(s)}{D_d(s)}\frac{\alpha_r}{s} \tag{4.5}$$

with

$$N_d(s) = G_d G_{ff}(1 + kG_1 e^{-s\tau_1}) \tag{4.6}$$

$$D_d(s) = G_1 G_c + G_c G_d - G_1 G_c e^{-s\tau_1} + kG_1 e^{-s\tau_1} + kG_1 G_c G_d + 1 \tag{4.7}$$

Substituting G_d, G_1 given in (1.6a) and the controller G_{ff} and G_c given in (4.3), we have,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \alpha_r$$

■

Lemma 4.2.

Consider the proposed observer scheme shown in Figure 3.1. Then, there exist a PI controller with two degree of freedom provided by (4.2) such that $\lim_{t\to\infty} y(t) = 0$, where $R(s) = 0$ and $D_0(s)$ is the a step input disturbance.

Proof. According to Figure 4.1, and by considering a PI-controller with two degree of freedom, it is obtained,

$$\frac{Y(s)}{D_0(s)} = \frac{G_d + G_1 G_c G_d + kG_1 G_d e^{-s\tau_1} - G_1 G_c G_d e^{-s\tau_1}}{G_1 G_c + G_c G_d - G_1 G_c e^{-s\tau_1} + kG_1 e^{-s\tau_1} + kG_1 G_c G_d + 1}$$

Applying the Final Value Theorem with $D_0(s) = \beta_r/s$, as input reference. From equation (4.4), we have,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\frac{N_{d2}(s)}{D_{d2}(s)}\frac{\beta_r}{s} \tag{4.8}$$

with

$$N_{d2}(s) = G_d + G_1 G_c G_d + kG_1 G_d e^{-s\tau_1} - G_1 G_c G_d e^{-s\tau_1} \tag{4.9a}$$

$$D_{d2}(s) = G_1 G_c + G_c G_d - G_1 G_c e^{-s\tau_1} + kG_1 e^{-s\tau_1} + kG_1 G_c G_d + 1. \tag{4.9b}$$

Substituting G_d, G_1 given in (1.6a) and the controller G_{ff} and G_c given in (4.3), we have,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 0 \tag{4.10}$$

■

From the previous discussions and results, the proposed methodology can be summarized as follows:

1. Make sure that the conditions of Theorem 3.1 are satisfied, that is, $G_r(s)$ a stable transfer function and $\tau < \tau_{un}$ for the unstable first order delayed plant.

2. Tune the parameter k using Corollary 2.1.

3. Design of a controller $J(s)$ based on the free delay model of the forward path. A PI or PID control based strategy can be considered (as the proposed PI controller with two degree of freedom).

4. Finally, to implement the general control structure as it is shown in Figure 4.1.

4.1.2 Delayed forward loop, $\tau_1 < 2\tau_{un}$.

For the second estimation strategy proposed in Section 3.1.2, it is proposed a general control strategy expressed as,

$$U(s) = J(s)(R(s) - \hat{\omega}_1 + \beta_d E_y(s)) - \hat{\omega}_2(s) \tag{4.11}$$

where by considering a controller $J(s)$ with an integral action, the term $\beta_d E_y(s)$ allows to obtain step disturbance rejecting $D_0(s)$. The corresponding control scheme is then shown in Figure 4.2. However, if it is desired to apply a PI controller with two degree of freedom, we have,

$$U(s) = R(s)G_{ff}(s) - G_c(s)(\widehat{\omega}_1(s) - \beta_d E_y(s)) - \widehat{\omega}_2(s), \tag{4.12}$$

where $G_{ff}(s)$ and $G_c(s)$ are given by (4.3). The parameter β_d, which can be computed as $\beta_d = 1 + g_1$, is used for guarantee step disturbance rejecting, $D_0(s)$. The results concerning to the step tracking reference and step disturbance rejection can be obtained in a similar way as Lemmas 4.1 and 4.2.

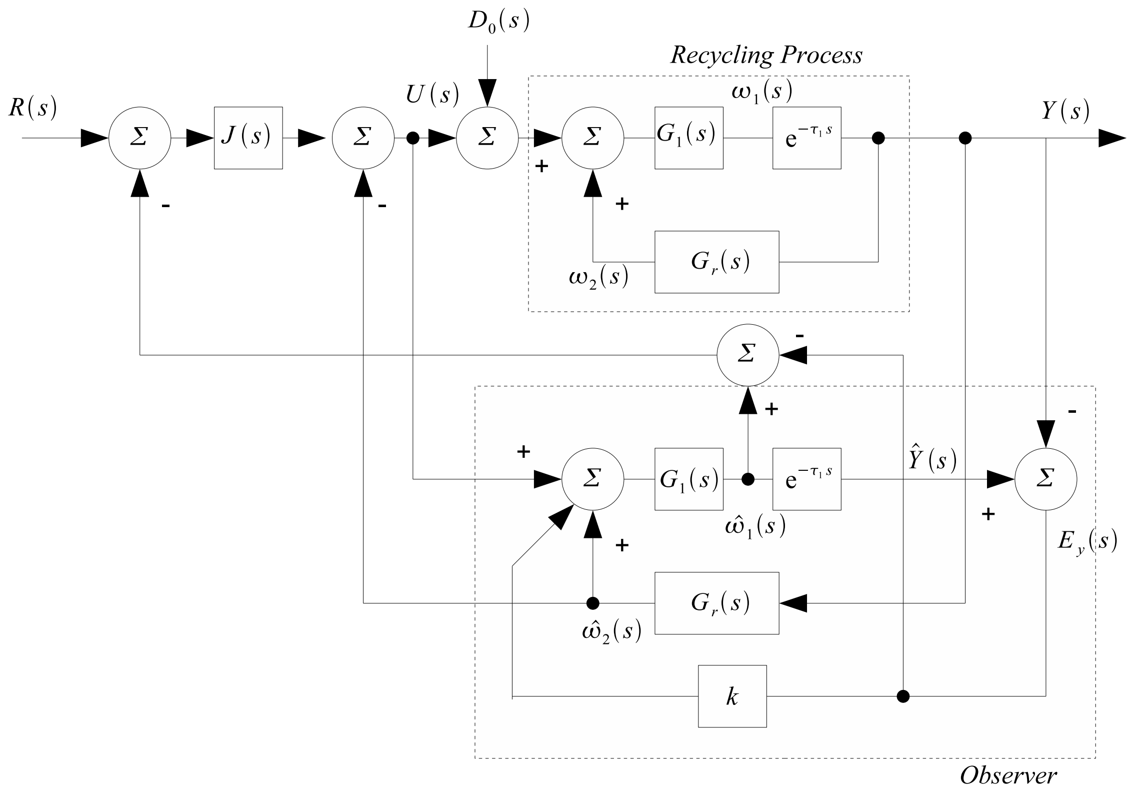

Figure 4.1: Proposed control schema

The proposed methodology is summarized as follows:

1. Fulfillment of the conditions of Theorem 3.2, $G_r(s)$ a stable transfer function and $\tau_1 < 2\tau_{un}$ for the unstable first order delayed at forward path.

2. Predictor stabilization. This can be achieve by tuning the parameter g_1, g_2 by means of Corollary A.1 and Algorithm A.1.

3. Design of a controller $J(s)$, based on the free delay model of the forward path. A PI or PID control based strategy can be considered as the proposed PI controller with two degree of freedom.

4. Finally, implementation of the general control structure proposed in Figure 4.2.

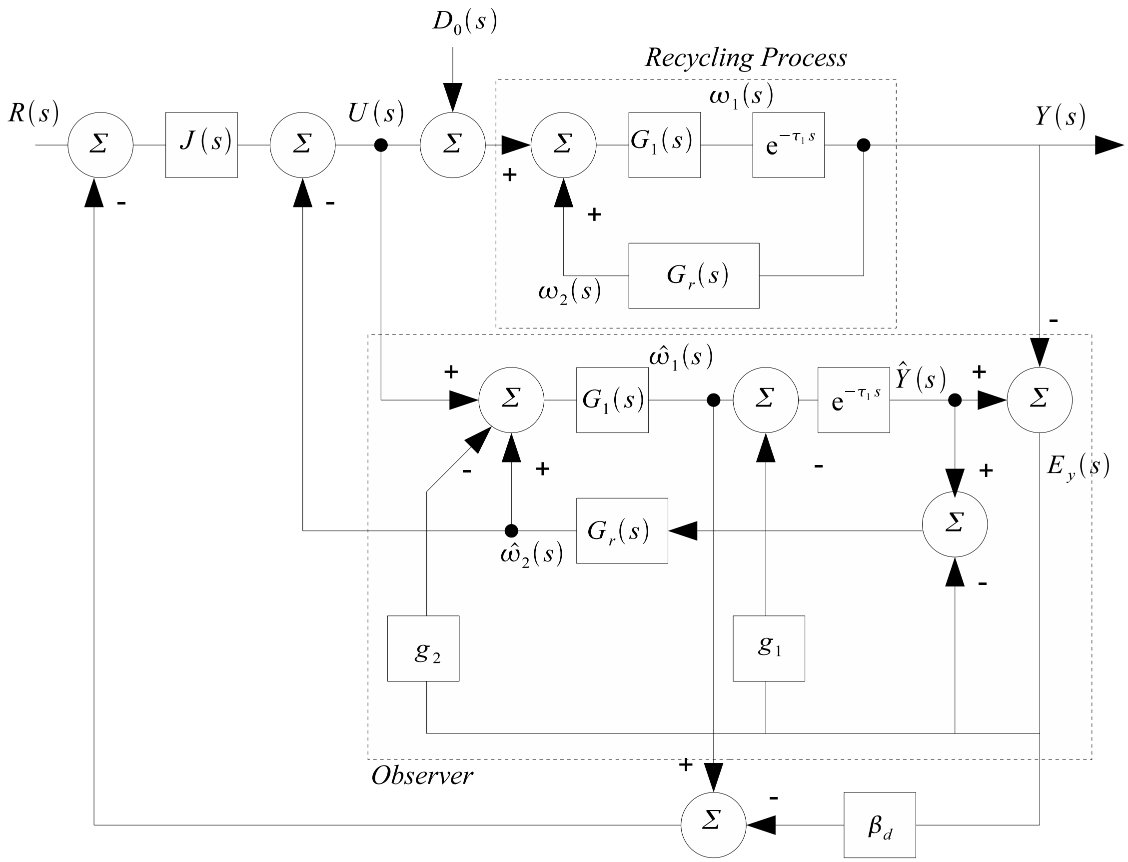

Figure 4.2: Control strategy proposed for recycling systems

4.2 Control for stable-poles and one unstable pole at forward path

Note that from the results obtained in this section the article "Control of Delayed Recycling Systems with an Unstable Pole at Forward Path" has been submitted to the American Control Conference 2012.

Consider the observer design given in Section 3.2. Consequently, a control law can be depicted as,

$$U(s) = J(s)(R(s) - \widehat{\omega}_1(s)\ ) - \widehat{\omega}_2(s) \tag{4.13}$$

The complete observer-controller is presented in Figure 4.3. Note that the control law given by (4.13) does not consider step disturbance rejecting neither a formal result with respect to step tracking reference is presented. However, formal results can be obtained in a similar way as in Section 4.1 by applying the required changes to the control law.

Remark 4.1 *Consider the forward loop transfer function* $G_1(s) = G_u(s)G_s(s)$*, where* $G_u = \frac{b}{s-a}$ *is the unstable part of* $G_1(s)$ *and* $G_s(s)$ *consists in stable poles of* $G_1(s)$*. Notice that the control law (4.13) suggests to feedback the internal signal* $\widehat{\omega}_1$*. However, note that there are more estimated signals that can be used for the control design, for instance the signal* $\widehat{\omega}_u$ *(see Figure 4.3). In this way, the internal signal* $\widehat{\omega}_u$ *can be used instead of* $\widehat{\omega}_1$*. Therefore, the controller* $J(s)$ *should be tuning based on* G_u *instead on the one based on the delay free model of the forward path,* $G_1(s)$*. Hence, this latest control proposal (based on the internal signal* $\widehat{\omega}_u$*) is less involved since it is easier to tune a controller for an unstable first order system than a higher order system.*

Now, from the previous discussions and results, the proposed methodology can be summarized as follows,

1. Make sure that the conditions of Theorem 3.3 are satisfied, that is, $G_r(s)$ a stable transfer function and $\lim_{t\to\infty} [\omega_i - \widehat{\omega}_i] = 0$, for $i = 1, 2$, for the unstable plant at direct loop.

2. Tune the parameter k. This can be done by taking into account the gain margin of the Nyquist diagram of $G_d(s)$.

3. Design of a controller $J(s)$ based on the delay free model of the forward path, $G_1(s)$. A PID control based strategy can be considered. Notice that since the internal states are estimated, it is possible to use an estimated state feedback for G_1 instead of the controller $J(s)$.

4. Finally, to implement the general control structure as it is shown in Figure 4.3.

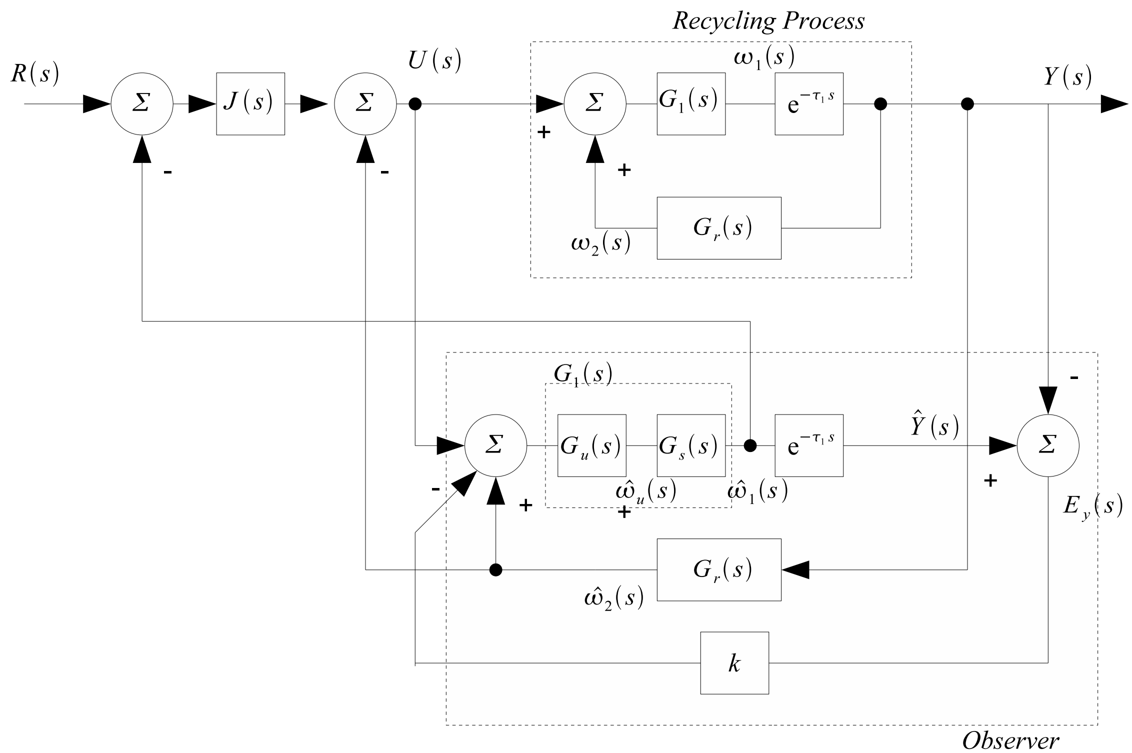

Figure 4.3: Proposed control schema

4.3 Control for generalized forward path

The proposed observer-controller schema of this Section will be submitted as a research article.

Taking into account the Case 1.3 and based on the estimation of internal signals ω_1, ω_2 we proceed to implement the ideas proposed in Section 1.1 using $\widehat{\omega}_1$ and $\widehat{\omega}_2$. Consider the observer design provided in Section 3.3 shown in Figure 3.5. First consider the general control strategy depicted in Figure 4.4, given by,

$$U(s) = C_c(s)(R(s)\eta - \hat{\omega}_1 + g_4 E_y(s)) - \hat{\omega}_2(s). \tag{4.14}$$

Then, the complete control scheme is presented in Figure 4.4. If a PI controller with two degree of freedom is used instead of a simple controller $C_c(s)$, the control law can be implemented as,

$$U(s) = R(s)\eta G_{ff}(s) - G_c(s)(\hat{\omega}_1 + g_4 E_y(s)) - \hat{\omega}_2(s) \tag{4.15}$$

where,

$$G_{ff}(s) = K(\sigma_c + \frac{1}{T_i s}) \tag{4.16a}$$

$$G_c(s) = K(1 + \frac{1}{T_i s}) \tag{4.16b}$$

The following results show that such scheme achieves step input tracking and a particular disturbance rejection action by using a PI controller with two degree of freedom, however the results can be also applied to a traditional PI controller $C_c(s)$.

Lemma 4.3.

Consider the proposed observer scheme shown in Figure 3.5. Then, there exist a PI controller with two degree of freedom given by (4.15) such that $\lim_{t\to\infty} y(t) = \alpha_r$ if $\eta = \frac{\alpha_1\alpha_2...\alpha_n}{\beta_1\beta_2...\beta_m}$, where $R(s) = \alpha_r/s$ is the step input reference and $D_0(s) = 0$.

Proof. Consider the transfer function $Y(s)/U(s)$ of the observer scheme shown in Figure 3.5 with a PI of the form (4.15). In this way, by applying the classical Final value theorem to $Y(s)$ when the input reference is given as $R(s) = \alpha_r/s$ and $\eta = \frac{\alpha_1\alpha_2...\alpha_n}{\beta_1\beta_2...\beta_m}$. It is an easy task to verify that,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \alpha_r \tag{4.17}$$

proving the result. ■

Lemma 4.4.

Consider the proposed observer scheme shown in Figure 3.5. Then, there exist a PI controller with two degree of freedom given by (4.15) such that $\lim_{t\to\infty} y(t) = 0$, where $R(s) = 0$ and $D_0(s)$ is the a step input disturbance if $g_4 = \delta_1\delta_2...\delta_{n-m}$, where δ_i are the relocated observer poles of $G_s(s)$.

Proof. Consider the transfer function $Y(s)/D_0(s)$ of the observer scheme shown in Figure 3.5 with a PI of the form (4.15). To verify the assertion of the lemma, the classical Final value theorem can be applied to $Y(s)$ when the disturbance signal is given as $D_0(s) = \beta_r/s$. It is an easy task to verify that under the condition $g_4 = \delta_1\delta_2...\delta_{n-m}$, it is obtained,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 0 \tag{4.18}$$

proving the result. ■

From the previous discussion and results, the proposed methodology is summarized as follows:

1. Make sure that the conditions of Theorem 3.5 are satisfied, that is, $G_r(s)$ a stable transfer function and $\tau_1 < \tau_{un}$ for the forward loop G_d of the plant.

2. Obtain the observer gains as follows: $G_3 = B_2$; G_2 is calculated by relocating the stable poles of $G_s(s)$ at the same position of zeros of $G_s(s)$, and the remaining stable poles should be relocated as far as possible to imaginary axe on the real half left plane (this can be done by using "place" or "acker" in Matlab); g_1 can be obtained from a frequency domain analysis. For instance, from Nyquist Diagram of the $G_u e^{-\tau_1 s}$ in cascade with a $\overline{G}_s(s) = \frac{1}{(s+\delta_1)(s+\delta_2)...(s+\delta_{n-m})}$, where δ_i are the relocated observer poles of $G_s(s)$.

3. Design of a controller $C_s(s)$ based on the unstable delay free model of the forward path $G_u(s)$. A PI or PID control based strategy can be considered.

4. If a controller with an integral action is considered, calculate the parameters

for tracking step references (η) and rejecting step disturbance (g_4) as Lemma 4.3 and Lemma 4.4 suggest.

5. Finally, implement the general control structure as it is shown in Figure 4.4.

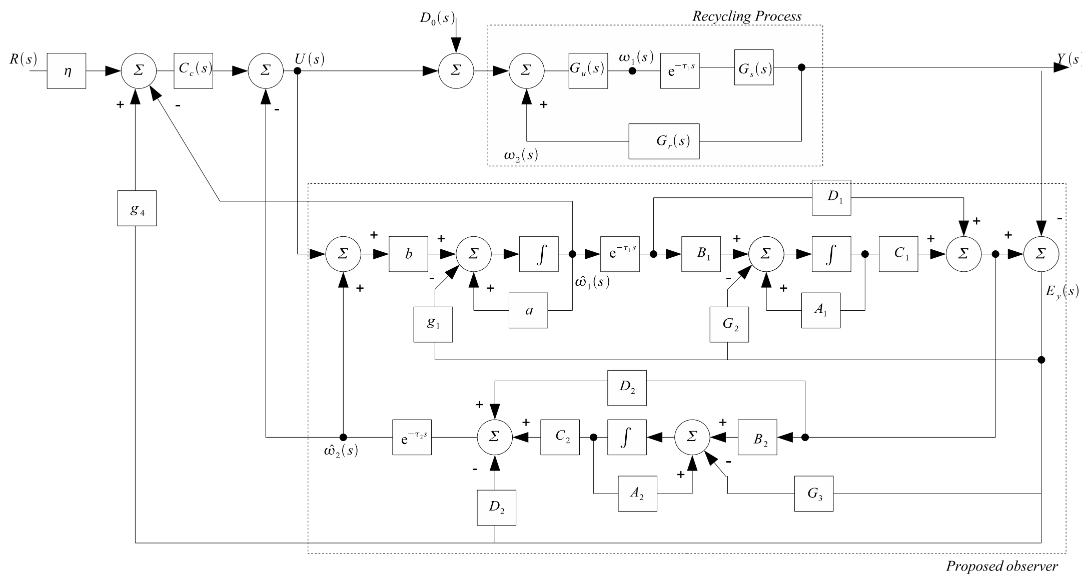

Figure 4.4: Proposed control strategy

4.4 Conclusions

In Chapter 3 the observer design for different classes of recycling systems have been provided. In this Chapter the complete control-observer strategies for recycling systems are presented. For the Case 1.1, a control law that considers the step tracking reference and rejecting disturbance rejection is provided. After this in Section 4.2, it is proposed a control law that ensures the step tracking reference for the Case 1.2. Finally in Section 4.3 it is presented an overall control-observer for a class of systems as defined in Case 1.3. This latest control-observer schema allows dealing with systems with an unstable pole, stable poles, zeros and time delay at direct loop. Therefore, this control strategy is focused to a case more general than the presented in Section 4.2. Also results concerning with the step tracking reference and step disturbance rejection are provided. A brief summary of how to apply each proposed control-observer strategy is given. The performance of the control schemas presented in this Chapter will be evaluated by means of numerical simulations in Chapter 6.

Chapter 5

Robustness

In the previous chapter, some control strategies have been presented and knowledge of the real process was considered. However, in practice, the control strategy should take into account a robustness analysis that provides at least stability limits with respect to model uncertainties. In particular, in observer design, the observer delay may be different from the system one, which may be unknown or difficult to measure. In this case, robustness analysis is done using procedure proposed in [51]. This analysis is applied under the assumption that the observer and controller for the nominal case have been designed.

Before define perturbations for the recycling system, as an example, let the nominal system be,

$$\dot{x}(t) = Ax(t) + Bx(t-\tau),$$

and the actual system as above but with τ replaced by τ_0, then $p(x(t)) = B[x(t-\tau_0) - x(t-\tau)]$. In this way, with $\theta = \tau_0 - \tau$, we get,

$$P_1(s,\theta) = B[e^{-\tau_0 s} - e^{-\tau s}] = Be^{-\tau s}[e^{-\theta s} - 1].$$

Under the same idea it is also possible to consider uncertainty on the matrix B. Let us consider the actual system matrix as B_0, then $p(x(t)) = B[x(t-\tau_0) - x(t-\tau)] + [B_0 - B]x(t-\tau_0)$ Assuming $B_\delta = B_0 - B$, it is obtained,

$$P_2(s,\theta,\delta) = e^{-\tau s}[B(e^{-\theta s} - 1) + B_\delta e^{-\theta s}].$$

The following sections presents an analysis of robustness with respect to time delay and process parameters for each control-observer strategy given in Chapter 4. The presentation of the analysis is organized as follows. For the control strategy given in Section 4.1 it is provided the complete methodology of the analysis based on [51]. Then, for the remaining cases the corresponding changes to the methodology are presented.

5.1 Robustness for the proposed control to Case 1.1

In this section the observer-controller strategy shown in Figure 4.1 (presented in Section 4.1.1) is analyzed when uncertainties in times delay are considered. Consider a state representation of the open-loop system with recycle in the nominal case (which can be obtained from the complete state representation of system-observer expressed in equation (3.2)),

$$\dot{x} = \overline{A}x + \overline{A_1}x(t-\tau_1) + \overline{A_2}x(t-\tau_2) + \overline{B}u(t) \tag{5.1a}$$

$$y = \overline{C_1}x(t-\tau_1) \tag{5.1b}$$

where

$$x = \begin{bmatrix} x_d & x_r \end{bmatrix}^T, \quad \overline{A} = \begin{bmatrix} A_d & 0 \\ 0 & A_r \end{bmatrix}, \quad \overline{A_1} = \begin{bmatrix} 0 & 0 \\ B_r C_d & 0 \end{bmatrix}, \tag{5.2a}$$

$$\overline{A_2} = \begin{bmatrix} 0 & B_d C_r \\ 0 & 0 \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} B_d \\ 0 \end{bmatrix}, \quad \overline{C_1} = \begin{bmatrix} C_d & 0 \end{bmatrix} \tag{5.2b}$$

Now, let define the perturbations on the time-delays for the recycling system. The nominal values of the delays are τ_1 and τ_2. Thus, the delay uncertainty for the recycling system is obtained as follows,

$$P(s,\theta) = \overline{A_1}e^{-s\tau_1}(e^{-s\theta_1} - 1) + \overline{A_2}e^{-s\tau_2}(e^{-s\theta_2} - 1), \tag{5.3a}$$

$$Q(s,\theta) = \overline{C_1}e^{-s\tau_1}(e^{-s\theta_1} - 1). \tag{5.3b}$$

Notice that the uncertainty expressed in (5.3) allows analyzing the plant uncertainty in both, direct and recycle paths. Furthermore, the uncertainties can be act with different proportion in times delay due to values of uncertainties θ_1 and θ_2. Also, the independent robustness analysis for each time delay τ_1 or τ_2 is possible (this can be done by removing τ_2 or τ_1 from equations (5.3a) and (5.3b), respectively). If uncertainties on both time delay and matrix ($\overline{A}$, $\overline{A}_1$ and $\overline{A}_2$) are considered, we have,

$$\begin{aligned} P(s,\theta,\delta) =& \overline{A}_\delta + e^{-s\tau_1}[\overline{A_1}(e^{-s\theta_1}-1) + \overline{A_1}_{\delta_1}e^{-s\theta_1}] + \\ & e^{-s\tau_2}[\overline{A_2}(e^{-s\theta_2}-1) + \overline{A_2}_{\delta_2}e^{-s\theta_2}], \qquad (5.4a) \\ Q(s,\theta) =& \overline{C_1}e^{-s\tau_1}(e^{-s\theta_1}-1). \qquad (5.4b) \end{aligned}$$

where, $\overline{A}$, $\overline{A_1}$ and $\overline{A_2}$ are the nominal matrix of the recycle system and the corresponding uncertainties are given by $\overline{A}_\delta$, $\overline{A_1}_{\delta_1}$ and $\overline{A_2}_{\delta_2}$. A state space representation in the Laplace domain of the controller-observer shown in Figure 4.1 can be expressed as,

$$\text{sX(s)} = \mathcal{A}\text{X(s)} + \mathcal{A}_1 e^{-s\tau_1}\text{X(s)} + \mathcal{A}_2 e^{-s\tau_2}\text{X(s)} + \mathcal{B}R(s) \qquad (5.5)$$

with X(s) $= \begin{bmatrix} e_x(s) & X(s) \end{bmatrix}^T$, $e_x(s) = \widehat{X}(s) - X(s)$,

$$\mathcal{A} = \begin{bmatrix} \overline{A} & GQ - P \\ -\overline{B}J\overline{K} & \overline{A} - \overline{B}J\overline{K} + P - \overline{B}JQ \end{bmatrix},$$

$$\mathcal{A}_1 = \begin{bmatrix} \overline{A}_1 - G\overline{C}_1 & 0 \\ \overline{B}J\overline{C}_1 & \overline{A}_1 \end{bmatrix}, \mathcal{A}_2 = \begin{bmatrix} \overline{A}_2 & 0 \\ -\overline{B}L & \overline{A}_2 - \overline{B}L \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 0 \\ \overline{B}C_c \end{bmatrix}.$$

For simplicity of the notation, it is taken into account the simplest case where G_d and G_r are first order plants. Then,

$$G = \begin{bmatrix} k & 1 \end{bmatrix}^T, \quad \overline{K} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 0 & C_r \end{bmatrix}, \qquad (5.7)$$

The characteristic equation of the system (5.5), is given by,

$$\gamma = \det \begin{bmatrix} sI - F + G\overline{C}_1 e^{-\tau_1 s} & P - GQ \\ M - \overline{B}J\overline{C}_1 e^{-\tau_1 s} & sI - F + M - P + \overline{B}JQ \end{bmatrix} \tag{5.8a}$$

$$= \det(sI - F + G\overline{C}_1 e^{-\tau_1 s}) \det(sI - F + M) \tag{5.8b}$$

$$\det(I + \psi^{-1}\Theta(s, \theta)) = 0 \tag{5.8c}$$

where

$$\gamma = \det(sI - \mathcal{A} - \mathcal{A}_1 e^{-\tau_1 s} - \mathcal{A}_2 e^{-\tau_2 s}), \tag{5.9}$$

$$F = \overline{A} + \overline{A}_1 e^{-\tau_1 s} + \overline{A}_2 e^{-\tau_2 s}, \tag{5.10}$$

$$M = \overline{B}J\overline{K} + \overline{B}L e^{-\tau_2 s}, \tag{5.11}$$

$$\tag{5.12}$$

and

$$\psi = \begin{bmatrix} sI - F + G\overline{C}_1 e^{-\tau_1 s} & 0 \\ M - \overline{B}J\overline{C}_1 e^{-\tau_1 s} & sI - F + M \end{bmatrix} \tag{5.13}$$

is the matrix corresponding to the combined observer-controller for nominal system, and $\Theta(s;\theta)$ collects the plant uncertainty.

If the closed loop quasi polynomials $\det(sI - F + G\bar{C}_1 e^{-\tau_1 s})$ and $\det(sI - F + M)$ are stable for a proper choice of G, K, L and J then the perturbed loop system remains stable if $\det(I + \psi^{-1}\Theta(s;\theta))$ does not change sign when s sweeps the imaginary axis. This yields to the criterion,

$$\begin{aligned} p_p &= \det \left[I + \begin{bmatrix} \widetilde{Q}_{pq} & \widetilde{Q}_p \end{bmatrix} \begin{bmatrix} P - GQ \\ \overline{B}JQ - P \end{bmatrix} \right] \\ &= \det \left[I + N_c(s) D_c(s) \right], \end{aligned} \tag{5.14}$$

where,

$$\widetilde{Q}_{pq} = -(sI - F + M)^{-1}(M - \overline{B}J\overline{C}_1 e^{-\tau_1 s})(sI - F + G\overline{C}_1 e^{-\tau_1 s})^{-1}, \tag{5.15}$$

$$\widetilde{Q}_p = (sI - F + M)^{-1}, \tag{5.16}$$

this only depends on the nominal system and observer/controller parameters. By using Rouche's theorem [52], it follows that the condition for stability is,

$$\|N_c(s)D_c(s,\theta)\|_\infty < 1. \tag{5.17}$$

Therefore, the applied controller-observer in the real case (5.5) preserves the closed loop stability for the uncertainties θ_1, θ_2, $\overline{A}_\delta$ $\overline{A_1}_{\delta_1}$ and $\overline{A_2}_{\delta_2}$ when (5.17) is satisfied.

The robustness analysis developed above can be summarized as follows,

1. A state space representation of the considered open-loop system is defined.

2. Perturbations on times delay are defined.

3. A state space representation of the proposed control-observer is provided. In such representation, the states vector considers the error (between the real system and the observer) and the states of the real system.

4. The characteristic equation of the proposed control-observer is also obtained.

5. From the obtained characteristic equation, it is possible to write the part that depends on the nominal model and the part that corresponds to uncertainty in a product of determinants. Then, the determinant that contains all plant uncertainty is identified.

6. Stability conditions of the perturbed observer-controller have been established. Such conditions are obtained from an analysis based on Rouche's Theorem of the determinant that contains all plant uncertainty mentioned in the previous step.

5.2 Robustness for the proposed control to Case 1.2

In this section it is presented a robustness analysis with respect to times delay uncertainties for the observer-controller strategy shown in Figure 4.3 (presented in Section 4.2). Note that the robustness analysis presented in the previous section could be applied for the strategy shown in Figure 4.3, however, in the present analysis a simpler control is used (where step tracking reference and step disturbance rejection are not addressed). As consequence, some expressions in the analysis are different with respect to the one provided in Section 5.1. Thus, consider a state representation of the open-loop system with recycle given by (5.1); the perturbations on times-delay depicted by (5.3) or uncertainties in both times-delay and parameters process given by (5.4) as in the case of the Section 5.1. Note that the state space of the open-loop system given by (5.1) is also valid for the Case 1.2, which is the case of our interest in the present analysis. Therefore, for the control-strategy analyzed in this section, a state space representation of the controller-observer shown in Figure 4.3 can be obtained as,

$$\mathrm{sX(s)} = \mathcal{A}\mathrm{X(s)} + \mathcal{A}_1 e^{-s\tau_1}\mathrm{X(s)} + \mathcal{A}_2 e^{-s\tau_2}\mathrm{X(s)} + \mathcal{B}R(s) \tag{5.18}$$

with $\mathrm{x(t)} = \begin{bmatrix} e_x(t) & x(t) \end{bmatrix}$, $e_x(t) = \widehat{x}(t) - x(x)$,

$$\begin{aligned} \mathcal{A} &= \begin{bmatrix} \overline{A} & GQ - P \\ -\overline{B}JK & \overline{A} - \overline{B}JK + P \end{bmatrix}, \mathcal{A}_1 = \begin{bmatrix} \overline{A}_1 - G\overline{C}_1 & 0 \\ 0 & \overline{A}_1 \end{bmatrix}, \\ \mathcal{A}_2 &= \begin{bmatrix} \overline{A}_2 & 0 \\ -\overline{B}L & \overline{A}_2 - \overline{B}L \end{bmatrix}, \end{aligned}$$

$G = \begin{bmatrix} k & 0 & 1 \end{bmatrix}^T$, $K = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$, $L = \begin{bmatrix} 0 & 0 & C_r \end{bmatrix}$. The characteristic equation of the system (5.18), is given by,

$$\begin{aligned} \gamma &= \det \begin{bmatrix} sI - F + G\overline{C}_1 e^{-\tau_1 s} & P - GQ \\ M & sI - F + M - P \end{bmatrix} \\ &= \det(sI - F + G\overline{C}_1 e^{-\tau_1 s}) \det(sI - F + M) \\ &\quad \det(I + \psi^{-1}\Theta(s,\theta)) = 0 \end{aligned}$$

where $\gamma = \det(sI - \mathcal{A} - \mathcal{A}_1 e^{-\tau_1 s} - \mathcal{A}_2 e^{-\tau_2 s})$, $F = \overline{A} + \overline{A}_1 e^{-\tau_1 s} + \overline{A}_2 e^{-\tau_2 s}$, $M = \overline{B}JK + \overline{B}Le^{-\tau_2 s}$ and $\psi = \begin{bmatrix} sI - F + G\overline{C}_1 e^{-\tau_1 s} & 0 \\ M & sI - F + M \end{bmatrix}$ is the matrix corresponding to the combined observer-controller for nominal system, and $\Theta(s;\theta)$ collects the plant uncertainty. By using the procedure depicted in previous section, it is not difficult to obtain the stability condition,

$$\|N_c(s)D_c(s,\theta)\|_\infty < 1. \tag{5.19}$$

with, $N_c = \begin{bmatrix} \widetilde{Q}_{pq} & \widetilde{Q}_{pq} \end{bmatrix}$, $D_c = \begin{bmatrix} P - GQ & -P \end{bmatrix}^T$ where,

$$\widetilde{Q}_{pq} = -(sI - F + M)^{-1} M (sI - F + G\overline{C}_1 e^{-\tau_1 s})^{-1}, \tag{5.20a}$$

$$\widetilde{Q}_p = (sI - F + M)^{-1}, \tag{5.20b}$$

5.3 Robustness for the proposed control to Case 1.3

In this section a robustness analysis for the observer-controller shown in Figure 4.4 (provided in Section 4.3) is addressed. Since the observer-controller strategy deals with a more general case of the recycling systems than in previous Sections (4.1 and 4.2), a more general state space representation of the open-loop system with recycle is required. Therefore, from the complete state representation of observer-predictor given by equation (3.41) a state space representation of the open-loop system with recycle in the nominal case can be expressed as,

$$\dot{x} = \overline{A}x + \overline{A_1}x(t-\tau_1) + \overline{A_2}x(t-\tau_2) + \overline{A_3}x(t-\tau_3) + \overline{B}u(t) \tag{5.21a}$$

$$y = \overline{C}x(t) + \overline{C_1}x(t-\tau_1) \tag{5.21b}$$

where $\tau_3 = \tau_1 + \tau_2$, $x = \begin{bmatrix} x_d & x_r \end{bmatrix}^T$, $\overline{C} = \begin{bmatrix} \mathcal{C} & 0 \end{bmatrix}$, $\overline{C_1} = \begin{bmatrix} \mathcal{C}_1 & 0 \end{bmatrix}$,

$$\overline{A} = \begin{bmatrix} \mathcal{A} & 0 \\ B_2\mathcal{C} & A_2 \end{bmatrix}, \overline{A_1} = \begin{bmatrix} \mathcal{A}_1 & 0 \\ B_2\mathcal{C}_1 & 0 \end{bmatrix},$$

$$\overline{A_2} = \begin{bmatrix} \mathcal{B}D_2\mathcal{C} & \mathcal{B}C_2 \\ 0 & 0 \end{bmatrix}, \quad \overline{A_3} = \begin{bmatrix} \mathcal{B}D_2\mathcal{C}_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} \mathcal{B} \\ 0 \end{bmatrix},$$

Let define the perturbations on the time-delays for the recycling system. The nominal values of the delays are τ_1 and τ_2. Thus, the delay uncertainty for the recycling system is obtained as follows,

$$\begin{aligned} P(s,\theta) &= \overline{A_1}e^{-s\tau_1}(e^{-s\theta_1}-1) + \overline{A_2}e^{-s\tau_2}(e^{-s\theta_2}-1) \\ &\quad + \overline{A_3}e^{-s\tau_3}(e^{-s\theta_3}-1), && (5.23a) \\ Q(s,\theta) &= \overline{C_1}e^{-s\tau_1}(e^{-s\theta_1}-1). && (5.23b) \end{aligned}$$

with $\theta_3 = \theta_1 + \theta_2$. If uncertainties on both time delay and matrix ($\overline{A}$, $\overline{A}_1$, $\overline{A}_2$ and $\overline{A}_3$) are considered, we have,

$$\begin{aligned} P(s,\theta,\delta) =& \overline{A}_\delta + e^{-s\tau_1}[\overline{A_1}(e^{-s\theta_1}-1) + \overline{A_{1\delta_1}}e^{-s\theta_1}] \\ &+ e^{-s\tau_2}[\overline{A_2}(e^{-s\theta_2}-1) + \overline{A_{2\delta_2}}e^{-s\theta_2}] \\ &+ e^{-s\tau_3}[\overline{A_3}(e^{-s\theta_3}-1) + \overline{A_{3\delta_3}}e^{-s\theta_3}], && (5.24a) \\ Q(s,\theta) =& \overline{C_1}e^{-s\tau_1}(e^{-s\theta_1}-1). && (5.24b) \end{aligned}$$

where, $\overline{A}$, $\overline{A_1}$, $\overline{A_2}$ and $\overline{A_3}$ are the nominal matrix of the recycle system and the corresponding uncertainties are given by $\overline{A}_\delta$, $\overline{A_{1\delta_1}}$, $\overline{A_{2\delta_2}}$ and $\overline{A_{3\delta_3}}$.

A state space representation in the Laplace domain of the controller-observer shown in Figure 4.4 can be expressed as,

$$\begin{aligned} \text{sX(s)} &= \overline{\mathbf{A}}\text{X(s)} + \overline{\mathbf{A}}_1 e^{-s\tau_1}\text{X(s)} + \overline{\mathbf{A}}_2 e^{-s\tau_2}\text{X(s)} + \overline{\mathbf{A}}_3 e^{-s\tau_2}\text{X(s)} \qquad (5.25) \\ &\quad + \overline{\mathbf{B}}R(s) \end{aligned}$$

with $\text{X(s)} = \begin{bmatrix} e_x(s) & X(s) \end{bmatrix}^T$, $e_x(s) = \widehat{X}(s) - X(s)$,

$$\overline{\mathbf{A}} = \begin{bmatrix} \overline{A} - G\overline{C} & GQ - P \\ \overline{B}C_c g_4 \overline{C} - \overline{B}C_c\overline{K} & \overline{A} - \overline{B}C_c\overline{K} + P - \overline{B}C_c g_4 Q \end{bmatrix},$$

$$\overline{\mathbf{A}}_1 = \begin{bmatrix} \overline{A}_1 - G\overline{C}_1 & 0 \\ \overline{B}C_c g_4 \overline{C}_1 & \overline{A}_1 \end{bmatrix}, \overline{\mathbf{A}}_2 = \begin{bmatrix} \overline{A}_2 - \overline{B}D_2\overline{C} & \overline{B}D_2 Q \\ -\overline{B}L & \overline{A}_2 - \overline{B}L \end{bmatrix},$$

$$\overline{\mathbf{A}}_3 = \begin{bmatrix} \overline{A}_3 - \overline{B}D_2\overline{C}_1 & 0 \\ 0 & \overline{A}_3 \end{bmatrix}, \overline{\mathbf{B}} = \begin{bmatrix} 0 \\ \overline{BC}\eta \end{bmatrix}.$$

where $G = \begin{bmatrix} g_1 & G_2 & 1 \end{bmatrix}^T, \overline{K} = \begin{bmatrix} 1 & \mathbf{0} \end{bmatrix}$ and $L = \begin{bmatrix} \mathbf{0} & C_2 \end{bmatrix}$. The characteristic equation of the system (5.25), is given by,

$$
\begin{aligned}
\gamma = \det &\begin{bmatrix} sI - F + M & T_{PQ} \\ V & sI - F + N + N_{PQ} \end{bmatrix} \\
&= \det(\psi(I + \psi^{-1} \begin{bmatrix} 0 & T_{PQ} \\ 0 & N_{PQ} \end{bmatrix})) \\
&= \det(sI - F + M) \det(sI - F + N) \\
&\det(I + \psi^{-1}\Theta(s, \theta)) = 0
\end{aligned} \tag{5.27a}
$$

where

$$
\begin{aligned}
\gamma &= \det(sI - \overline{\mathbf{A}} - \overline{\mathbf{A}}_1 e^{-\tau_1 s} - \overline{\mathbf{A}}_2 e^{-\tau_2 s} - \overline{\mathbf{A}}_3 e^{-\tau_3 s}), \\
F &= \overline{A} + \overline{A}_1 e^{-\tau_1 s} + \overline{A}_2 e^{-\tau_2 s} + \overline{A}_3 e^{-\tau_3 s}, \\
M &= G(\overline{C} + \overline{C}_1 e^{-\tau_1 s}) + \overline{B} D_2 (\overline{C} e^{-\tau_2 s} + \overline{C}_1 e^{-\tau_3 s}), \\
N &= \overline{B} C_c \overline{K} + \overline{B} L e^{-\tau_2 s}, \\
N_{PQ} &= \overline{B} C_c g_4 Q - P, \\
T_{PQ} &= P - GQ - \overline{B} D_2 Q e^{-\tau_2 s}, \\
V &= \overline{B} C_c (\overline{K} - g_4 \overline{C} - g_4 \overline{C}_1 e^{-\tau_1 s}) + \overline{B} L e^{-\tau_2 s},
\end{aligned}
$$

and

$$
\psi = \begin{bmatrix} sI - F + M & 0 \\ V & sI - F + N \end{bmatrix}
$$

is the matrix corresponding to the combined observer-controller for nominal system, and $\Theta(s;\theta)$ collects the plant uncertainty. Following the procedure suggested in Section 5.1, it is not difficult to obtain the stability condition,

$$
\|N_c(s) D_c(s, \theta)\|_\infty < 1. \tag{5.29}
$$

where $N_c = \begin{bmatrix} \widetilde{Q}_1 & \widetilde{Q}_2 \end{bmatrix}$, $D_c = \begin{bmatrix} T_{PQ} & N_{PQ} \end{bmatrix}^T$ and

$$\widetilde{Q}_1 = -(sI - F + N)^{-1} V (sI - F + M)^{-1}, \tag{5.30a}$$

$$\widetilde{Q}_2 = (sI - F + N)^{-1}, \tag{5.30b}$$

Therefore, the applied controller-observer in the real case (5.25) preserves the closed loop stability for the uncertainties $\overline{A}_\delta$ $\overline{A_1}_{\delta_1}$, $\overline{A_2}_{\delta_2}$, $\overline{A_3}_{\delta_3}$, θ_1, θ_2 and θ_3 when (5.29) is satisfied.

Chapter 6

Simulation Results

6.1 Case 1.1.

In this section, some academic examples show the performance of observer based control strategies proposed in Section 4.1

6.1.1 Proposed control schema for $\tau_1 < \tau_{un}$

Example 6.1 *In this example the control schema shown in Figure 4.1 is used. Then, let us consider a simple reaction $A \rightarrow B$ in a reactor-separator system with recycle. We now consider a reactor-separator system as discussed in [53] by considering times-delay at direct and forward loops. In this way, the open-loop linear model can be written in the form of (5.1) where the state x_d is the temperature of the reactor (T) and x_r is the concentration of component A (C_A). The manipulated input is the jacket reactor temperature (T_j). Also, the constant matrix are given by,*

$$\bar{A} = \begin{bmatrix} \frac{F}{V}(1-\lambda) - \frac{F}{V} - \frac{UA}{V\rho c_p} + \frac{(-\Delta H)}{\rho c_p} C_{As} k_{ps} & 0 \\ 0 & \frac{F}{V}(1-\lambda) - \frac{F}{V} + k_s \end{bmatrix}, \; \bar{A}_1 = \begin{bmatrix} 0 & 0 \\ C_{As} k_{ps} & 0 \end{bmatrix},$$

$$\bar{A}_2 = \begin{bmatrix} 0 & 0 \\ C_{As} k_{ps} & 0 \end{bmatrix}, \; \bar{B} = \begin{bmatrix} \frac{UA}{V\rho c_p} \\ 0 \end{bmatrix}$$

where $k_s = k_0 \exp(-E_a/RT_s)$ and $k_{ps} = k_s(E_a/RT_s^2)$. It is assumed that τ_1 is the time delay due to temperature measurement and τ_2 the time lag by transport. The

Operating Volume (V)	500 ft^3
Operating Flowrate (F)	2000 ft^3/hr
Reactor Diameter (D_r)	7.5 ft
Overall heat-transfer coefficient (U)	492.3192 $Btu/(hr\ ft^2\ {}^{\circ}F)$
Heat transfer area through reactor wall (A)	47.1238 ft^2
Preexponential factor (k_0)	$16.96x10^{12}$ hr^{-1}
Activation energy (E_a)	32400 $Btu/lbmol$
Ideal gas constant (R)	1.987 $Btu/lbmol^{\circ}F$
Heat of reaction ($-\Delta H$)	39000 $Btu/lbmol\ PO$
Density of coolant (ρ)	53.25 lb/ft^3
Heat capacity of coolant (c_p)	1 $Btu/(lb^{\circ}F)$
Operating concentration (C_{As})	0.066 $lbmol/ft^3$
Operating temperature (T_s)	$560.77^{\circ}R$
Forward loop time delay (τ_1)	0.1 hr
Backward loop time delay (τ_2)	0.2 hr
Recirculation coefficient (λ)	0.5

Table 6.1: Constant parameters for Example 6.1

output matrix is $\bar{C}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$, *since it is considered temperature (first state) to be measured. The transfer functions of direct and recycle paths are given by,*

$$G_d = \frac{\overline{B}(1,1)}{s + \overline{A}(1,1)} e^{-\tau_1 s}, \quad G_r = \frac{\overline{B}(1,1)^{-1}\overline{A}_1(2,1)\overline{A}_2(1,2)}{s + \overline{A}(2,2)} e^{-\tau_2 s} \tag{6.1}$$

Taking into account the parameters given by Table 6.1 it is possible to apply the proposed control schema shown in Figure 4.1 as follows. Since conditions of Theorem 3.1 are satisfied a set of proportional gain can be calculated as $8.18 < k < 13.19$ *(see Corollary 2.1 and Remark 2.1). For the simulation it is used* $k = 9$*. The two degree of freedom PI controller is implemented with* $K = 41$, $\sigma_c = 0.5$ *and* $T_i = 0.142$.

In order to analyze the robustness with respect to time delays, Figure 6.1 shows the stability condition given by (5.17), where the uncertainties in time delays $\theta_1 = 0.012$ *and* $\theta_2 = 0.04$ *are considered. In this case, such combination of uncertainties gives as result a stable closed loop system since* $\|N_c(s)D_c(s,\theta)\|_\infty = 0.9872 < 1$*. Taking into account the time delay uncertainty mentioned above, also in Figure 6.1 it is presented the stability condition (5.17) when all parameters of the model are different from the* *nominal. In this case, we consider the following uncertainties,*

$$\overline{A}_{\delta} = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.08 \end{bmatrix}, \quad \overline{A}_{1\delta_1} = \begin{bmatrix} 0 & 0 \\ -0.11 & 0 \end{bmatrix}, \quad \overline{A}_{2\delta_2} = \begin{bmatrix} 0 & 0.025 \\ 0 & 0 \end{bmatrix}, \tag{6.2}$$

As it is seen, this set of uncertainty satisfies the stability condition (5.17) and therefore the stability of the closed loop system is also assured. Now, in order to evaluate the output signal evolution, some numerical simulations are presented. It is considered a positive unit step input and initial conditions in the process and the observer of magnitude 0.1 *and* 0.2 *units, respectively. In Figure 6.2, a continuous line shows the output response when it is considered the exact knowledge of the model parameters; a dashed line presents the output signal when the time delays* τ_1 *and* τ_2*, are increased by* 10%*, respectively; uncertainties on the poles of the process are considered i.e., the actual direct unstable pole* $s_1 = 7.3$ *and the actual stable recycle pole* $s_2 = -6.2$ *(when the nominal poles are* $s_1 = 7.1318$ *and* $s_1 = -6$*). Also, a step disturbance* $d(t)$ *with a magnitude of* 0.3 *acting at* $3hr$ *is considered. Then, Figure 6.3 shows the corresponding estimation error at the output* $e_y(t)$*, when it is considered exact knowledge of the model parameters, a positive unit step input and initial conditions in the process and the observer of magnitude* 0.1 *and* 0.2 *units, respectively. From Figure 6.2 and Figure 6.3 it can be seen the observer predictor convergence and the adequate behavior of the output response.*

Example 6.2 *Consider now the recycled system (1.1) with,*

$$G_d = \frac{1}{s-0.25}e^{-2s}, \quad G_r = \frac{10}{(s+1)(s+2)}e^{-2s}. \tag{6.3}$$

In this case, the observer design is implemented by considering $k = 0.3$*. The control feedback (4.2) is obtained by considering,*

$$G_{ff}(s) = 1.5\left(0.4 + \frac{1}{4s}\right) \text{ and } G_c(s) = 1.5\left(1 + \frac{1}{4s}\right) \tag{6.4}$$

Let us consider that time-delays uncertainties act with $\theta_1 = 0.016$ *and* $\theta_2 = 0.05$*. Thus, the stability condition given by (5.17) is shown in Figure 6.4. As it is seen, such combination of uncertainties gives a stable closed loop system since*

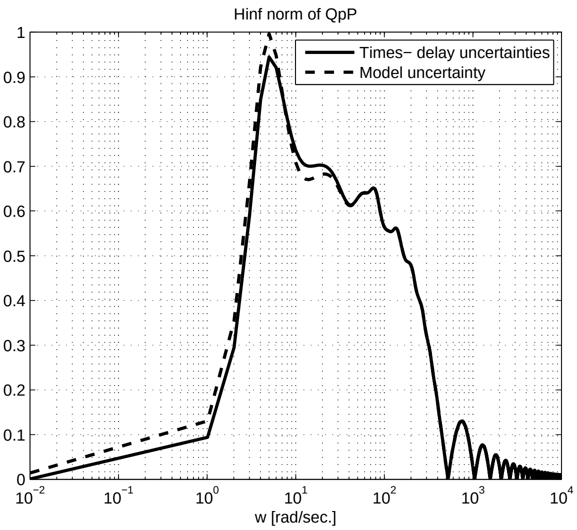

Figure 6.1: $\|N_c(s)D_c(s,\theta)\|_\infty$ for $\theta_1 = 0.012$ and $\theta_2 = 0.04$.

$\|N_c(s)D_c(s,\theta)\|_\infty = 0.9840 < 1$.

Then, it is considered a step disturbance $D(s) = -0.05$ *acting at* $t = 40$ *sec. Figure 6.5 shows the evolution of the output signal when considering a zero initial conditions (continuous line) and for the case when the initial condition in the recycle path is set at 0.01 (dashed line). Figure 6.6 shows the error output signal* $e_{\omega_2}(t)$ *when it is considered* $D(s) = 0$ *and a small initial condition of magnitude 0.07 is presents in the backward path process. Again in figures 6.5 and 6.6 the observer predictor convergence and good control performance is provided.*

6.1.2 Proposed control schema for $\tau_1 < 2\tau_{un}$.

Here the observer-controller shown in Figure 4.2 is used.

Example 6.3 *Consider the process is given by,*

$$G_d = \frac{3}{(s-0.1)}e^{-15s}, \tag{6.5a}$$

$$G_r = \frac{4(s+10)}{(s+0.5)(s+2)}e^{-3s}. \tag{6.5b}$$

The observer parameters are given as $g_1 = 0.65$,$g_2 = 0.056$ *and* $\beta_d = 1.65$. *Also the*

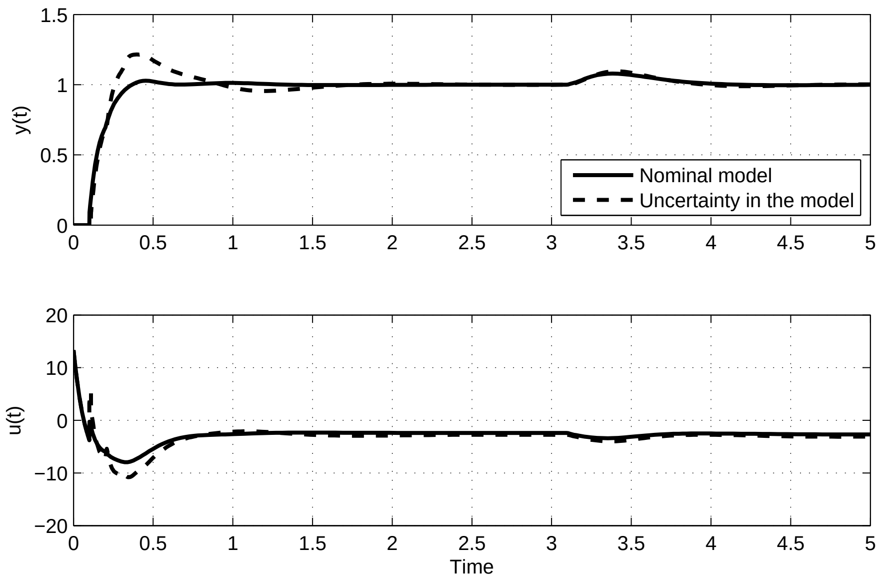

Figure 6.2: Output and control signals for different initial condition.

control given by (4.12) is implemented with $K = 0.2$, $T_i = 10$ *and* $\sigma_c = 0.1$. *Thus, the output of the system is shown in Figure 6.7 under initial conditions equal to zero as well as different from zero i.e., when the initial condition in the forward loop of the process is* 0.01. *Also, small step disturbance is considered acting at* 200 *sec. As it is seen a robustness analysis for the control strategy used in this section is not provided. However, in Figure 6.8 it is shown the output signal when forward time-delay of the process is* $\tau_1 = 15.3$. *Finally, the convergence of the signals is presented in Figure 6.9, where the initial condition of* 0.001 *in both forward and backward paths.*

6.2 Case 1.2.

In this section, an academic example show the performance of observer based control strategy proposed in Section 4.2 and shown in Figure 4.3.

Example 6.4 *Consider the recycled time delay system given by,*

$$G_d = \frac{1}{(s-1)(s+10)}e^{-0.5s}, \quad G_r = \frac{1}{s+1}e^{-2s}. \tag{6.6}$$

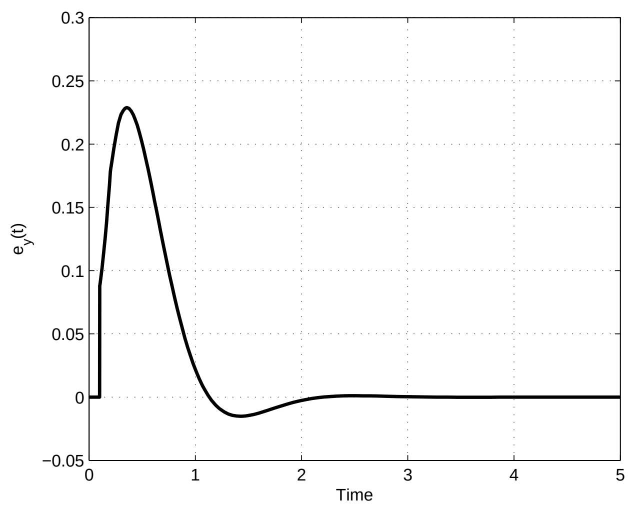

Figure 6.3: Estimation error $e_y(t)$ by considering different initial condition.

Following the procedure above described, it is obtained a proportional gain $k = 12$. *The free delay direct path can be stabilized by a two degree of freedom PID [48], obtaining a general feedback of the form,*

$$U(s) = R(s)G_{ff}(s) - G_c(s)\widehat{\omega}_1(s) - \widehat{\omega}_2(s) \tag{6.7}$$

with

$$G_{ff}(s) = 160\left(0.3 + \frac{1.6}{s} + 0.1s\right), \tag{6.8}$$

$$G_c(s) = 160\left(1 + \frac{1.6}{s} + 0.1s\right). \tag{6.9}$$

In order to analyze the robustness with respect to time delays, Fig. 6.10 shows the stability condition given by (5.19), where the uncertainties in time delays $\theta_1 = 0.005$ *and* $\theta_2 = 0.23$ *are considered. In this case, such combination of uncertainties gives as result a stable closed loop system since* $\|N_c(s)D_c(s,\theta)\|_\infty = 0.9903 < 1$.

Now, in order to evaluate the output signal evolution some numerical simulations are presented. It is considered a positive unit step input and initial conditions in the process and the observer of magnitude 0.4 *units and* 0.2 *units, respectively. In Figure 6.11, a continuous line shows the output response when it is considered the*

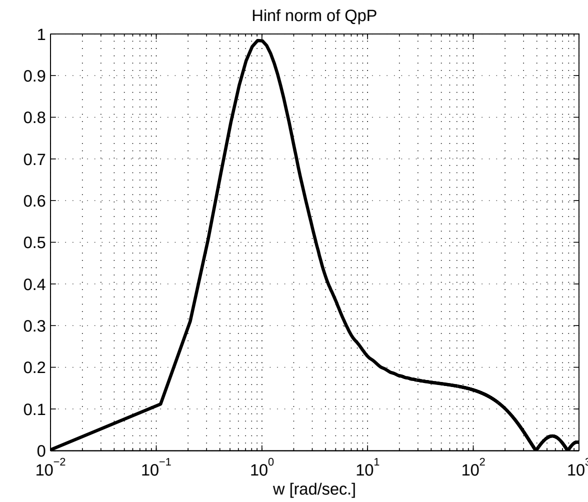

Figure 6.4: $\|N_c(s)D_c(s,\theta)\|_\infty$ for $\theta_1 = 0.016$ and $\theta_2 = 0.05$

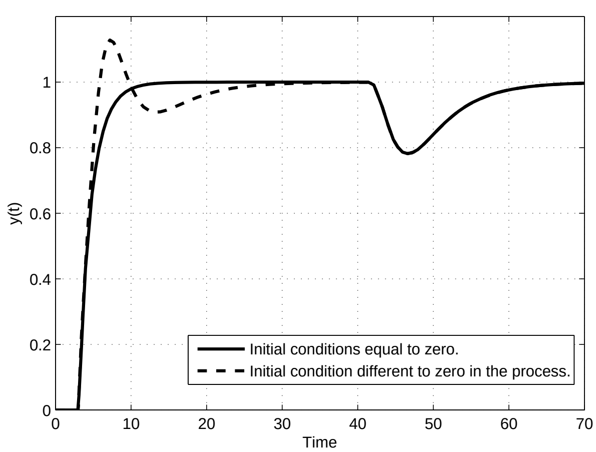

Figure 6.5: Output signal in example 6.2

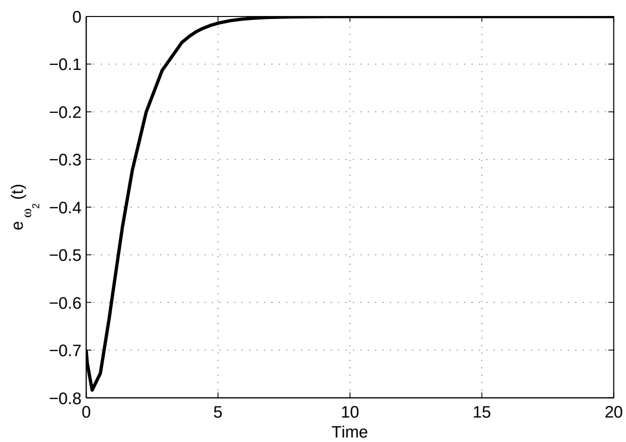

Figure 6.6: Estimation error $e_\omega(t)$, example 6.2

exact knowledge of the model parameters; a dashed line presents the output signal when the time delays τ_1 *and* τ_2*, are increased by* 8% *and* 15%*, respectively. Then, Figure 6.12 shows the corresponding estimation error at the output* $e_y(t)$*, for the mentioned cases. From Figure 6.11 and Figure 6.12 it can be seen the observer predictor convergence and the well behavior of the control based on estimated signals.*

6.3 Case 1.3.

In this Section the proposed control schema shown in Figure 4.4 is evaluated by means two academic examples.

Example 6.5 *Consider the recycled time delay system given by,*

$$G_d = \frac{1}{(s-1)(s+2)(s+3)}e^{-0.7s} \tag{6.10a}$$

$$G_r = \frac{1}{s+2}e^{-s}. \tag{6.10b}$$

In what follows the gains of the observer are calculated. A state space representation

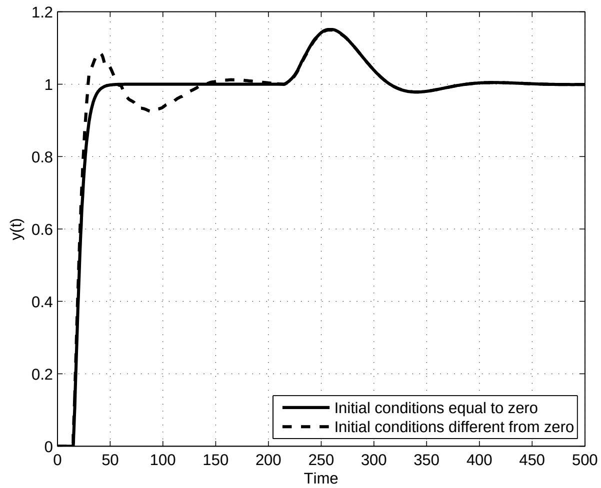

Figure 6.7: Output signal, Example 6.3.

of the stable part of G_d *is given by (3.30), with*

$$A_1 = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix} B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{6.11a}$$

$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} D_1 = 0 \tag{6.11b}$$

The stable poles of (6.11) are relocated at $\delta_1 = 100$ *and* $\delta_2 = 100$ *with* $G_2 = \begin{bmatrix} 195 & 9409 \end{bmatrix}^T$. *A state space representation of the recycle loop can be given as* $A_2 = -2$, $B_2 = 1$, $C_2 = 1$ *and* $D_2 = 0$. *Thus the observer gain* G_3 *is given as* $G_3 = B_2 = 1$ *and* $g_1 = 12000$. *It is proposed to use a PI with two degree of freedom. Therefore the control law (4.15) is implemented with* $g_4 = 10000$, $\eta = 6$, $K = 6$, $T_i = 0.96$ *and* $\sigma_c = 0.2$. *In order to show the behavior of the output signal it is considered perfect knowledge of the process and a step disturbance with a magnitude of* 0.25 *acting at* 25 *sec. Hence, in Figure 6.13, it is presented a comparison when initial conditions equal to zero are set against initial condition different from zero i.e., it is considered an initial condition of* 0.2 *at forward loop and and* 0.5 *at recycle path. Under these conditions, in Figure 6.14, it is presented the otput performance when uncertainties in both times-delay of* $\pm 5\%$ *and* $\pm 10\%$ *are considered. Finally, in order to show the effectiveness of the convergence in Figure 6.15 the errors in the*

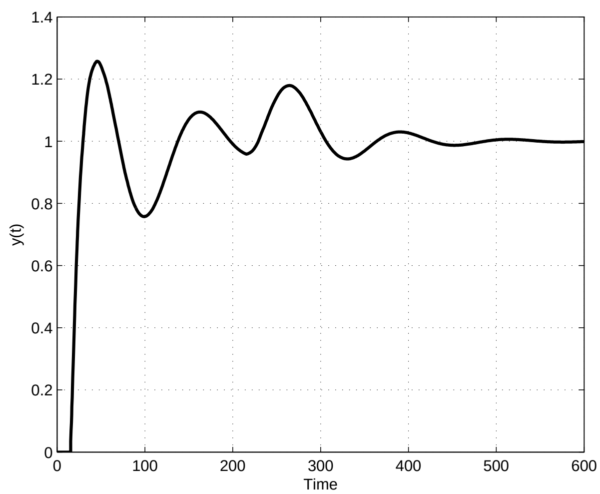

Figure 6.8: Output signal evolution, example 6.3

output $e_y(t)$ *and an internal signal* $e_{\omega_2}(t)$ *are presented when only initial conditions different from zero are used.*

Example 6.6 *Consider the recycle system given by (1.1), with,*

$$G_d = \frac{3(s+1)}{(s-1)(s+2)(s+3)} e^{-0.5s} \tag{6.12a}$$

$$G_r = \frac{10(s+1)}{(s+2)(s+3)} e^{-0.5s} \tag{6.12b}$$

The observer gains are calculated as follows. Let consider a state space representation of the stable part of G_d*, which can be expressed as (3.30), with*

$$A_1 = \begin{bmatrix} -5 & -6 \\ 1 & 0 \end{bmatrix} B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{6.13a}$$

$$C_1 = \begin{bmatrix} 1 & 1 \end{bmatrix} D_1 = 0 \tag{6.13b}$$

The stable poles of (6.13) are relocated at $\{-1, -100\}$ *i.e.,* $\delta_1 = 1$ *and* $\delta_2 = 100$ *with*

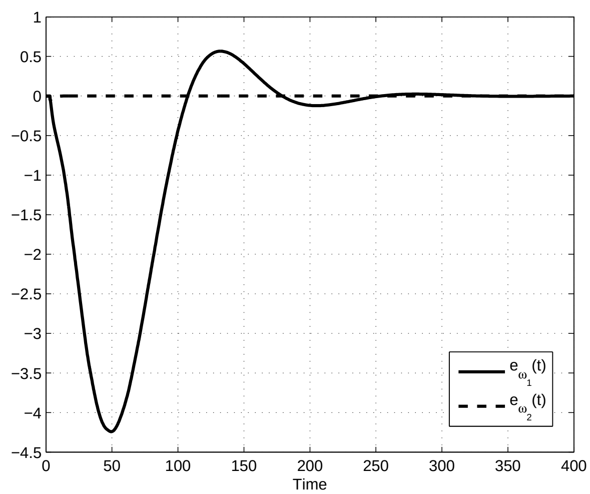

Figure 6.9: Output signal, Example 6.3.

$G_2 = \begin{bmatrix} 95 & 1 \end{bmatrix}^T$. *A state space representation of the recycle loop can be given as,*

$$A_2 = \begin{bmatrix} -5 & -6 \\ 1 & 0 \end{bmatrix} B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{6.14a}$$

$$C_2 = \begin{bmatrix} 10 & 10 \end{bmatrix} D_2 = 0 \tag{6.14b}$$

Thus the observer gain G_3 *is given as* $G_3 = B_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ *and* $g_1 = 50$ *is chosen from* $33.5 < g_1 < 83$. *It is proposed to use a PI with two degree of freedom. Therefore the control law (4.15) is implemented with* $g_4 = 100$, $\eta = 6$, $K = 2$, $T_i = 1$ *and* $\sigma_c = 0.1$.

Hence, the output signal is evaluated by considering perfect knowledge of the process and a step disturbance with a magnitude of 0.2 *acting at* 10 *sec. In Figure 6.16, it is presented a comparison when initial conditions are equal to zero against initial condition different from zero i.e., it is considered an initial condition of* 0.1 *at forward loop. Also, under these conditions, in Figure 6.17, it is presented the output performance when uncertainties on both times-delay of* $\pm 5\%$ *are considered. As it is seen from Figure 6.17, the system remains stable even when some uncertainties in the process are set. Finally, in order to show the effectiveness of the convergence in Figure 6.18 the errors in the output* $e_y(t)$ *and an internal signal* $e_{\omega_2}(t)$ *are presented*

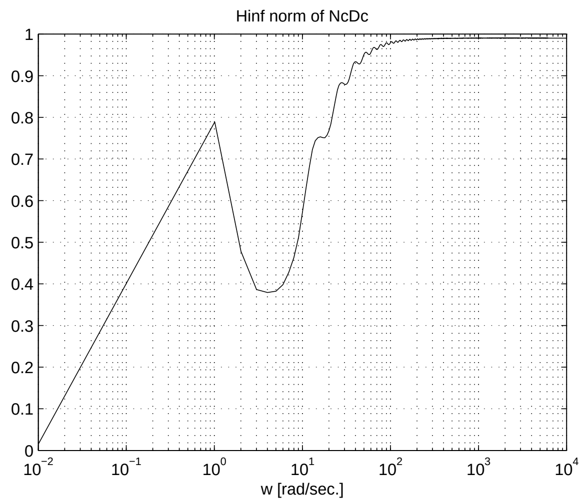

Figure 6.10: $\|N_c(s)D_c(s,\theta)\|_\infty$ for $\theta_1 = 0.005$ and $\theta_2 = 0.23$

when only initial conditions different from zero are used i.e., 0.1 *at forward loop and* 0.1 *at backward loop in the process.*

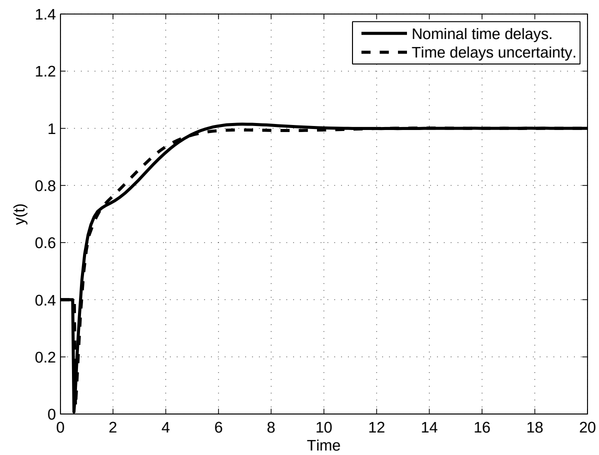

Figure 6.11: Output signal with different initial condition in process and observer.

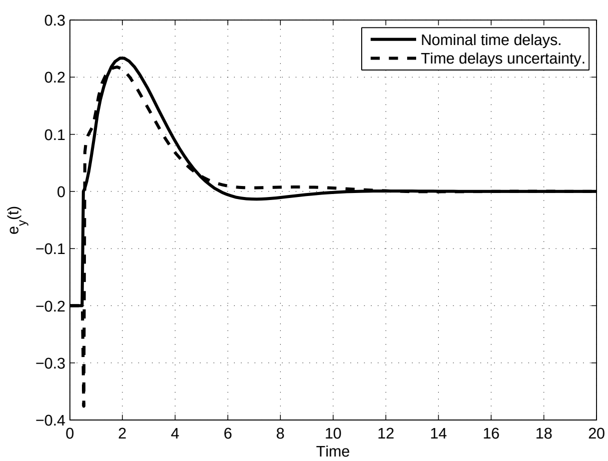

Figure 6.12: Estimation error $e_y(t)$ by considering different initial conditions.

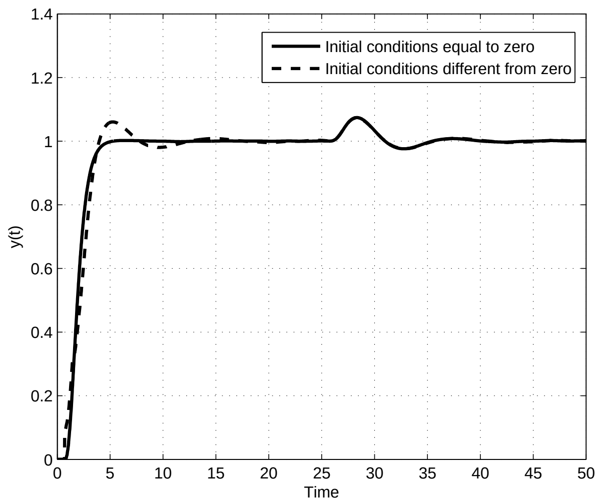

Figure 6.13: Output signal for Example 6.5.

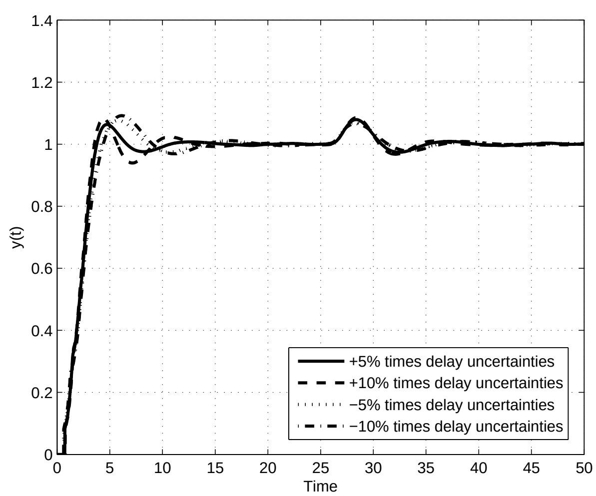

Figure 6.14: Output signal for Example 6.5.

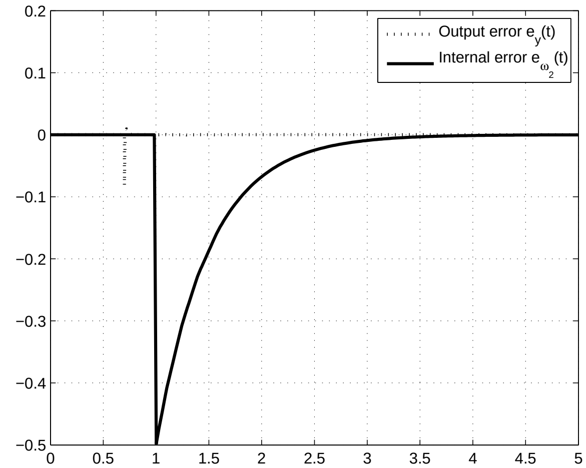

Figure 6.15: Errors $e_y(t)$ and $e_{\omega_2}(t)$ for Example 6.5.

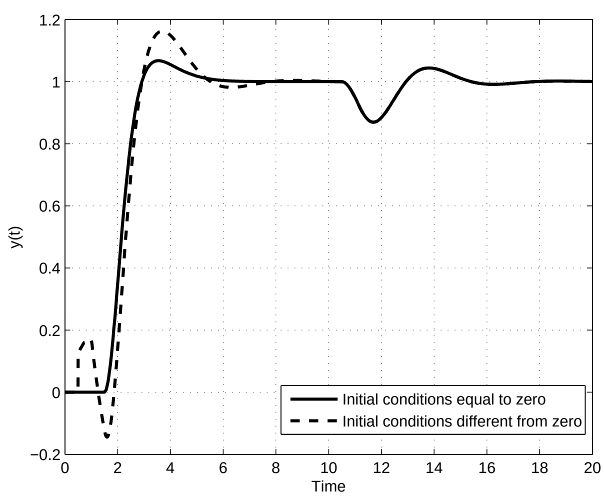

Figure 6.16: Output signal evolution, Example 6.6.

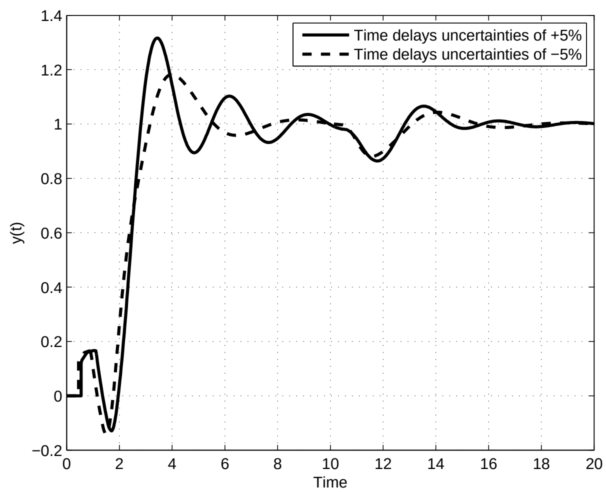

Figure 6.17: Output signal when time-delays uncertainties are considered.

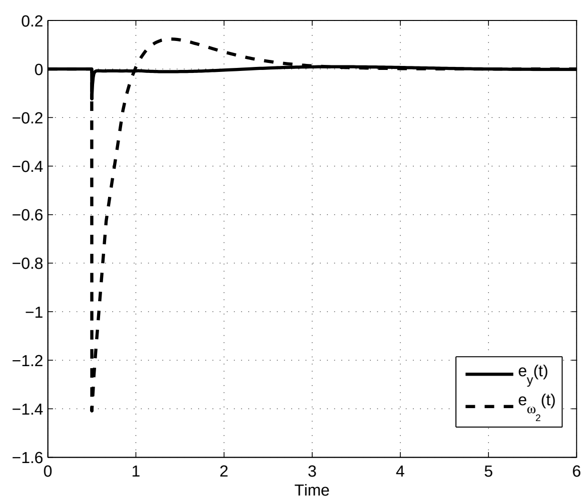

Figure 6.18: Estimation errors, Example 6.6.

Chapter 7

General Conclusions

This work presents new results on design of observer-controller for systems with time delay. As preliminary results, some existing results concerning with the static output feedback are presented in Chapter 2. Also, in Chapter 2 four new control schemas for systems with time delay at the input/output are presented. In particular, the case of unstable FOPTD systems, special attention to large time delay is paid. The first control strategy deals with systems satisfying $\tau < \tau_{un}$. Then, two control strategies (referred as second and third, respectively) allows to stabilize systems satisfying $\tau < 2\tau_{un}$. The second control schema is based on a special output feedback (Section 2.4.1). Although in second proposed control schema step disturbance rejection and step tracking reference are not easy to implement, the simplicity of the structure and design is notably when is compared with other recent proposals in literature. Then, based on an observer approach a third control strategy is given (Section 2.4.2). Such control strategy overcomes the limitations of the step disturbance rejection and step tracking reference imposed by the second proposal. With a different approach a fourth control strategy (Section 2.5) is proposed, which is also based on an observer design together with a P, PI or PID controller. This strategy allows to stabilize systems satisfying $\tau < 3\tau_{un}$ when a P or PI controller is used. Then, if a PID controller is regarded the stability condition is improved to $\tau < 4\tau_{un}$. It should be highlighted that up to our best knowledge this latest stability condition has not been obtained in recent literature. Also, step tracking reference and disturbance rejection is also solved for this particular schema. In order to show the effectiveness of the proposed control schemas, some numerical simulations are presented. In Table 7 it is presented a summary of the proposed control schemas mentioned above.

On the other hand, as main results of this work, control schemas for recycling systems with time delay at both forward and backward loops are presented. Using recycle in unstable processes with significant time delay leads to a challenging control problem. In this work this problem has been addressed for three particular cases where one unstable pole with significant delay is present in the forward path. The presentation of the results is provided as follows. In Chapter 3 the prediction strategies are obtained where explicit conditions for the construction of a stabilizing observer based controller scheme for such class of systems are presented. The obtained stability conditions depends on the unstable time constant and time delay associated to direct path of recycling system (see Table 7). The observer-prediction strategy is used in all cases to estimate some internal variables of the process required for: i) remove the dynamics of backward loop in the recycling process and ii) design a stabilizing control law for the free delay model of the forward path. Then, in Chapter 4 the control laws are proposed. As a result, in Chapter 4 the complete proposed observer-controller strategies are given.

It should be pointed out that the developed control strategies are for different class of recycling systems at direct loop. In this way, the observer design for each case is also different. The first proposed schema considers unstable FOPTD system at direct loop, achieving stabilize systems satisfying $\tau_1 < \tau_{un}$ (Section 4.1.1) and $\tau_1 < 2\tau_{un}$ (Section 4.1.2). The second control strategy (Section 4.2) can be used for systems with an unstable pole, several stable poles and time delay at forward loop. Finally in order to improve the stability conditions and to consider a more general case of recycling systems (with respect to direct loop), a third control schema (Section 4.3) is provided. Such control strategy allows dealing with systems with one unstable pole, several stable poles, several zeros in left half plane and time delay at forward loop.

In order to illustrate the improvement in the observer design of the third control strategy with respect to the second one, let consider the case of system with one unstable pole, several stable poles and time delay at forward path. This is, any of the two control structures can be applied for this task. Hence, for the second

Table 7.1: Summary of the proposed control strategies.

System	Stability Condition	Approach strategy	Tracking reference	Disturbance rejection	Robustness analysis
Unstable FOPTD system $G = \frac{b}{s-a}e^{-\tau s}$ with $\tau_{un} = 1/a$	$\tau < 2\tau_{un}$	Output feedback			✓
		Observer and PI	✓	✓	✓
	$\tau < 3\tau_{un}$	Observer and P or PI	✓	✓	
	$\tau < 4\tau_{un}$	Observer and PID	✓	✓	
Recycle system $G_d = \frac{b}{s-a}e^{-\tau_1 s}$ $G_r = \frac{N(s)}{D(s)}e^{-\tau_2 s}$	$\tau_1 < \tau_{un}$	Observer-Controller	✓	✓	✓
	$\tau_1 < 2\tau_{un}$		✓	✓	
Recycle system $G_d = \frac{b}{(s-a)(s+\bar{b}_1)...(s+\bar{b}_m)}e^{-\tau_1 s}$ $G_r = \frac{N(s)}{D(s)}e^{-\tau_2 s}$	$\tau_1 < \tau_{un} - \sum_{i=1}^{m}\frac{1}{b_i}$				✓
Recycle system $G_d = \frac{N_1(s)}{D_1(s)}\frac{b}{s-a}e^{-\tau_1 s}$ $G_r = \frac{N(s)}{D(s)}e^{-\tau_2 s}$	$\tau_1 < \tau_{un}$		✓	✓	✓

proposed control the stability condition is,

$$\tau_1 < \frac{1}{a} - \frac{1}{b} \tag{7.1}$$

while for the third one is,

$$\lim_{t\to\infty} [\omega_i - \widehat{\omega}_i] = 0, \text{ for } i = 1, 2, \tag{7.2}$$

From condition (7.1), it is clear that the stability condition is limited by the location of the open forward loop of the recycling systems. This limitation is overcome when the third control strategy is used since from condition 7.2, one can move the poles such that the condition is close to the maximal condition $\tau_1 < \tau_{un}$.

Also note that the first control schema can be seen as a particular case of the second one. However the second proposed control schema cannot be seen as a particular case of the third one. On the other hand, as it is well known, time delay uncertainties can affect closed loop stability of recycled systems. To prevent this possibility a robustness analysis has been developed for each control strategy proposed (Chapter 5). Also in Chapter 6 some numerical simulations are presented in order to show the performance of the control schemas. Finally in Table 7 a summary of all new proposed control strategies in this work is presented.

Future work

As a perspective of the results presented in this work, we can mention:

- ▷ In this work bounds on the parameters involved in the control-observer strategies were provided. However, an analysis for the "best" values for such parameters would be welcome.

- ▷ Once some theoretical results have been presented, a natural next step is to apply the proposed observer-controller to a physical plant in order to evaluate the performance.

- ▷ In this work some control structures were presented for unstable FOPTD systems with large time delay. In this way, as a future work we can consider to extend the approach to systems with recycle, i.e., design control schemas that allow dealing with unstable FOPTD system at forward loop with a larger time delay.

- To modify/extend the proposed control strategies for a more generalized system where zeros in the right half plane in both direct and recycle loops were considered.

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Appendix A

Complementary Results

A.1 Static output feedback (Lemma 2.1)

Lemma A.1.

Consider the unstable input-output delay system

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} = \frac{b}{s-a}e^{-\tau s}, \quad a > 0 \tag{A.1}$$

with a proportional output feedback

$$U(s) = R(s) - kY(s) \tag{A.2}$$

where $R(s)$ is the new reference input. There exist a proportional gain k such that the closed loop system

$$\frac{Y(s)}{R(s)} = \frac{be^{-\tau s}}{s - a + kbe^{-\tau s}} \tag{A.3}$$

is stable if and only if $\tau < \frac{1}{a}$.

Proof. The proof use the well known fact that a discrete time model derived from a continuous time system is equal to its continuous counterpart if the sampling period

$T \to 0$. It is carried out by discretizing the system and then showing that all the poles remain inside the unitary circle when the sampling period tends to zero iff $\tau < \frac{1}{a}$.

Discretizing model (A.1) using a zero order hold and a sampling period $T = \frac{\tau}{n}$ with $n \in \mathbb{N}$, it is obtained,

$$G(z) = \frac{b}{a}\frac{(e^{aT} - 1)}{z^n(z - e^{aT})} \tag{A.4}$$

Model (A.4) in closed loop with the (discretized) output feedback (A.2) produces the characteristic equation,

$$p(z) = z^n(z - e^{aT}) + k\frac{b}{a}(e^{aT} - 1) = 0 \tag{A.5}$$

Let us to analyze the root locus of (A.5). Open loop system has n poles at the origin and one at $z = e^{aT}$. Then, there exist $n+1$ branches to infinity, $n-1$ of them starting at the origin and going directly to infinity. The two remaining branches starting at a breaking point z_1 located over the real axis between the origin and $z = e^{aT}$ (this situation is illustrated in Figure A.1 for the case $n = 5$). z_1 can be found by considering the equation,

$$\frac{dk}{dz} = \frac{d}{dz}\left[-\frac{z^n(z - e^{aT})}{\frac{b}{a}(1 - e^{aT})}\right] = 0,$$

producing the equation,

$$(n+1)z^n - nz^{n-1}e^{aT} = 0,$$

which has $n-1$ roots at the origin and one at $z_1 = \frac{n}{n+1}e^{a\frac{\tau}{n}}$. If the breaking point z_1 over the real axis is located inside the unit circle, the closed loop system could have a region of stability, otherwise the system is unstable for any k.
The stability properties of the continuous system (A.3) are obtained by considering the limit as $n \to \infty$, or equivalently, when $T \to 0$, this is,

$$\lim_{n\to\infty} z_1 = \lim_{n\to\infty} \frac{n}{n+1}e^{a\frac{\tau}{n}} = 1. \tag{A.6}$$

It is important to note that any point $s = \theta$, over the real axis on the complex plane

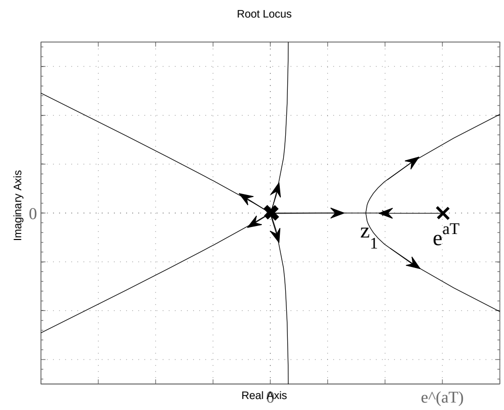

Figure A.1: Root locus of equation (A.5) for $n = 5$.

s is mapped to $z = e^{\theta T}$ on the z plane and as a consequence this point converges to $z = 1$ when T tends to zero. Notice also that any real point $s = \theta$ on the left half side of the complex plane ($\theta < 0$) is mapped to a point $e^{\theta T}$ that tends to one over the stable region of the z plane. On the contrary, if θ is on the right side of the complex plane over the real axis ($\theta > 0$), the point $e^{\theta T}$ tends to one over the unstable region. Then, from (A.5), it is not difficult to see that if $a\tau < 1$ (i.e., $\tau < 1/a$) there exists a gain k that stabilizes the closed loop system (i.e., the limit tends to one from the left). In the case that $a\tau \geq 1$ it is not possible to get k that stabilize the system.

Then, if the remaining $n-1$ roots are into the unit circle, the closed loop is stable. Let us now prove that the remaining $n-1$ roots are into the unitary circle if and only if $a\tau < 1$. Assume that $a\tau \leq 1$ and to take into account the continuous case, the characteristic equation (A.5) is modified as,

$$\begin{aligned}\lim_{n\to\infty} p(z) &= \lim_{n\to\infty}\left[z^n(z - e^{aT}) + k\frac{b}{a}(e^{aT} - 1)\right] \\ &= \lim_{n\to\infty}\left[z^n(z - e^{a\frac{\tau}{n}}) + k\frac{b}{a}(e^{a\frac{\tau}{n}} - 1)\right] \\ &= (z-1)\lim_{n\to\infty} z^n\end{aligned}$$

from where it is stated that while one pole is on the neighborhood of $z = 1$, the remaining poles are in a neighborhood of the origin. Then, we can finally state that the system can be stabilized iff $a\tau < 1$. ■

A.2 Proof of Corollary 2.1

Assume that $\tau < \frac{1}{a}$ and take into account the discretized system given by (A.4). Analyzing the root locus associated to such discrete system, it is possible to see that the open loop system has n poles at the origin and one at $z = e^{aT}$ without finite zeros. Then, there are $n-1$ branches going to infinity and a pair converging to a point on the real axis located between the origin and $z = 1$(stability region). Note that if $k = 0$ the system is unstable. The gain k that takes the systems to the border of the stability region ($z = 1$) is obtained by evaluating k for $z = 1$, this is,

$$k = -\frac{z^n(z - e^{aT})}{\frac{b}{a}(1 - e^{aT})}\bigg|_{z=1} = \frac{a}{b} \tag{A.7}$$

Then by Lemma 2.1 the proof is concluded.

A.3 Proof of Lemma 2.2

[47] Let us consider the Lemma 2.1. There is a gain k such that the system $\frac{\beta}{s-a}e^{-\tau s}$ is closed loop stable if and only if $\tau < \frac{1}{a}$. An analysis in the frequency domain confirms this result. Fig. A.2 shows the Nyquist diagram for a system satisfying $\tau < \frac{1}{a}$. The Nyquist stability criterion states that when the loop is closed with a gain k, the system will be stable iff $0 = N + P$, with P the number of poles $G(s)$ in the right half plane and N the number of clockwise round trips to the point -1 (N negative in counterclockwise) in the Nyquist diagram. In this case there is a gain that stabilizes the system since there is one tour counterclockwise to the point -1. When $\tau < \frac{1}{a}$ is not satisfied, there is not detour in counterclokwise. The phase as a function of frequency ω is given by $\angle G(j\omega) = -(\pi - tg^{-1}\frac{\omega}{a}) - tg^{-1}\omega\tau$. It can be shown that the condition $\tau < \frac{1}{a}$ is equivalent to ask that the angle path tap at least one point (for some frequency) with a value exceeding $-\pi$, that is $\angle G(j\omega) > -\pi$. Let us now analyze the system under consideration given by (2.7). It is evident that with $\tau < \frac{1}{a}$ and the parameter b small enough there is a k that stabilizes the system, since the

Nyquist condition remains the same (one counterclockwise detour to the point -1). Now we have,

$$\angle G(j\omega) = -(\pi - tg^{-1}\frac{\omega}{a}) - tg^{-1}\frac{\omega}{b} - tg^{-1}\omega\tau.$$

Growing the value of parameter b, the bound that forms the trajectory conterclockwise decreses until extinction (See Fig. A.3). As for small frequencies $tg^{-1}\omega\varphi \approx \omega\varphi$, starting from $\angle G(j\omega) > -\pi$ it is not difficult to conclude the relation $\tau < \frac{1}{a} - \frac{1}{b}$.

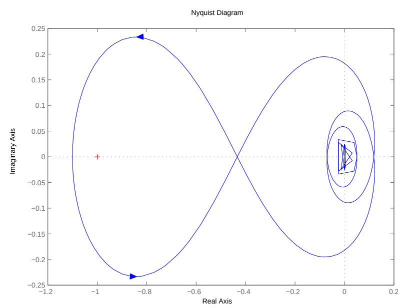

Figure A.2: Nyquist diagram when $\tau < \frac{1}{a}$

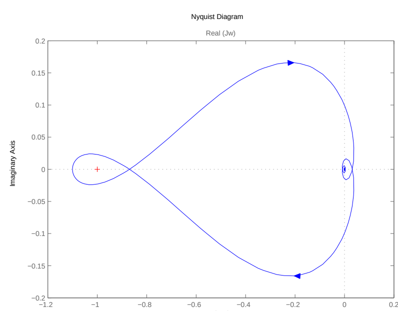

Figure A.3: Nyquist Diagram when rising the b value.

A.4 Proof of Lemma 2.3

[47] Let us consider the Lemma 2.1. There is a gain k such that the system $\frac{b}{s-a}e^{-\tau s}$ is closed loop stable if and only if $\tau < \frac{1}{a}$. An analysis in the frequency domain confirms

this result. Fig. A.2 shows the Nyquist diagram for a system satisfying $\tau < \frac{1}{a}$. The Nyquist stability criterion states that when the loop is closed with a gain k, the system will be stable iff $0 = N + P$, with P the number of poles $G(s)$ in the right half plane and N the number of clockwise round trips to the point -1 (N negative in counterclockwise) in the Nyquist diagram. In this case there is a gain that stabilizes the system since there is one tour counterclockwise to the point -1. When $\tau < \frac{1}{a}$ is not satisfied, there is not detour in counterclokwise. The phase as a function of frequency ω is given by $\angle G(j\omega) = -(\pi - tg^{-1}\frac{\omega}{a}) - tg^{-1}\omega\tau$. It can be shown that the condition $\tau < \frac{1}{a}$ is equivalent to ask that the angle path tap at least one point (for some frequency) with a value exceeding $-\pi$, that is $\angle G(j\omega) > -\pi$. Let us now analyze the system under consideration given by (2.7), with $n = 1$. It is evident that with $\tau < \frac{1}{a}$ and the parameter c_1 small enuogh there is a k that stabilizes the system, since the Nyquist condition remains the same (one counterclockwise detour to the point -1). Now we have,

$$\angle G(j\omega) = -(\pi - tg^{-1}\frac{\omega}{a}) - tg^{-1}\frac{\omega}{c_1} - tg^{-1}\omega\tau.$$

Growing the value of parameter c_1, the bound that forms the trajectory conterclockwise decreses until extinction (See Fig. A.3). As for small frequencies $tg^{-1}\omega\varphi \approx \omega\varphi$, starting from $\angle G(j\omega) > -\pi$ it is not difficult to conclude the relation $\tau < \frac{1}{a} - \frac{1}{c_1}$. If we consider now $n = 2$, under the assumption $\tau < \frac{1}{a} - \frac{1}{c_1} - \frac{1}{c_2}$, and with c_2 small enough, there is k that stabilizes the system (Nyquist condition remains one counterclockwise loop to the point -1). We have now,

$$\angle G(j\omega) = -(\pi - tg^{-1}\frac{\omega}{a}) - tg^{-1}\frac{\omega}{c_1} - tg^{-1}\frac{\omega}{c_2} - tg^{-1}\omega\tau.$$

Now, growing the value of parameter c_2, the loop that forms the detour counterclockwise decreses until extinction. Again considering $tg^{-1}\omega\varphi \approx \omega\varphi$, from $\angle G(j\omega) > -\pi$ it is not difficult to conclude the relation $\tau < \frac{1}{a} - \frac{1}{c_1} - \frac{1}{c_2}$. This analysis can be generalizated to any $n \in \mathbb{R}$ concluding that there is a gain k such that the closed loop system given by (2.9) is BIBO stable if and only if $\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}$

A.5 PI with two degree of freedom

The traditional tuning methods of PI/PID controllers induce a zero on the closed loop system that produces an undesirable overshoot. To improve the tracking properties of the system together with an adequate overshoot response and set time reduction, it has been proposed in the literature a two degree of freedom control scheme [48], also known as PI-setpoint weighting tuning. Following this approach, let us consider an unstable FOPTD system given by

$$\frac{Y(s)}{U(s)} = \frac{b}{s-a}e^{-\tau s} \tag{A.8}$$

and the partition

$$\frac{W(s)}{U(s)} = \frac{b}{s-a} \tag{A.9}$$

In what follows it is assumed that the signal $W(s)$ is available in the system. Thus, the PI controller tuning can be done by considering the delay free model. In this way, the PI-controller proposed by [48] is given by,

$$u(t) = K\left[(e_p(t) + \frac{1}{T_i}\int_0^t e(s)ds\right] \tag{A.10}$$

with a modified proportional error given by, $e_p(t) = \sigma_c r(t) - w(t)$ and an integral error of the form $e(t) = r(t) - w(t)$. Under these conditions, feedback (A.10) can be rewritten in a two-degree-of-freedom structure as,

$$U(s) = R(s)G_{ff}(s) - W(s)G_c(s) \tag{A.11}$$

where,

$$G_{ff}(s) = K(\sigma_c + \frac{1}{sT_i}) \text{ and } G_c(s) = K(1 + \frac{1}{sT_i}). \tag{A.12}$$

The tuning of the PI controller is based is based on a pole placement strategy that attempts to find a controller that gives a desired closed-loop behavior. For doing this, consider the free-delay transfer function given by (A.9) in closed loop with the

PI compensator (A.10). It is obtained the characteristic equation,

$$s^2 + (bK - a)\, s + \frac{bK}{T_i} = 0. \tag{A.13}$$

By considering the general characterization of a second order system in terms of the relative damping parameter ζ and the natural frequency ω_0, equation (A.13) takes the standard form, $s^2+2\zeta\omega_0 s+\omega_0^2 = 0$, from where it is possible to obtain $K = \frac{2\zeta\omega_0+a}{b}$ and $T_i = \frac{2\zeta\omega_0+a}{\omega_0^2}$.

Notice that the transfer function from the setpoint input to the process output has a zero at $s = -\frac{1}{\sigma_c T_i}$. Following Astrom et al. [48], to avoid excessive overshoot in the response; parameter σ_c has to be chosen so that the zero is located to the left of the dominant closed-loop poles. A reasonable value is $\sigma_c = \frac{1}{\omega_0 T_i}$, which places the zero at $s = -\,\omega_0$. Also, the integral time T_i can be approximated for a sufficiently large ω_0 as $T_i = \frac{2\zeta}{\omega_0}$, and thus, independent of the process dynamics.

Remark A.1 *At the begining of this Section it is assumed that the signal $W(s)$ is an available signal in the system, which in practice is unreal. However, in the control strategies proposed in this work the design of the observers is used in order to estimate this signal. This is the reason why one can design the PI controller as if signal $W(s)$ were available. In the proposed control strategies the estimated signal $\hat{W}(s)$ is used to implement the PI controller.*

A.6 Output injection feedback-Eurojournal-

Lemma A.2.

Consider the delayed system given by

$$\frac{Y(s)}{U(s)} = \frac{b}{s-a}e^{-\tau s} \tag{A.14}$$

and the static output injection scheme shown in Figure A.4. There exist constants g_1 and g_2 such that the closed loop system

$$\frac{Y(s)}{U(s)} = \frac{be^{-\tau s}}{(s-a)(1+g_1 e^{-\tau s}) + g_2 b e^{-\tau s}} \tag{A.15}$$

is stable if and only if $\tau < \frac{2}{a}$.

Proof. In order to state the stability conditions for the system (A.15), consider now a discrete time version of the original plant (A.14) together with the output injection scheme given in Figure A.4. To carry out this task, it is assumed that there exist a sampling period T that satisfies the condition $T = \frac{\tau}{n}$ for an integer n and that a zero order hold is located at the input of the system. Under these conditions, the discrete time closed-loop transfer function is,

$$\frac{Y(z)}{U(z)} = \frac{(b/a)(e^{aT}-1)}{(z-e^{aT})(z^n+g_1) + g_2(b/a)(e^{aT}-1)}, \tag{A.16}$$

with the characteristic polynomial given by,

$$p_1(z) = (z-e^{aT})(z^n+g_1) + g_2(b/a)(e^{aT}-1). \tag{A.17}$$

The proof of the theorem is based on demonstrate that all roots in (A.17) lie inside the unit circle when it is considered $\lim_{n\to\infty}\frac{\tau}{n}$, i.e., when the sampling period T tend to zero (the continuous case), if and only if, $\tau < \frac{2}{a}$.

To begin with, consider first the simple case when $g_1 = 0$ in (A.16), this produces the characteristic equation,

$$(z - e^{aT})z^n - g_2(b/a)(e^{aT} - 1) = 0. \tag{A.18}$$

The root locus diagram ([54]) associated to (A.18) shows that the open loop system has n poles at the origin and one at $z = e^{aT}$. Then, there exist $n+1$ branches to infinity, $n-1$ of them starting at the origin and going directly to infinity. The two remaining branches starting at a breaking point z_1 located over the real axis between the origin and $z = e^{aT}$ (this situation is illustrated in Figure A.1 for the case $n = 5$). z_1 can be found by considering the equation,

$$\frac{dg_2}{dz} = \frac{d}{dz}\left[-\frac{z^n(z - e^{aT})}{\frac{b}{a}(1 - e^{aT})}\right] = 0,$$

that produces,

$$(n+1)z^n - nz^{n-1}e^{aT} = 0,$$

which has $n-1$ roots at the origin and one at,

$$z_1 = \frac{n}{n+1}e^{a\frac{\tau}{n}}.$$

If the breaking point z_1 over the real axis is located inside the unit circle, the closed loop system could have a region of stability, otherwise will be unstable for any g_2. The stability properties of the continuous system (A.15) are obtained by considering the limit as $n \to \infty$, or equivalently, when $T \to 0$, this is,

$$\lim_{n\to\infty} z_1 = \lim_{n\to\infty} \frac{n}{n+1}e^{a\frac{\tau}{n}} = 1. \tag{A.19}$$

It is important to note that any point $s = \theta$, over the real axis on the complex plane s is mapped to $z = e^{\theta T}$ on the z plane and as a consequence this point converges to $z = 1$ when T tends to zero. Notice also that any real point $s = \theta$ on the left half side of the complex plane ($\theta < 0$) is mapped to a point $e^{\theta T}$ that tends to one over the stable region of the z plane. On the contrary, if θ is on the right side of the complex plane over the real axis ($\theta > 0$), the point $e^{\theta T}$ tends to one over the unstable region. Then, from (A.18), it is not difficult to see that if $a\tau < 1$ (i.e., $\tau < 1/a$) there exists a gain g_2 that stabilizes the closed loop system (i.e., the limit tends to one from the left). In the case that $a\tau \geq 1$ (always considering $g_1 = 0$) it is not possible to get g_2 that stabilize the system.

Consider now the case $g_1 \neq 0$. Applying again a root locus analysis for system (A.16) and its characteristic equation (A.17), as g_1 grows from zero, the breaking point over the real axis moves in the root locus diagram (indeed, goes to the left). This point can be found by taking into account the equation,

$$\frac{dg_2}{dz} = \frac{d}{dz}\left[-\frac{(z-e^{aT})(z^n+g_1)}{(b/a)(e^{aT}-1)}\right] = 0, \tag{A.20}$$

yielding,

$$(n+1)z^n - nz^{n-1}e^{aT} + g_1 = 0. \tag{A.21}$$

Expression (A.21) corresponds to the characteristic equation of a fictitious system of the form,

$$\begin{aligned} \tfrac{Y(z)}{V(z)} &= G(z) \\ &= \frac{1/(n+1)}{z^n - z^{n-1}e^{aT}n/(n+1)} \\ &= \frac{1/(n+1)}{z^{n-1}(z-e^{aT}n/(n+1))} \end{aligned} \tag{A.22}$$

in closed loop with the feedback,

$$V(z) = U(z) - g_1 Y(z). \tag{A.23}$$

The open loop system (A.22) has $n-1$ root at the origin and one at

$$z = \frac{n}{n+1}e^{a\frac{\tau}{n}}.$$

If the breaking point over the real axis is located inside the unit circle, the closed loop system (A.22)-(A.23) could have a region of stability (once proved that the others $n-2$ poles are inside the unitary circle), otherwise the system will be unstable for any g_1. This point can be found by considering,

$$\frac{dg_1}{dz} = \frac{d}{dz}\left[-\frac{z^{n-1}\{z - e^{aT}n/(n+1)\}}{1/(n+1)}\right] = 0, \tag{A.24}$$

that produces,

$$z^{n-2}(z - \frac{n-1}{n+1}e^{aT}) = 0,$$

which has $n-2$ roots at the origin and one at,

$$z = \frac{n-1}{n+1}e^{a\frac{\tau}{n}}.$$

As previously, the stability properties of the equivalent continuous system (A.15) are obtained by considering the limit as $n \to \infty$, or equivalently, when $T \to 0$. That is,

$$\lim_{n\to\infty} z = \lim_{n\to\infty} \frac{n-1}{n+1}e^{a\frac{\tau}{n}} = 1.$$

Again, since this limit point is located on the stability boundary, in this case it is possible to see that if $a\tau \leq 2$ (i.e., the limit tends to one from the left) there exists a gain g_1 that places the breaking point (two poles) inside the unit circle in the original discrete Root Locus diagram. Then, if the remaining $n-1$ roots are into the unit circle, the closed loop system is stable. In the case that $a\tau > 2$ it is not possible to stabilize the system by static output injection (i.e., the limit goes to one from the right). Let us now prove that the remaining $n-1$ roots are into the unitary circle if and only if $a\tau < 2$. Assume that $a\tau \leq 2$ and to take into account the continuous case, the characteristic equation (A.17) it is modified as,

$$\begin{aligned}\lim_{n\to\infty} p_1(z) &= \lim_{n\to\infty}[(z - e^{a\frac{\tau}{n}})(z^n + g_1) \\ &\quad + g_2(b/a)(e^{a\frac{\tau}{n}} - 1)] \\ &= (z-1)\lim_{n\to\infty}(z^n + g_1) = 0\end{aligned}$$

from where it is stated that while one pole is on the neighborhood of $z = 1$, the remaining poles are in a neighborhood of the points $(-g_1)^{1/n}$, inside the unit circle producing a stable closed loop system if, as it was previously stated, it is satisfied, $g_1 < 1$. From equation (A.24),

$$g_1 = -\frac{z^n\{z - e^{aT}n/(n+1)\}}{1/(n+1)},$$

then if $z = 1$,

$$\begin{aligned} g_1 &= -\frac{\{1 - e^{aT}n/(n+1)\}}{1/(n+1)} \\ &= -(n + 1 - ne^{aT}). \end{aligned}$$

Taking into account the continuous case as previously done, it is obtained,

$$\begin{aligned} \lim_{n\to\infty} g_1 &= \lim_{n\to\infty} -\left(n + 1 - ne^{a\tau/n}\right) \\ &= a\tau - 1. \end{aligned} \tag{A.25}$$

As $g_1 < 1$ is a necessary condition for the stability, $a\tau - 1 < 1$, then $a\tau < 2$. ■

Remark A.2 *Note that the analytical idea of the previous proof can be applied for systems with superior order than the system (2.1). However it is not easy to get stability conditions with respect to time delay since more variables should be taken into account as well as not only one breaking point is appearing over the real axis on the complex plane.*

It is not an easy task to get the stability region associated with gains g_1 and g_2, however a useful and practical result in order to compute the parameters involved on the structure shown in Figure A.4 is the following one.

Corollary A.1.

Consider the static output injection scheme shown in Figure A.4. If $\tau < \frac{2}{a}$, then the parameters g_1 and g_2 that stabilize the closed loop system (A.15) satisfy the inequalities,

$$a\tau - 1 < g_1 \leq a\tau - 1 + \sigma,$$

$$\frac{a}{b}(g_1 + 1) < g_2 \leq \frac{a}{b}(g_1 + 1) + \bar{\sigma},$$

for some constants $\sigma, \bar{\sigma} > 0$.

Proof. From equation (A.25) in the proof of Lemma A.2 we have:

$$\lim_{n\to\infty} g_1 = a\tau - 1.$$

Therefore if $\tau < \frac{2}{a}$, there exist g_2 that stabilizes the closed loop system (2.38), with $a\tau - 1 < g_1 \leq a\tau - 1 + \sigma$ for $\sigma > 0$.

Now, from equation (A.20),

$$g_2 = -\frac{(z - e^{aT})(z^n + g_1)}{(b/a)(e^{aT} - 1)},$$

then, if $z = 1$,

$$g_2 = \frac{g_1 + 1}{(b/a)} = (a/b)(g_1 + 1).$$

Therefore, the gain g_2 can be obtained by considering the condition $\frac{a}{b}(g_1+1) < g_2 < \frac{a}{b}(g_1 + 1) + \bar{\sigma}$, for some $\bar{\sigma} > 0$. ■

From Corollary A.1 it is now possible to give a recursive algorithm in order to obtain stabilizing parameters g_1 and g_2. This procedure can be given as follows.

Algorithm A.1 *Step 0:*

1. *Define* $\sigma_0 = 0.6(2/a - \tau)$ *and* $\bar{\sigma}_0 = 0.02(2/a - \tau)$.

Step i:

1. *Define* $\sigma_i = \sigma_{i-1}/2$ *and* $\bar{\sigma}_i = \bar{\sigma}_{i-1}/2$.

2. *Obtain* g_2 *and* g_1 *as,*

$$g_2 = \frac{a}{b}(a\tau + \sigma_i) + \bar{\sigma}_i,$$
$$g_1 = a\tau - 1 + \sigma_i.$$

If at the step i *it is obtained an unstable closed loop system for the obtained* g_1 *and* g_2*, proceeds to step* $i+1$*. The algorithm ends when the obtained closed loop system is stable.*

Notice that even if the resulting closed loop dynamic for the obtained g_1 and g_2 results stable it is possible to continue the algorithm without breaking the stability properties of the system in order to improve the general closed-loop response.

A.7 Stability of a polynomial with two times delay.

For the sake of the completness in this Section some results provided in [50] are adressed. Also some comments related to the robustness analysis applied to our control strategy provided in Section 2.4.2 are given. In this way, the stability of the characteristic quasipolinomial,

$$p(s) = p_0(s) + p_1(s)e^{-\tau s} + p_2(s)e^{-\tau_0 s} = 0, \tag{A.26}$$

is analyzed. In particular, the change of stability as the delays τ and τ_0 vary is studied. The approach used in such work is more geometric rather than purely algebraic. In that follows, some results provided in [50] are presented in order to get an idea of the analysis, later such results are applied to a robustness analysis for the proposed control schema.

Let $\mathcal{T}$ denote the set of all points (τ, τ_0) in $\mathbb{R}_+^2$ such that $p(s)$ has at least one zero on the imaginary axis. Any $(\tau, \tau_0) \in \mathcal{T}$ is known as a crossing points, is known as the stability crossing curves. The polynomial given by (2.32) can be rewritten as,

$$a(s) = a(s, \tau, \tau_0) = 1 + a_1(s)e^{-\tau s} + a_1(s)e^{-\tau_0 s}$$

with $a_l(s) = p_l(s)/p_0(s)$, $l = 1, 2$. As we can see, for given τ and τ_0, as long as $p_0(s)$ does not have imaginary zeros, $p(s)$ and $a(s)$ share all the zeros in a neighborhood of the imaginary axis. Then, we have the following proposition,

Proposition A.1 *[50] For each ω, $\omega \neq 0$, $p_0(j\omega) \neq 0$, $s = j\omega$ can be a solution of $p(s, \tau, \tau_0) = 0$ for some $(\tau, \tau_0) \in \mathbb{R}_+^2$ if and only if,*

$$|a_1(j\omega)| + |a_2(j\omega)| \geq 1, \tag{A.27}$$

$$-1 \leq |a_1(j\omega)| - |a_2(j\omega)| \leq 1. \tag{A.28}$$

For $\omega \neq 0$ satisfying $p_0(j\omega) = 0$, $s = j\omega$ can be a zero of $p(s, \tau, \tau_0)$ for some $(\tau, \tau_0) \in \mathbb{R}_+^2$ if and only if

$$|p_1(j\omega)| = |p_2(j\omega)| \tag{A.29}$$

Let Ω be the set of all $\omega > 0$ which satisfy (A.27) and (A.28) if $p_0(j\omega) \neq 0$ and (A.29) if $p_0(j\omega) = 0$. We will refer to Ω as the crossing set. It contains all the ω such that some zero(s) of $p(s, \tau, \tau_0)$ may cross the imaginary axis at $j\omega$. Then, for any given $\omega \in \Omega$, $p_l(j\omega) \neq 0$, $l = 0, 1, 2$, one may easily find all the pairs of (τ, τ_0) satisfying $a(s) = 0$, as follows,

$$\tau = \tau^{u\pm}(\omega) = \frac{\angle a_1(j\omega) + (2u-1)\pi \pm \theta_1}{\omega} \geq 0, u = u_0^{\pm}, u_0^{\pm} + 1, u_0^{\pm} + 2, ..., \tag{A.30}$$

$$\tau_0 = \tau_0^{v\pm}(\omega) = \frac{\angle a_2(j\omega) + (2v-1)\pi \mp \theta_1}{\omega} \geq 0, v = v_0^{\pm}, v_0^{\pm} + 1, v_0^{\pm} + 2, ..., \tag{A.31}$$

where $\theta_1, \theta_2 \in [0, \pi]$ can be calculated by,

$$\theta_1 = \cos^{-1}\left(\frac{1 + |a_1(j\omega)|^2 - |a_2(j\omega)|^2}{2\,|a_1(j\omega)|}\right),$$

$$\theta_2 = \cos^{-1}\left(\frac{1 + |a_2(j\omega)|^2 - |a_1(j\omega)|^2}{2\,|a_2(j\omega)|}\right),$$

and $u_0^+, u_0^-, v_0^+, v_0^-$ are the smallest possible integers (may be negative, or may depend on ω) such that the corresponding $\tau^{u_0^+}$, $\tau^{u_0^-}$, $\tau_0^{v_0^+}$, $\tau_0^{v_0^-}$ calculated are nonnegative. Notice, $u_0^+ \leq u_0^-$, $v_0^+ \geq v_0^-$.

The set of ω satisfying (A.27) and (A.28) consists of a finite number of intervals of finite length. Let these intervals be Ω_k, $k = 1, 2, ..., N$, ordered from the left to rigth. Then,

$$\Omega = \bigcup_{k=1}^{N} \Omega_k.$$

It should be noted that $0 \notin \Omega$ even if $\omega = 0$ satisfies (A.27) and (A.28). Indeed, if (A.27) and (A.28) are satisfied for $\omega = 0$ and sufficiently small positive value, then, $\Omega_1 = (0, \omega_1^r]$, and we will let $\omega_1^l = 0$ in this case. Otherwise, $\Omega_1 = [\omega_1^l, \omega_1^r]$, $\omega_1^l = 0$. Let

$$\mathcal{T}_{u,v}^{\pm} = \bigcup_{\omega \in \Omega_k} \mathcal{T}_{\omega,u,v}^{\pm} = \{(\tau^{u\pm}(\omega), \tau_0^{v\pm}(\omega)) \mid \omega \in \Omega_k\}$$

and

$$\mathcal{T}^k = \bigcup_{u=-\infty}^{\infty} \bigcup_{v=-\infty}^{\infty} \left(\mathcal{T}_{u,v}^{+k} \cup \mathcal{T}_{u,v}^{-k}\right) \cap \mathbb{R}_+^2$$

$$\mathcal{T} = \bigcup_{k=1}^{N} \mathcal{T}^k$$

A careful examination of (A.30) and (A.31) allows to arrive at the following list,

Type 1. $|a_1(j\omega)| - |a_2(j\omega)| = 1$ is satisfied. In this case, $\theta_1 = 0$, $\theta_2 = \pi$, and $\mathcal{T}_{u,v}^{+k}$ is connected with $\mathcal{T}_{u,v-1}^{-k}$ at this end.

Type 2. $|a_2(j\omega)| - |a_1(j\omega)| = 1$ is satisfied. In this case, $\theta_1 = \pi$, $\theta_2 = 0$, and $\mathcal{T}_{u,v}^{+k}$ is connected with $\mathcal{T}_{u+1,v}^{-k}$ at this end.

Type 3. $|a_1(j\omega)| + |a_2(j\omega)| = 1$ is satisfied. In this case, $\theta_1 = \theta_2 = 0$, and $\mathcal{T}_{u,v}^{+k}$ is connected with $\mathcal{T}_{u,v}^{-k}$ at this end.

Type 0. $\omega_k^l = 0$, which simply means that $\omega = 0$ satisfies both inequalities (A.27) and (A.28). In this case, as $\omega \to 0$, $\mathcal{T}_{u,v}^{+k}$ and $\mathcal{T}_{u,v}^{-k}$ approach infinity with asymptotes defined in [50].

According to the types of Ω_k, $\mathcal{T}^k$ may have different shapes, as specified in the following proposition,

Proposition A.2 *[50] Under the standing assumption that $p_l(s) \neq 0$, $l = 1, 2$, the stability crossing curves $\mathcal{T}^k$ corresponding to Ω_k, $\omega_k^l \neq 0$ must be an intersection of $\mathbb{R}_+^2$ with a series of curves belonging to one of the following categories: A) A series of closed curves; B) A series of spiral oriented either horizontally, vertically or diagonally; C) A series of open ended curves with both ends approaching to ∞.*

Now, consider the system given by (2.1), the proposed predictor shown in Figure 2.10 with $G_{ff} = 1$, $g_3 = 0$ and $G_c = \overline{k}$ which leads to a control law $U(s) = \overline{k}\hat{W}(s)$. In this way, the closed loop characteristic equation is given by,

$$p_A(s) = p_a(s) + p_b(s)e^{-\tau s} + p_c(s)e^{-\tau_0 s} = 0, \tag{A.32}$$

where,

$$\begin{aligned} p_a(s) &= s^2 + (b\overline{k} - 2a)s + a(a - b\overline{k}), \\ p_b(s) &= g_1 s^2 + \left[b(g_2 + \overline{k}g_1) - 2ag_1\right] s + \left[a^2 g_1 - ab(g_2 + \overline{k}g_1)\right], \\ p_c(s) &= b^2\overline{k}g_2, \end{aligned}$$

τ and τ_0, are the time delay in the observer and the real time delay in the process, respectively. Now, from the results presented above, it is possible to identify the regions of (τ, τ_0) in $\mathbb{R}_+^2$ such that $p_A(s)$ is stable. As we can see, the characteristic equation (A.32) is a particular case of (2.32). In this case, from (A.32) it is posible to see that this class of system has only one interval $\Omega_1 = [\omega_1^l, \omega_1^r]$ and as a consequence only $\mathcal{T}^1$. Also, such $\mathcal{T}^1$corresponding to Ω_1 is the type 23, which correspond to a form of spiral-like curves with horizontal axes. For the type 23, $\mathcal{T}_{u,v}^{+k}$ is connected to $\mathcal{T}_{u+1,v}^{-k}$ at ω_1^l, and the other end of $\mathcal{T}_{u+1,v}^{-k}$ is connected to $\mathcal{T}_{u+1,v}^{-k}$ at ω_1^r, and so on.

Then, Figure A.5 shows $\mathcal{T}^1$ for the characteristic equation (A.32). Note that a stable region is indicated as the delays τ and τ_0 vary. This stable region illustrate the set of (τ, τ_0) that the proposed controller-observer is able to stabilize. Furthermore, from this Figure, for a fixed value of τ, it is possible to obtain the range of τ_0, $[\tau_{0\,\min}, \tau_{0\,\max}]$, such that the characteristic equation (A.32) remains stable.

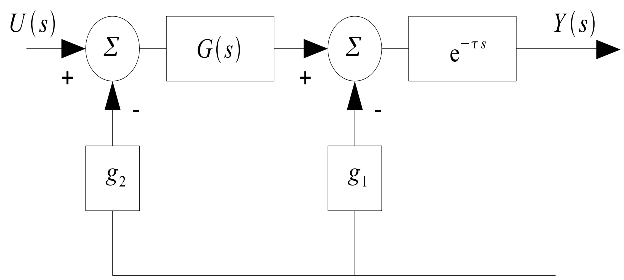

Figure A.4: Static output injection.

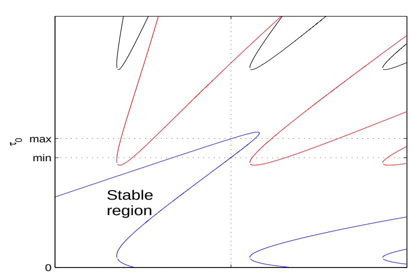

Figure A.5: $\mathcal{T}^1$ for the characteristic equation (A.32)

Appendix B

Publications

B.1 Control based on an observer schema

Control basado en un esquema observador para sistemas de primer orden con retardo. J.F. Marquez-Rubio, B. del Muro-Cuéllar, M. Velasco-Villa and J. Alvarez-Ramírez. In Revista Mexicana de Ingeniería Química. Vol. 9, No. 1 (2010) pp 43-52. ISSN 1665-2738

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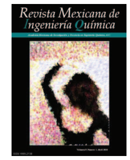

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CONTROL BASADO EN UN ESQUEMA OBSERVADOR PARA SISTEMAS DE PRIMER ORDEN CON RETARDO

CONTROL BASED IN AN OBSERVER SCHEME FOR FIRST-ORDER SYSTEMS WITH DELAY

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Resumen

Este trabajo considera el problema de estabilización y control de sistemas lineales con retardo en el lazo directo. Como es bien sabido, el análisis de estabilidad de éste tipo de sistemas se dificulta debido al término de retardo considerado. Para resolver el problema de estabilización, como primer paso se presentan las condiciones que aseguran la estabilidad del sistema en lazo cerrado con una retroalimentación estática de salida. Las condiciones de estabilidad se utilizan para diseñar un esquema observador (predictor) que proporciona una convergencia adecuada de la señal de predicción. El esquema propuesto presenta una configuración similar al tradicional Predictor de Smith sin los requerimientos de estabilidad de la planta que impone este último enfoque. El esquema observador se complementa con el uso de un compensador tipo PI para asegurar el seguimiento de referencias tipo escalón y el rechazo del mismo tipo de perturbaciones.

Palabras clave: retardo de tiempo, estabilización, predictor de Smith.

Abstract

This work considers the problem of stabilization and control of first order linear systems with time delay at direct path. As it is well known, the stability analysis of this kind of systems becomes difficult due to the term dead time considered. To solve the stabilization problem as a first step, the conditions that assure the stability of the systems in closed-loop with a proportional feedback are presented. These conditions are used in order to design an observer (predicting) scheme that provides adequate convergent error. The proposed scheme results similar to the traditional Smith Predictor without stability demands in the process that such approach require. The observer scheme is complemented by the use of a PI compensator to follow step references signals and disturbances rejecting of the same sort.

Keywords: time delay, stabilization, Smith Predictor, observer, root locus diagram.

1. Introducción

Los tiempos de retardo aparecen comúnmente en el modelado de diferentes clases de procesos. En particular, los procesos inestables con retardo de tiempo son frecuentemente encontrados en los procesos químicos e industriales, tales como, tanques de almacenamiento de líquido (Liu *y col.*, 2005a), reactores continuos tipo tanque agitado (CSTR) (Liu *y col.*, 2005b), reactores químicos discontinuos (Liu *y col.*, 2005a), reactores químicos irreversibles-exotérmicos (Luyben, 1988), bioreac-

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tores no lineales (Kavdia y Chidambaram, 1996), proceso de polimerización continuo en emulsión (Semino, 1994), etc.

Los retardos de tiempo deben su origen a diversas circunstancias, tales como transporte de material, los efectos de los lazos de reciclo e incluso en la aproximación de sistemas de alto orden a sistemas de bajo orden (Skogestad, 2003; Kolmanovskii y Myshkis, 1992). Desde la perspectiva de control, los retardos son un caso desafiante que debe ser superado diseñando estrategias de control que proporcionen un comportamiento aceptable del sistema en lazo cerrado y por supuesto estable. Se han desarrollado diversas estrategias de control para tratar a los retardos. El enfoque más simple consiste en ignorar el término de retardo, diseñar un controlador para el proceso libre del retardo y aplicar el control diseñado al proceso retardado. Es claro que este método solo funciona en el caso de procesos que cuentan con un retardo suficientemente pequeño. Por otro lado, para retardos de magnitud no despreciable, cuando la ley de control es aplicada a través de una computadora digital, la discretización del proceso con retardo a la entrada o salida (para un retardo τ) produce una función de transferencia racional en la variable compleja z, libre de retardo (Astrom y Wittenmark, 1997). En este caso el periodo de muestreo T, debe satisfacer la relación $T = \tau/n$, para cualquier n entero.

Cuando se considera el caso de control en tiempo continuo, el operador de retardo $e^{-\tau s}$ puede ser aproximado a través de una expansión en series de Taylor o mediante la aproximación de Padé. De esta manera, el sistema puede ser visto como un sistema de fase no mínima con una función de transferencia racional en la variable compleja s (Marshall, 1979; Hu y Wang, 2002). Una segunda clase de estrategias consiste en contrarrestar los efectos del retardo de tiempo a través de estrategias que intentan predecir los efectos de la entrada actual para una salida futura. La estrategia de predicción más usada es el Predictor de Smith (PS) (Smith, 1957; Palmor, 1996), el cual proporciona una estimación de la salida futura (señal antes de ser retardada) a través de un esquema tipo observador (ver Fig. 1). La principal limitación del PS original se debe a que el esquema de predicción no cuenta con una etapa de estabilización, lo cual restringe su aplicación a sistemas estables en lazo abierto. Para resolver este problema se han reportado diversos trabajos que abordan el caso de procesos con un integrador y un retardo de tiempo largo (Liu *y col.*, 2005a; Astrom *y col.*, 1994; Matausek y Micic, 1996; Normey y Camacho, 2001; Ingimundarson y Hagglund, 2002). Con la misma intención, se han reportado varios trabajos basados en modificaciones al PS, que abordan el caso de sistemas inestables (Liu *y col.*, 2005b; Xiang, 2005; Torrico y Normey, 2005; Seshagiri y Chidambaram, 2005; Normey y Camacho, 2008, 2009).

Este trabajo se enfoca al problema de regulación de sistemas lineales con retardo a la entrada de primer orden inestables mediante un esquema observador (predictor) en tiempo continuo. La motivación de tratar con esta clase de sistemas se debe a que si bien este es un primer paso antes de abordar sistemas de orden superior, adicionalmente, en algunos casos, los sistemas de alto orden se pueden aproximar por sistemas de primer orden en cascada con un elemento de retardo (Skogestad, 2003). Esta es la principal razón por la cual algunos trabajos en la literatura se enfocan al diseño de estrategias de control para tales sistemas de bajo orden. Por ejemplo, Seshagiri *y col.* (2007), presentan una eficiente modificación del PS para controlar sistemas inestables de primer orden con retardo de tiempo. Dicha metodología está restringida para retardos menores que 1.5 veces la constante de tiempo inestable. Con una perspectiva diferente, Normey y Camacho, (2008), proponen una modificación al PS original para tratar sistemas de primer orden inestable con retardo. Usando una estructura similar, en Normey y Camacho (2009), el resultado se extiende a sistemas de alto orden con retardo . En ambos trabajos se aplica un análisis de robustez y se concluye que para sistemas inestables con retardo, el sistema en lazo cerrado puede ser inestabilizado con un valor infinitesimal de error en el modelo, i.e., la robustez es dependiente de la relación τ/τ_{un}, donde τ es el retardo de tiempo y τ_{un} es la constante de tiempo inestable. Adicionalmente, para los esquemas de control propuestos en Normey y Camacho (2008, 2009), puede probarse fácilmente que para el caso de procesos inestables no se garantiza la estabilidad interna del sistema, pues una pequeña condición inicial diferente de cero en el proceso inestabilizan al sistema en lazo cerrado.

En este trabajo se considera una nueva y simple metodología que permite mostrar claramente las propiedades de estabilidad del sistema en lazo cerrado en tiempo continuo, a partir de propiedades de su representación en tiempo discreto. La equivalencia entre las dos representaciones se obtiene al considerar un periodo de muestreo T tal que $T \to 0$.

La estrategia de control propone el diseño de un esquema observador para obtener la estimación de la señal localizada entre el proceso libre de re-

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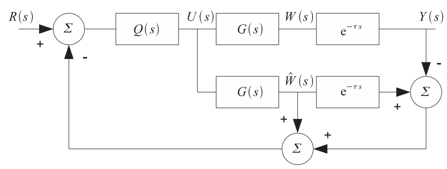

Fig. 1: Esquema del compensador Predictor de Smith

tardo y el término de retardo. Esta estrategia permite mostrar de manera simple la estabilidad interna del sistema y la convergencia de la predicción. Una vez mostrada la efectividad de la estrategia de predicción, se propone el uso de un controlador PI en una configuración de dos grados de libertad con el fin de resolver el problema de regulación y al mismo tiempo permitir el rechazo de perturbaciones tipo escalón.

El trabajo está organizado de la siguiente manera; en la Sección 2 se presenta la clase de sistemas considerados en este trabajo y una breve introducción al tradicional PS. En la Sección 3 se presenta el estimador propuesto y se analiza la convergencia del error. Posteriormente, la Sección 4 se dedica al diseño del controlador PI propuesto y se muestra el rechazo de perturbaciones tipo escalón para el sistema en lazo cerrado. En la Sección 5 se muestra el desempeño de la estrategia de control, mediante algunas simulaciones digitales y finalmente se dan algunas conclusiones en la Sección 5.

2. Clase de sistemas

Considere la clase de sistemas lineales una-entrada una-salida (UEUS) con retardo de tiempo a la entrada,

$$\frac{Y(s)}{U(s)} = \frac{N(s)}{D(s)}e^{-\tau s} = G(s)e^{-\tau s}, \quad (1)$$

donde $U(s)$ y $Y(s)$ son las señales de entrada y salida respectivamente, $\tau \geq 0$ es el retardo de tiempo que se supone conocido. $N(s)$ y $D(s)$ son polinomios en la variable compleja s y $G(s)$ es la función de transferencia libre de retardo.

Note que en relación con la clase de sistemas de la Ec. (1), una estrategia de control tradicional basada en una retroalimentación de salida de la forma,

$$U(s) = [R(s) - Y(s)]Q(s) \quad (2)$$

produce un sistema en lazo cerrado,

$$\frac{Y(s)}{R(s)} = \frac{Q(s)G(s)e^{-\tau s}}{1 + Q(s)G(s)e^{-\tau s}}, \quad (3)$$

donde el término $e^{-\tau s}$ localizado en el denominador de la función de transferencia en la Ec. (3), dificulta el análisis de estabilidad (Hale y Verduyn, 1993) debido al número infinito de polos del sistema en lazo cerrado.

La función de transferencia en lazo cerrado de la estructura del clásico PS mostrada en la Fig. 1, esta dada por,

$$\frac{Y(s)}{R(s)} = \frac{Q(s)G(s)}{1 + Q(s)G(s)}e^{-\tau s}. \quad (4)$$

Entonces, el PS proporciona internamente una estimación futura de la señal $y(t)$, la cual es utilizada para una retroalimentación determinada. Desafortunadamente, el esquema clásico del PS está restringido para el caso de plantas estables (Smith, 1957; Palmor, 1996). Cabe mencionar que si se intenta aplicar el PS a un sistema inestable, este esquema de compensación no es capaz de estabilizar el sistema en lazo cerrado ya que los polos inestables de la planta original pertenecen a la ecuación característica del sistema en lazo cerrado (Palmor, 1996).

Para tratar el caso de sistemas inestables, algunos autores han propuesto modificaciones al PS original y presentan soluciones adecuadas para casos particulares (Liu *y col.*, 2005b; Astrom *y col.*, 1994; Matausek y Micic, 1996; Xiang *y col.*, 2005; Torrico y Normey, 2005; Seshagiri y Chidambaram, 2005; Normey y Camacho, 2008; Seshagiri *y col.*, 2007; Majhi y Atherton, 1998). En la siguiente Sección, se propone reemplazar el compensador tradicional de Smith mediante un esquema observador que en el caso inestable permite estabilizar al sistema en lazo cerrado. El esquema propuesto se diseña con base en la teoría de observadores tradicionales. De esta manera, para obtener una estimación adecuada de la señal de salida retardada, es suficiente el modelo de la planta y una ganancia estática. Obsérvese que el esquema de predicción propuesto posee una estructura más simple que algunos de los esquemas presentados recientemente en la literatura, ver por ejemplo Seshagiri y Chidambaram (2005); Normey y Camacho (2008, 2009); Seshagiri *y col.* (2007).

En este trabajo se considera la noción clásica de estabilidad relacionada con la función de transferencia i.e., la estabilidad del sistema depende de la posición de los polos en el semiplano izquierdo del plano complejo para el caso continuo y dentro del círculo unitario en el caso discreto.

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3. Estrategia de estimación

La estrategia de predicción propuesta en este trabajo se muestra en la Fig. 2, donde como en el caso del PS se describe un módulo de estimación así como un controlador diseñado a partir del proceso libre de retardo. A continuación se presentan condiciones necesarias y suficientes para la existencia de un estimador de la señal $w(t)$ (ver Fig. 2). Además, se propone una simple metodología para obtener explícitamente dicho estimador cuando sea posible.

Considérese un sistema inestable con retardo en el lazo directo,

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} = \frac{b}{s-a}e^{-\tau s}, \tag{5}$$

donde $a > 0$. Note que $\tau_{un} = a^{-1}$, puede verse como la constante de tiempo inestable del proceso.

El siguiente resultado describe las condiciones que aseguran la estabilidad en lazo cerrado para el sistema con retroalimentación estática de la variable de salida.

Lema 1 *Considere el sistema retardado dado por la Ec. (5) y una retroalimentación proporcional de la salida,*

$$U(s) = R(s) - kY(s), \tag{6}$$

donde $R(s)$ es la nueva entrada de referencia. Entonces, existe k tal que el sistema en lazo cerrado, ecs. (5)-(6),

$$\frac{Y(s)}{R(s)} = \frac{be^{-\tau s}}{s-a+kbe^{-\tau s}} \tag{7}$$

es BIBO estable (bounded-input bounded-output) si y solo si $\tau < \frac{1}{a}$.

Demostración. Considere la discretización del proceso continuo dado por la Ec. (5) utilizando un retenedor de orden cero y un periodo de muestreo $T = \frac{\tau}{n}$ con $n \in \mathbb{N}$. De esta forma se obtiene,

$$G(z) = \frac{b}{a}\frac{(e^{aT}-1)}{z^n(z-e^{aT})}. \tag{8}$$

El sistema de la Ec. (8) en lazo cerrado con una retroalimentación estática de salida de la forma de la Ec. (6) produce la ecuación característica,

$$z^n(z-e^{aT}) + k\frac{b}{a}(e^{aT}-1) = 0. \tag{9}$$

El problema se reduce a mostrar que todas las raíces de la Ec. (9) están dentro del círculo unitario si y solo si $\tau < \frac{1}{a}$, cuando $T \to 0$, equivalentemente cuando se considera $\lim_{n\to\infty} \frac{\tau}{n}$. Considere ahora el diagrama del lugar geométrico de las raíces de la Ec. (9) (Evans, 1954). El sistema en lazo cerrado tiene n polos en el origen y uno en $z = e^{aT}$, dado que no existen polos finitos, tenemos $n+1$ trayectorias hacia el infinito, $n-1$ empiezan en el origen y las dos trayectorias restantes empiezan en un punto localizado entre el origen y $z = e^{aT}$. Este punto puede ser localizado fácilmente considerando,

$$\frac{dk}{dz} = \frac{d}{dz}\left[-\frac{z^n(z-e^{aT})}{\frac{b}{a}(e^{aT}-1)}\right] = 0.$$

Esto produce,

$$p(z) = (n+1)z^n - nz^{n-1}e^{aT},$$

que tiene $n-1$ raíces en el origen y una en $z = \frac{n}{n+1}e^{a\frac{\tau}{n}}$. Si el punto de ruptura sobre el eje real se encuentra dentro del circulo unitario, entonces el sistema en lazo cerrado tiene una región de estabilidad, de lo contrario el sistema es inestable para cualquier k.
Las propiedades de estabilidad del sistema equivalente continuo de las ecs. (5)-(6) son obtenidas considerando el límite cuando $n \to \infty$, o equivalente, cuando $T \to 0$, esto es,

$$\lim_{n\to\infty} z = \lim_{n\to\infty} \frac{n}{n+1}e^{a\frac{\tau}{n}} = 1. \tag{10}$$

Debido a que este punto está localizado en el límite de la estabilidad, es fácil ver que si $a\tau < 1$ (i.e., $\tau < \tau_{un}$), el límite tiende a uno por la izquierda. Entonces, existe k que estabiliza al sistema en lazo cerrado. En el caso $a\tau \geq 1$, no es posible estabilizar al sistema a través de una retroalimentación estática de salida (i.e., el límite tiende a uno por la derecha y un par de polos están fuera del círculo unitario).

Para los $n-1$ polos restantes, de la ecuación característica dada por la Ec. (9), tomando en cuenta el caso continuo equivalente $n \to \infty$, se obtiene,

$$\begin{aligned}\lim_{n\to\infty} p(z) &= \lim_{n\to\infty}\left[z^n(z-e^{aT}) + k\frac{b}{a}(e^{aT}-1)\right]\\ &= \lim_{n\to\infty}\left[z^n(z-e^{a\frac{\tau}{n}}) + k\frac{b}{a}(e^{a\frac{\tau}{n}}-1)\right]\\ &= (z-1)\lim_{n\to\infty} z^n.\end{aligned}$$

Por lo tanto, en este caso se confirma que un polo esta localizado en $z = 1$ y el resto de los polos están en el origen. De lo anterior, es claro que cuando un polo esta localizado en una vecindad del punto $z = 1$, todos los demás polos están en una vecindad del origen. Entonces, se muestra finalmente que el sistema puede ser estabilizado si y solo si $a\tau < 1$. ■

Observación 1 *En la prueba del lema anterior se hace uso del hecho que, el modelo en tiempo discreto de un sistema en tiempo continuo, coincide con el sistema continuo cuando el tiempo de muestreo $T \to 0$, considerando un periodo de muestreo y un dispositivo de retención.*

Observación 2 *Note que la estabilidad del sistema de la forma de la Ec. (7), se ha estudiado en la literatura (Kolmanovskii y Myshkis, 1992; Hale y Verduyn, 1993; Niculescu, 2001; Silva y Bhattacharyya, 2005; Shafiei Z., Shenton A.T., 1994) y la prueba del Lema 1 puede obtenerse también considerando diferentes enfoques como el de respuesta en frecuencia, la D-descomposición, o por el clásico Método de Pontryagin. Sin embargo, en este trabajo se ha presentado un enfoque novedoso para la obtención de dicho resultado, el cual es la base del esquema observador propuesto más adelante.*

Una vez garantizada la existencia de la ganancia k, a continuación se presenta un resultado que nos permite calcular de una manera práctica y sencilla un valor de k, que estabilice al sistema dado por la Ec. (7).

Lema 2 *Considere el sistema dado por la Ec. (5) con $\tau < \frac{1}{a}$. Entonces existe $k \in R^+$ dado en la Ec. (6) que estabiliza el sistema en lazo cerrado de las ecs. (5)-(6). Más aún, k satisface los límites $\alpha < k < \beta$, con $\alpha = \frac{a}{b}$ y para alguna constante $\beta > \frac{a}{b}$.*

Demostración. Suponga que $\tau < \frac{1}{a}$ y como en el Lema 1 considere la discretización del sistema de la Ec. (5) con $T = \frac{\tau}{n}$, $n \in \mathbb{N}$ dada por la Ec. (8). En el diagrama del lugar geométrico de las raíces asociado con el sistema discreto, es posible ver que en la configuración en lazo abierto el sistema tiene n polos en el origen y uno en $z = e^{aT}$ sin ceros finitos. Por esta razón, habrá $n-1$ trayectorias hacia el infinito y un par convergen a un punto sobre el eje real localizado entre el origen y $z = 1$.

Note que si $k = 0$, el sistema es inestable. La ganancia k que toma el acotamiento de la región de estabilidad ($z = 1$), se obtiene evaluando k con $z = 1$, esto es,

$$k = -\left.\frac{z^n(z - e^{aT})}{\frac{b}{a}(1 - e^{aT})}\right|_{z=1} = \frac{a}{b}.$$

Entonces con $k = a/b$ el sistema es marginalmente estable. Para concluir la prueba, note que el caso continuo se obtiene considerando nuevamente $n \to \infty$, (i.e., $T \to 0$) y por lo tanto, como $\tau < \frac{1}{a}$ existe una region de estabilidad, i.e., el sistema es estable para $\alpha < k < \beta$, con $\alpha = a/b$ y para alguna constante $\beta > a/b$. ■

Observación 3 *A partir de un análisis en el dominio de la frecuencia, no es muy dificil determinar con precisión el valor de β dado en el Lema anterior. De hecho, tal valor esta dado por $\beta = \frac{a}{b}\sqrt{1 + (\frac{\omega}{a})^2}$, donde ω es tal que satisface la relación $\frac{\omega}{a} = \tan(\omega\tau)$ para $0 < \omega < \frac{\pi}{2\tau}$. La utilidad del Lema 2 estriba en que cualquier $k = \frac{a}{b} + \epsilon$, con un $\epsilon > 0$ estabiliza el sistema en lazo cerrado de las ecs. (5)-(6) para ϵ suficientemente pequeño.*

Observación 4 *Nótese que conforme $\tau \to 0$, se tiene que $\omega \to \infty$ y por lo tanto la magnitud de k también tiende infinito, lo cual coincide con las propiedades de estabilidad de un sistema de la forma de las ecs. (5)-(6) libre de retardo, i.e., con $\tau = 0$.*

Podemos ahora presentar el resultado principal de este trabajo; las condiciones para la existencia de un esquema predictor como el descrito en la Fig. 2 para la clase de sistemas inestables con retardo dados en la Ec. (5).

Teorema 1 *Considere el esquema observador dado en la Fig. 2. Existe una ganancia $k \in R$ tal que $\lim_{t\to\infty} [\widehat{w}(t) - w(t)] = 0$ si y solo si $\tau < \frac{1}{a}$.*

Demostración. Considere el esquema observador dado en la Fig. 2, la dinámica completa de dicho esquema puede escribirse como,

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\widehat{x}}(t) \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} x(t) \\ \widehat{x}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ bk & -bk \end{bmatrix} \begin{bmatrix} y(t) \\ \widehat{y}(t) \end{bmatrix} + \begin{bmatrix} b \\ b \end{bmatrix} u(t)$$

$$\begin{bmatrix} y(t+\tau) \\ \widehat{y}(t+\tau) \end{bmatrix} = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \begin{bmatrix} x(t) \\ \widehat{x}(t) \end{bmatrix}.$$

Definiendo el error de predicción como $e_x(t) = \hat{x}(t) - x(t)$ es fácil obtener,

$$\dot{e}_x(t) = ae_x(t) - kbe_x(t-\tau).$$

Por lo tanto, a partir del Lema 1, se demuestra el resultado. ■

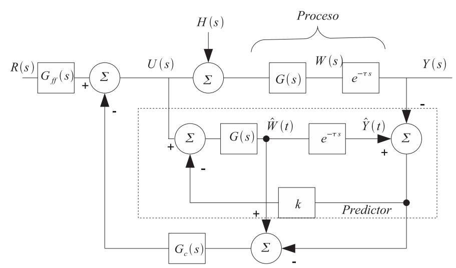

Fig. 2: Esquema de control propuesto

4. Regulación y rechazo de perturbaciones

Una vez establecido el esquema de predicción, la estructura de control se complementa con el uso de una acción Proporcional-Integral. Obsérvese que la estrategia de control puede ser implementada independientemente de la estrategia de estimación y por lo tanto, el uso control PI no es forzoso.

Por otro lado, más adelante se muestra que la estrategia de control propuesta permite rechazar perturbaciones de tipo escalón.

4.1. Acción Proporcional-Integral

Los métodos tradicionales de sintonización de los controladores PI/PID inducen un cero en el sistema en lazo cerrado que produce un sobreimpulso. Por esta razón, para mejorar las propiedades de seguimiento, una respuesta de sobreimpulso adecuada y reducción de tiempo de establecimiento, en la literatura se propone un esquema de control con dos grados de libertad (Astrom y Hagglund, 1995), también conocido como "PI-setpoint weighting". Siguiendo este enfoque, el controlador PI propuesto está dado por,

$$u(t) = K\left[e_p(t) + \frac{1}{T_i}\int_0^t e(s)ds\right] \qquad (11)$$

con una modificación en el error proporcional dada por, $e_p(t) = \sigma r(t) - y(t)$ y un error integral de la forma, $e(t) = r(t) - y(t)$. Bajo estas condiciones, la retroalimentación expresada por ls Ec. (11) puede ser escrita en una estructura con dos grados de libertad, como,

$$U(s) = R(s)G_{ff}(s) - Y(s)G_c(s) \qquad (12)$$

donde,

$$G_{ff}(s) = K(\sigma + \frac{1}{sT_i}) \qquad (13)$$

$$G_c(s) = K(1 + \frac{1}{sT_i}). \qquad (14)$$

Acorde con la idea de predicción de Smith, la sintonización del controlador PI está basada en el modelo del proceso libre de retardo. En este caso, se propone el diseño de dicho control a través de la reubicación de polos, la cual permite encontrar un controlador que proporciona las especificaciones deseadas en lazo cerrado. Para esto, considere la función de transferencia libre de retardo $G(s)$ dada en la Ec. (5) en lazo cerrado con un compensador PI dado por la Ec. (11). Así, se obtiene la ecuación característica,

$$s^2 + (bK - a)\,s + \frac{bK}{T_i} = 0. \qquad (15)$$

Considerado la caracterización general de un sistema de segundo orden en términos del parámetro de amortiguamiento ζ y la frecuencia natural ω_0, la Ec. (15) toma la forma estándar, $s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$, de donde es posible obtener, $K = \frac{2\zeta\omega_0 + a}{b}$ y $T_i = \frac{2\zeta\omega_0 + a}{\omega_0^2}$.

Note que la función de transferencia entrada-salida del proceso tiene un cero en $s = -\frac{1}{\sigma T_i}$. Para minimizar el sobreimpulso excesivo en la respuesta, Astrom y Hagglund (1995), proponen utilizar el parámetro σ tal que el cero se localice a la izquierda de los polos dominantes en lazo cerrado. Un valor razonable es $\sigma = \frac{1}{\omega_0 T_i}$, el cual ubica al cero en $s = -\omega_0$. También, el tiempo integral T_i puede ser aproximado para un valor grande de ω_0 como, $T_i = \frac{2\zeta}{\omega_0}$, y así, el cero del controlador PI es independiente de las dinámicas dominantes del sistema.

4.2. Rechazo de perturbaciones.

Para mostrar el rechazo de perturbaciones tipo escalón de la estrategia de control propuesta se presenta el siguiente resultado.

Lema 3 *Sea* $\tau < \frac{1}{a}$ *y considere el esquema de control mostrado en la Fig. 2, junto con el controlador PI descrito en la Ec. (11) anteriormente. Sea* $H(s)$ *una perturbación de tipo escalón y la entrada de referencia* $R(s) = 0$. *Entonces,* $\lim_{t\to\infty} y(t) = 0$.

Demostración. Considere nuevamente la estrategia de control de la Fig. 2. La función de transferencia $Y(s)/H(s)$ está dada por,

$$\frac{Y(s)}{H(s)} = \frac{G(s)e^{-\tau s}\left[1 + G(s)(J(s) - G_c(s)e^{-\tau s})\right]}{1 + (G(s)(J(s) - G_c(s)G(s)e^{-\tau s}))}. \tag{16}$$

Donde $J(s) = ke^{-\tau s} + G_c(s)$. Aplicando el teorema del valor final a la salida $Y(s)$ de la Ec. (16) y considerando una perturbación $H(s) = \eta/s$ (η constante), obtenemos,

$$\lim_{s\to 0} sY(s) = 0$$

Así, la estrategia de control es capaz de rechazar perturbaciones tipo escalón. ■

La metodología de control propuesta, que tiene como propósito estabilizar y al mismo tiempo mejorar la respuesta del sistema en lazo cerrado, se resume a continuación:

1. El cumplimiento del Teorema 1 ($\tau < \frac{1}{a}$). Este hecho garantiza la existencia del esquema de predicción.

2. Estabilización del predictor para asegurar estabilidad interna del sistema. Esto puede lograrse sintonizando k como se propone usando el resultado del Lema 2.

3. Diseño del controlador mediante la retroalimentación de la salida libre de retardo estimada. Una opción es utilizar la estrategia del control "PI setpoint weighting" (referida en la Fig. 2), o bien utilizar cualquier otro control deseado que estabilice el proceso libre de retardo $G(s)$.

5. Resultados en simulación

La metodología propuesta se evalúa a través de ejemplos comparativos tomados de la literatura reciente, (Normey y Camacho, 2009; Seshagiri *y col.*, 2007).

Ejemplo 1. Considere el control de concentración en un reactor inestable citado en Normey y Camacho (2009). El sistema en lazo abierto está dado por,

$$\frac{Y(s)}{U(s)} = \frac{3{,}433}{103{,}1s - 1} e^{-20s}.$$

Los controladores propuestos por Normey y Camacho (2009), para este proceso son, $C(s) = \frac{3{,}29(43{,}87s+1)}{43{,}87s}$, $F(s) = \frac{20s+1}{43{,}87s+1}$ y $F_r(s) = \frac{(20s+1)^2(93{,}16s+1)}{(43{,}87s+1)(26s+1)^2}$ (ver Normey y Camacho, 2009). La metodología propuesta en este trabajo sugiere estabilizar al observador con, $k = \frac{a}{b} + \varepsilon$. De esta manera, se escoge $\varepsilon = 0{,}4087$, por lo tanto $k = 0{,}7$. Los parámetros del controlador PI son, $K = 22{,}6$, $T_i = 20$ y $\sigma = 0{,}5$.

Con estos controladores, se compara el comportamiento de los dos esquemas de control, considerando una entrada escalón unitario y una perturbación $H(s)$ de tipo escalón, actuando a los $t = 300$ *seg*.

La Fig. 3, muestra las respuestas del sistema en lazo cerrado, considerando el conocimiento perfecto del proceso. Note que el método propuesto proporciona una mejor respuesta al rechazo de perturbación que la estructura propuesta por Normey y Camacho (2009). Note además que el problema de estabilidad interna se ve reflejado en el estado estacionario de la respuesta de la metodología dada en Normey y Camacho (2009). En la Fig. 4, se considera una condición inicial mínima en el error de salida, i.e., $y(t) - \widehat{y}(t) = 0{,}01$, para ambas estructuras. Es evidente que el esquema de control propuesto en Normey y Camacho (2009), presenta una dinámica inestable en el error de salida mientras que la estrategia de control aquí propuesta conserva una dinámica estable aún en estas condiciones. De esta manera se verifica nuevamente el problema de estabilidad interna que presenta el esquema de control en Normey y Camacho (2009).

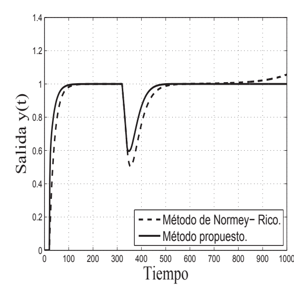

Fig. 3: Respuesta de salida considerando conocimiento exacto de los parámetros, Ejemplo 1.

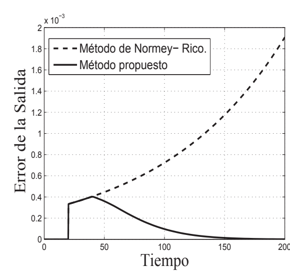

Fig. 4: Error de salida considerando condiciones iniciales diferentes de cero, Ejemplo 1.

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Ejemplo 2. Considere el sistema de primer orden con retardo dado por,

$$\frac{Y(s)}{U(s)} = \frac{1}{s-1} e^{-0,4s}.$$

Para este sistema, Seshagiri *y col.*, (2007) proponen los siguientes controladores $k_c = 6$, $\tau_i = 0{,}96$, $k_d = 2{,}078$, $\epsilon = 0{,}5$ y $\tau_f = 0{,}56$ (ver Seshagiri *y col.*, 2007). El esquema de control propuesto en el presente trabajo sugiere $k = 1{,}5$. Para obtener la misma velocidad del seguimiento de referencia, tenemos los parámetros del control PI, $K = 6$, $T_i = 0{,}96$ y $\sigma = 0{,}5$.

En la Fig. 5 se evalúa la respuesta del sistema en lazo cerrado para las dos estrategias de control considerando el conocimiento exacto del proceso, una entrada escalón unitario y una perturbación $H(s)$ tipo escalón de magnitud $-0{,}5$ unidades, actuando a los 15 *seg*. Note que el tiempo de recuperación del sistema con respecto a la perturbación es el mismo.

Cuando se pone en operación un sistema en la práctica, es bien sabido que no siempre es posible medir las condiciones iniciales del sistema para programar al modelo del sistema con las mismas condiciones iniciales del proceso. Por tal motivo, las estrategias de control diseñadas deben considerar este problema. De esta manera, la Fig. 6 muestra la comparación de las respuestas de salida para el esquema propuesto y la estructura presentada en Seshagiri *y col.*, (2007), considerando una condición inicial en el proceso. Obsérvese que la respuesta de la estructura de Seshagiri *y col.*, (2007) presenta sobreimpulsos, que en ciertos casos son indeseables, mientras que la estrategia propuesta muestra dichos sobreimpulsos atenuados considerablemente. De cualquier manera, ambas estrategias son estables en estado estacionario aún bajo estas condiciones. Finalmente en la Fig. 7, se muestra la respuesta de ambas estrategias considerando una variación paramétrica de +15 % en el retardo del proceso τ.

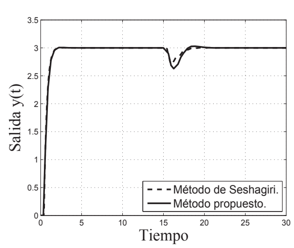

Fig. 5: Respuesta de salida considerando conocimiento exacto de los parámetros, Ejemplo 2.

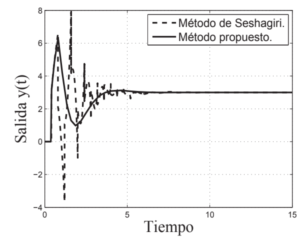

Fig. 6: Respuesta de salida considerando condiciones iniciales en el proceso, Ejemplo 2.

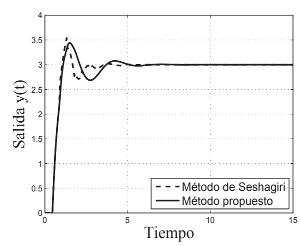

Fig. 7: Respuesta de salida considerando una variación parámetrica en el retardo de +15 %, Ejemplo 2.

Conclusiones

Los procesos inestables con retardo de tiempo representan comúnmente un problema de control difícil de abordar. De hecho, la existencia de un retardo de tiempo suficientemente grande representa el peor escenario en el caso de regulación, debido a los problemas de inestabilidad asociados a este fenómeno. En este trabajo se presentan condiciones explícitas bajo las cuales es posible la construcción de un predictor de salida considerando una configuración observador para una clase de sistemas de primer orden con retardo en el lazo directo. La predicción de salida es complementada con el uso de un controlador PI de dos grados de libertad, el cual es capaz de reducir sobreimpulsos en la respuesta de salida y permite rechazar perturbaciones de tipo escalón. La estrategia de control es simple y fácil de sintonizar. Para evaluar el desempeño de la estrategia de control propuesta, se presentan simulaciones comparativas con estrategias de control reportadas recientemente.

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B.2 Stabilization Strategy for First Order with large time-delay

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STABILIZATION STRATEGY FOR UNSTABLE FIRST ORDER LINEAR SYSTEMS WITH LARGE TIME-DELAY

J. F. Márquez-Rubio, B. del Muro-Cuéllar, M. Velasco-Villa, and J. Álvarez-Ramírez

ABSTRACT

This work considers the problem of stabilization of a class of unstable first order linear systems subject to a large input-output time delay. Necessary and sufficient conditions are stated to guarantee the stability of the closed loop delayed system by means of a compensation scheme based on two static gains and an induced delay term. The proof of the main result is derived by considering a discrete time approach that, under adequate assumptions, allows to conclude the stability condition for the continuous time case.

Key Words: Linear systems, input time-delay, stabilization.

I. INTRODUCTION

Time delays, appearing in the modeling of different classes of systems (chemical processes, manufacturing chains, economy, etc.) are originated by several mechanisms like material transport, recycling loops or even by the approximation of a high order system by means of a lower dimension one [1, 2]. From the control viewpoint, time delays become a challenging situation that should be affronted to yield acceptable closed-loop stability and performance. Several control strategies have been developed to deal with time delays. The simplest one consists in ignoring the effects of the time delay, designing a compensator for the delay-free process and applying the obtained controller to the actual delayed system. It is clear that this method works only in the case of stable processes with sufficiently small time delay. A second approach consists in the approximation of the delay operator by means of a Taylor or Pade series expansions that leads to non-minimum-phase process with rational transfer function representation. With the same stability purpose analysis, some works have applied the Rekasius substitution in order to obtain stability results, see for instance [3, 4].

A different class of compensation strategies consist of counteracting the time delay effects by means of schemes intended to predict the effects of current inputs in future outputs. The Smith prediction compensator (SPC) ([5, 6]) is the most common prediction strategy considered in the literature that provides a future output estimation by means of a type of open-loop observer scheme. The main limitation of the original SPC is the fact that the prediction scheme does not have a stabilization step, which restricts its application to open-loop stable plants. To alleviate this problem, some modifications to the original structure, and the ability to handle processes with an integrator and large time delay, have been reported [7–9]. Some extensions to the non-stable case have been also reported, see for instance [10, 11] and the references included there in.

The case of first order plus dead time unstable processes has been analyzed in [12], where a stability

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analysis based on [13] is used in order to calculate all stabilizing values of proportional controllers for such systems. In [14] the well known D-partition technique to estimate the stabilization limits of PID controllers is used. For unstable processes with dominant unstable time-constant τ_{un} and under the action of a PID control, the analysis leads to the constraints of the type $\tau<\tau_{un}$, where τ is the process time delay [15]. Stabilization conditions independent of the delay value have been reported in [16]. A complete set of PID-controllers for time-delay systems has been analyzed in [17] where different bounds for stabilization for first order dead time systems were provided. On the other hand, in [18] it is determined the set of stabilizing parameters of low-order controllers (P, PI, PID) for SISO linear time-invariant high order plants with time delay. The results are based on an extension of the Hermite-Bielhler Theorem. Necessary and sufficient conditions for the stability are given. However since the analysis consists mainly in satisfying a certain interlacing property, stability conditions with respect to time delay are not provided.

With a different perspective, in [19] a modification to the original Smith compensator is proposed, in order to deal with unstable first order delayed systems. Using a similar structure, the result is extended to delayed high order systems [20]. In both works, a robustness analysis is done concluding that for unstable dead time dominant systems, the resulting closed-loop can be destabilized with an infinitesimal value of the modeling error, i.e., that robustness is strongly dependent on the relationship τ/τ_{un}. For the control scheme proposed in [19, 20], it can be easily proven that in the case of unstable plants, the internal stability is not guaranteed. In fact, it is obtained an unstable dynamics for the estimation error and, as a result, a minimal initial condition error between the original plant and the model produces an internal unbounded signal.

A compensation strategy for large input time delays is presented in [21], where the provided method is able to stabilize first order unstable systems with time delay satisfying, $\tau \leq 1.5\tau_{un}$.

This work focuses on the stabilization problem of linear first-order unstable systems with time delays in the input channel. The main motivation to deal with this class of systems is based on the fact that, in some cases, high order systems can be approximated with first or second order system with time delays that could be considered as a first step toward studying the stability properties of high-order unstable delayed plants [1, 21].

The stabilization problem for first order unstable processes with significant large time delay at the direct path ($\tau>\tau_{un}$) is addressed, stating in particular, a stabilization problem as the one posed in [19]. It is shown that our control strategy produces a stable closed loop system able to solve the regulation problem without the initial condition problem of [19, 20] described above and without the use of a Smith prediction strategy. In addition it is shown that our method allows a stability condition $\tau \leq 2\tau_{un}$ instead of the one obtained in [21] restricted to $\tau \leq 1.5\tau_{un}$. The closed loop stability of the proposed scheme is analyzed by means of a discrete time representation of the systems when the sampling period T approaches zero.

The paper is organized as follows. Section II resumes the main problem when dealing with unstable time delay systems. Section III presents the main result concerning the case of first order unstable systems with significant large time delay at the direct path. The performance of the overall stabilization strategy is evaluated in Section IV by means of numerical simulations. Finally, Section V presents some conclusions.

II. CLASS OF SYSTEMS

Consider the class of single-input single-output (SISO) linear systems with delay at the input:

$$\frac{Y(s)}{V(s)} = \frac{N(s)}{D(s)}e^{-\tau s} = G(s)e^{-\tau s}, \tag{1}$$

where $V(s)$ and $Y(s)$ are the input and output signals respectively, $\tau \geq 0$ is the time delay, $N(s)$ and $D(s)$ are polynomials in the complex variable s and $G(s)$ is the delay-free transfer function. Notice that with respect to the class of systems (1), a traditional control strategy based on an output feedback of the form,

$$V(s) = [R(s) - Y(s)]Q(s), \tag{2}$$

produces a closed loop system given by,

$$\frac{Y(s)}{R(s)} = \frac{Q(s)G(s)e^{-\tau s}}{1+Q(s)G(s)e^{-\tau s}}, \tag{3}$$

where the term $e^{-\tau s}$ located at the denominator of the transfer function (3) leads to a system with an infinite number of poles and consequently, it is obtained a system where the stability properties should be carefully stated.

From the classical structure of the SPC depicted in Fig. 1, the closed loop transfer function is given by,

$$\frac{Y(s)}{R(s)} = \frac{Q(s)G(s)}{1+Q(s)G_1(s)+T(s)}e^{-\tau s},$$

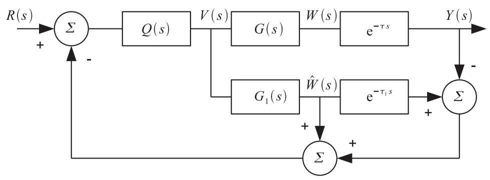

Fig. 1. Classical Smith Predictor Compensator structure.

with,

$$T(s) = Q(s)G_1(s)e^{-\tau_1 s} - Q(s)G(s)e^{-\tau s}.$$

It is easily seen that under ideal conditions, this is, under the exact knowledge of the plant parameters ($G(s) = G_1(s)$ and $e^{-\tau s} = e^{-\tau_1 s}$), the transfer function $Y(s)/R(s)$ of the closed loop system is obtained as,

$$\frac{Y(s)}{R(s)} = \frac{Q(s)G(s)}{1+Q(s)G(s)}e^{-\tau s}. \quad (4)$$

The SPC provides the future estimation, τ units of time ahead, of the signal $y(t)$ that could be used in a specific feedback scheme. Unfortunately, the original SPC is restricted to the case of stable plants [5, 6]. The case of unstable open loop plant has been also analyzed by several authors and consequently, it is possible to find modifications to the original compensator that result on adequate solutions for some particular cases, see for instance the results reported in [7, 8, 10, 11, 22]. In the following section, we will propose a control scheme for unstable systems that yields stable closed-loop operation.

In this work, the notion of stability is used in the classical sense when dealing with transfer functions, that is, it is assumed that a continuous system is stable when all the roots of its characteristic equation are on the left half complex plane or inside the unitary circle when considering the corresponding property for discrete time systems.

III. STABILIZATION STRATEGY FOR SYSTEMS WITH LARGE TIME DELAY

Consider the unstable input-output delay system,

$$\frac{Y(s)}{V(s)} = G(s)e^{-\tau s} \quad (5)$$

with

$$G(s) = \frac{b}{s-a} = \frac{ba^{-1}}{a^{-1}s-1},$$

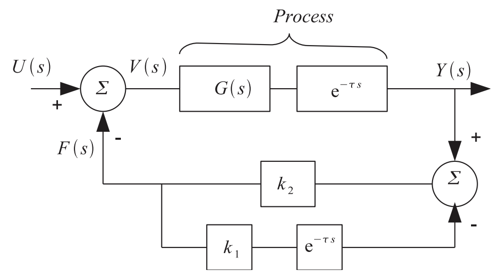

Fig. 2. Proposed stabilization scheme.

where $a > 0$. Notice that $\tau_{un} = a^{-1}$ can be seen as the unstable time-constant of the process.

In order to improve the stability properties of system (5) with respect to a proportional feedback of the form (2) in what follows it is proposed a stabilization scheme based on two proportional gains together with an induced time delay on the feedback loop. This particular array is depicted in Fig. 2.

Notice that, from Fig. 2, the feedback function $f(t)$ satisfies the difference equation,

$$f(t) = -k_1k_2 f(t-\tau) + k_2 y(t). \quad (6)$$

At a first glance, it is evident that the implicit discrete nature of feedback (6) imposes the restriction,

$$|k_1k_2| < 1, \quad (7)$$

in order to satisfy the stability conditions of the difference equation (6).

The closed loop transfer function of the system depicted in Fig. 2, with $G(s)$ as in (5), can be expressed as,

$$\frac{Y(s)}{U(s)} = \frac{G(s)e^{-\tau s}}{1+G(s)e^{-\tau s}\frac{k_2}{1+k_1k_2e^{-\tau s}}} = \frac{be^{-\tau s}(1+k_1k_2e^{-\tau s})}{(s-a)(1+k_1k_2e^{-\tau s})+k_2be^{-\tau s}}. \quad (8)$$

The complete stability condition for the overall control scheme given in Fig. 2 is stated in the following result.

Theorem 1. Consider the delayed system (5) and the feedback scheme shown in Fig. 2. There exist constants k_1 and k_2 such that the corresponding closed loop system given in equation (8), is stable if and only if $\tau < \frac{2}{a}$.

Proof. The result of the theorem will be proven in an indirect way by considering the alternate system

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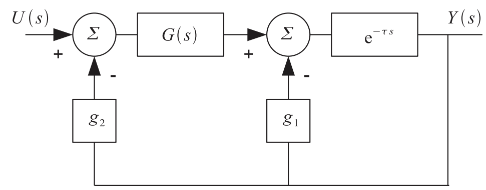

Fig. 3. Alternative stabilization structure.

depicted in Fig. 3 where its corresponding closed loop system is obtained as,

$$\frac{Y(s)}{U(s)}=\frac{be^{-\tau s}}{(s-a)(1+g_1e^{-\tau s})+g_2be^{-\tau s}}.$$

Note that in this equation, its corresponding characteristic equation is equivalent to the one derived from equation (8) when considering $g_1=k_1k_2$ and $g_2=k_2$.

In order to state the stability conditions for system (8), consider now a discrete time version of the original plant (5) together with the output injection scheme given in Fig. 3. To carry out this task, it is assumed that there exist a sampling period T that satisfies the condition $T=\frac{\tau}{n}$ for an integer n and that a zero order hold is located at the input of the system. Under these conditions, it is obtained,

$$\frac{Y(z)}{U(z)}=\frac{(b/a)(e^{aT}-1)(z^n+1/g_1)}{(z-e^{aT})(z^n+g_1)+g_2(b/a)(e^{aT}-1)}, \quad (9)$$

producing the characteristic polynomial,

$$p_1=(z-e^{aT})(z^n+g_1)+g_2(b/a)(e^{aT}-1). \quad (10)$$

The proof of the theorem is based on demonstrate that all roots in (10) lie inside the unit circle when it is considered, $\lim_{n\to\infty}\frac{\tau}{n}$, if and only if, $\tau<\frac{2}{a}$.

To begin with, consider first the simple case when $g_1=0$ in (9), this produces the characteristic equation,

$$(z-e^{aT})z^n+g_2(b/a)(e^{aT}-1)=0. \quad (11)$$

The root locus diagram ([23]) associated to (11) shows that the open loop system has n poles at the origin and one at $z=e^{aT}$. Then, there exist $n+1$ branches to infinity, $n-1$ of them starting at the origin and going directly to infinity. The two remaining branches starting at a breaking point z_1 located over the real axis between the origin and $z=e^{aT}$ (this situation is illustrated in Fig. 4 for the case $n=5$). z_1 can be found by considering the equation,

$$\frac{dg_2}{dz}=\frac{d}{dz}\left[-\frac{z^n(z-e^{aT})}{\frac{b}{a}(1-e^{aT})}\right]=0,$$

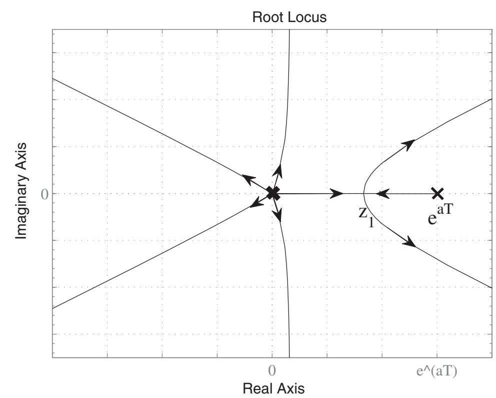

Fig. 4. Root locus of equation (11) for $n=5$.

that produces,

$$(n+1)z^n-nz^{n-1}e^{aT}=0,$$

which has $n-1$ roots at the origin and one at,

$$z_1=\frac{n}{n+1}e^{a\frac{\tau}{n}}.$$

If the breaking point z_1 over the real axis is located inside the unit circle, the closed loop system could have a region of stability, otherwise will be unstable for any g_2. The stability properties of the continuous system (8) are obtained by considering the limit as $n\to\infty$, or equivalently, when $T\to 0$, this is,

$$\lim_{n\to\infty} z_1=\lim_{n\to\infty}\frac{n}{n+1}e^{a\frac{\tau}{n}}=1. \quad (12)$$

It is important to note that any point $s=\theta$, over the real axis on the complex plane s is mapped to $z=e^{\theta T}$ on the z plane and as a consequence this point converges to $z=1$ when T tends to zero. Notice also that any real point $s=\theta$ on the left half side of the complex plane ($\theta<0$) is mapped to a point $e^{\theta T}$ that tends to one over the stable region of the z plane. On the contrary, if θ is on the right side of the complex plane over the real axis ($\theta>0$), the point $e^{\theta T}$ tends to one over the unstable region. Then, from (11), it is not difficult to see that if $a\tau<1$ (i.e., $\tau<1/a$) there exists a gain g_2 that stabilizes the closed loop system (i.e., the limit tends to one from the left). In the case that $a\tau\geq 1$ (always considering $g_1=0$) it is not possible to get g_2 that stabilize the system.

Consider now the case $g_1\neq 0$. Applying again a root locus analysis for system (9) and its characteristic equation (10), as g_1 grows from zero, the breaking point over the real axis moves in the root locus diagram (indeed, goes to the left). This point can be found by taking into account the equation,

$$\frac{dg_2}{dz}=\frac{d}{dz}\left[-\frac{(z-e^{aT})(z^n+g_1)}{(b/a)(e^{aT}-1)}\right]=0, \qquad (13)$$

yielding,

$$(n+1)z^n-nz^{n-1}e^{aT}+g_1=0. \qquad (14)$$

Expression (14) corresponds to the characteristic equation of a fictitious system of the form,

$$\begin{aligned}\frac{Y(z)}{V(z)}&=G(z)\\&=\frac{1/(n+1)}{z^n-z^{n-1}e^{aT}n/(n+1)} \qquad (15)\\&=\frac{1/(n+1)}{z^{n-1}(z-e^{aT}n/(n+1))}\end{aligned}$$

in closed loop with the feedback,

$$V(z)=U(z)-g_1Y(z). \qquad (16)$$

The open loop system (15) has $n-1$ root at the origin and one at

$$z=\frac{n}{n+1}e^{a\frac{\tau}{n}}.$$

If the breaking point over the real axis is located inside the unit circle, the closed loop system (15)–(16) could have a region of stability (once proved that the others $n-2$ poles are inside the unitary circle), otherwise the system will be unstable for any g_1. This point can be found by considering,

$$\frac{dg_1}{dz}=\frac{d}{dz}\left[-\frac{z^{n-1}\{z-e^{aT}n/(n+1)\}}{1/(n+1)}\right]=0, \qquad (17)$$

that produces,

$$z^{n-2}\left(z-\frac{n-1}{n+1}e^{aT}\right)=0,$$

which has $n-2$ roots at the origin and one at,

$$z=\frac{n-1}{n+1}e^{a\frac{\tau}{n}}.$$

As previously, the stability properties of the equivalent continuous system (8) are obtained by considering the limit as $n\to\infty$, or equivalently, when $T\to 0$. That is,

$$\lim_{n\to\infty}z=\lim_{n\to\infty}\frac{n-1}{n+1}e^{a\frac{\tau}{n}}=1.$$

Again, since this limit point is located on the stability boundary, in this case it is possible to see that if $a\tau\le 2$ (i.e., the limit tends to one from the left) there exists a gain g_1 that places the breaking point (two poles) inside the unit circle in the original discrete Root Locus diagram. Then, if the remaining $n-1$ roots are into the unit circle, the closed loop system is stable. In the case that $a\tau>2$ it is not possible to stabilize the system by static output injection (i.e., the limit goes to one from the right). Let us now prove that the remaining $n-1$ roots are into the unitary circle if and only if $a\tau<2$. Assume that $a\tau\le 2$ and to take into account the continuous case, the characteristic equation (10) it is modified as,

$$\begin{aligned}\lim_{n\to\infty}p_1(z)&=\lim_{n\to\infty}[(z-e^{a\frac{\tau}{n}})(z^n+g_1)\\&\quad+g_2(b/a)(e^{a\frac{\tau}{n}}-1)]\\&=(z-1)\lim_{n\to\infty}(z^n+g_1)=0\end{aligned}$$

from where it is stated that while one pole is on the neighborhood of $z=1$, the remaining poles are in a neighborhood of the points $(-g_1)^{1/n}$, inside the unit circle producing a stable closed loop system if, as it was previously stated, it is satisfied, $g_1<1$. From equation (17),

$$g_1=-\frac{z^n\{z-e^{aT}n/(n+1)\}}{1/(n+1)},$$

then if $z=1$,

$$\begin{aligned}g_1&=-\frac{\{1-e^{aT}n/(n+1)\}}{1/(n+1)}\\&=-(n+1-ne^{aT}).\end{aligned}$$

Taking into account the continuous case as previously done, we obtain

$$\begin{aligned}\lim_{n\to\infty}g_1&=\lim_{n\to\infty}-(n+1-ne^{a\tau/n})\\&=a\tau-1. \qquad (18)\end{aligned}$$

As $g_1<1$ is a necessary condition for the stability, $a\tau-1<1$, then $a\tau<2$. □

Remark 2. Note that the analytical idea of the previous proof can be applied for systems with superior order than the system (5). However it is not easy to get stability conditions with respect to time delay since more variables should be taken into account as well as not only one breaking point is appearing over the real axis on the complex plane.

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A useful practical result in order to compute the parameters involved on the control scheme is the following.

Corollary 3. Consider the control scheme described in Fig. 2. If $\tau<\frac{2}{a}$, then the parameters k_1 and k_2 that stabilize the closed loop system (8) satisfy,

$$a\tau-1<k_1k_2\leq a\tau-1+\sigma,$$

for some constant $\sigma>0$, and

$$\frac{a}{b}(k_1k_2+1)<k_2\leq\frac{a}{b}(k_1k_2+1)+\bar{\sigma},$$

for some constant $\bar{\sigma}>0$.

Proof. Taking into account that $g_1=k_1k_2$ and $g_2=k_2$, from equation (18) in the proof of the Theorem 1, we have,

$$\lim_{n\to\infty} g_1=\lim_{n\to\infty}-(n+1-ne^{a\tau/n})$$
$$=a\tau-1.$$

Therefore if $\tau<\frac{2}{a}$, there exist g_2 that stabilizes the closed loop system (8), with

$$a\tau-1<g_1\leq a\tau-1+\sigma$$

for some $\sigma>0$.

Now, from equation (13),

$$g_2=-\frac{(z-e^{aT})(z^n+g_1)}{(b/a)(e^{aT}-1)}.$$

Then, if $z=1$,

$$g_2=\frac{g_1+1}{(b/a)}=(a/b)(g_1+1).$$

The gain g_2 can be computed as,

$$\frac{a}{b}(g_1+1)<g_2\leq\frac{a}{b}(g_1+1)+\bar{\sigma},$$

for some $\bar{\sigma}>0$. □

Remark 4. It should be pointed out that the closed loop stability of equation (8) can be stated by considering the continuous time approach, presented in ([24]) where the conditions are stated by considering the set of finite poles of the equivalent transfer function of the free delay system ($\tau=0$) and its behavior when the input delay is not null. However, the results are significatively different, in ([24]) the stability property of the system is obtained for fixed g_1 and g_2 (or k_1 and k_2). In our case, the conditions are given explicitly for the system parameters τ and $1/a$ and a practical and easy way is proposed to obtain the stabilizing controller parameters k_1 and k_2 as stated in Corollary 3.

From Corollary 3 it is now possible to state a recursive algorithm in order to obtain stabilizing parameters k_1 and k_2. This procedure can be given as follows.

Algorithm 5.
Step 0:

1. Define $\sigma_0=0.6(2/a-\tau)$ and $\bar{\sigma}_0=0.02(2/a-\tau)$.

Step i:

1. Define $\sigma_i=\sigma_{i-1}/2$ and $\bar{\sigma}_i=\bar{\sigma}_{i-1}/2$.
2. Obtain k_2 and k_1 as,

$$k_2=\frac{a}{b}(a\tau+\sigma_i)+\bar{\sigma}_i,$$

$$k_1=\frac{a\tau-1+\sigma_i}{k_2}.$$

If at the step i it is obtained an unstable closed loop system for k_1 and k_2, proceed to step $i+1$. The algorithm ends when the obtained closed loop system is stable.

Notice that even when the resulting closed loop dynamic for the obtained k_1 and k_2 is stable, it is possible to continue the algorithm without breaking the stability properties of the system in order to improve the general closed loop response.

IV. SIMULATION RESULTS

The proposed methodology will be now illustrated by means of an academic example.

Example 1. Consider the unstable input delay system given by:

$$\frac{Y(s)}{V(s)}=\frac{6}{s-1}e^{-\tau s}. \tag{19}$$

with $\tau=1.5$.

From Theorem 1, it is clear that there exist gains k_1 and k_2 that stabilize the closed loop system depicted in Fig. 2 since the time delay satisfy $\tau<\frac{2}{a}$.

For the simulation experiments it is considered $\sigma=0.1$ and $\bar{\sigma}=0.0033$ and therefore it is obtained $k_2=0.27$ and $k_1=2.22$. In Fig. 5 it is depicted the behavior of the stabilized system by means of the evolution of the output signal $y(t)$. To carry out this experiment it was assumed the exact knowledge of the plant parameters.

Let us consider now the particular control strategy presented by Seshagiri ([21]) and implemented here by considering the parameters design $\lambda=0.7$, $\theta_m=1.5$,

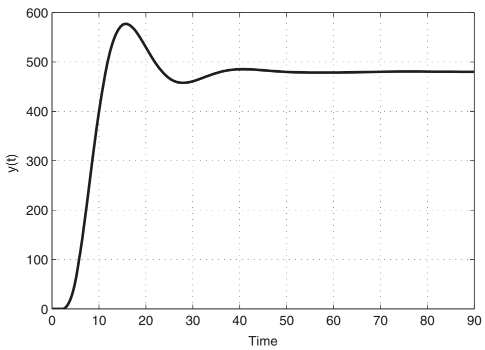

Fig. 5. Output signal in Example 1.

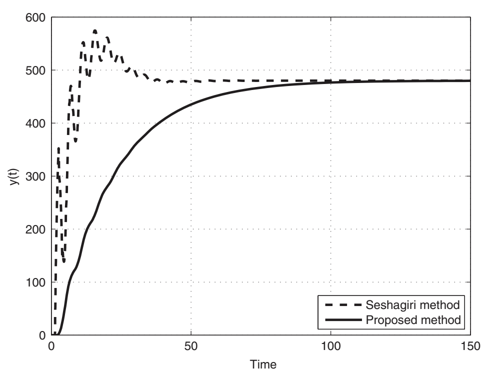

Fig. 6. Time delay uncertainty in the process of −13%.

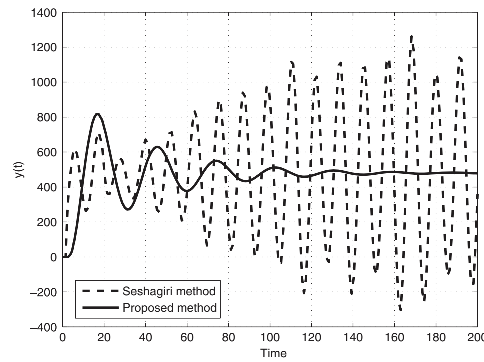

Fig. 7. Time delay uncertainty in the process of +3%.

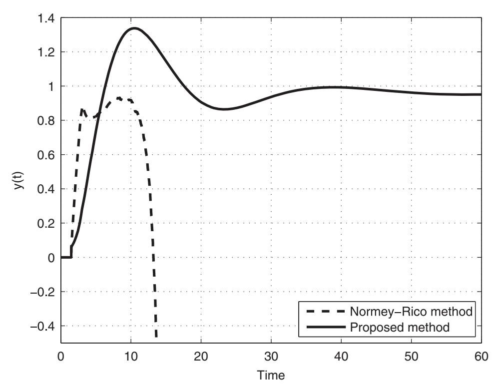

Fig. 8. Output response under initial condition different to zero.

$k_c = 0.4841$, $\tau_i = 3.2021$, $\varepsilon = 0.35$ and $k_d = 0.1701$. To evaluate the output signal evolution on both schemes it is considered uncertainties acting on the time delay. Figures 6 and 7 compare the output response of the strategies by considering −13% and +3%, respectively. It can can be seen that the method proposed in this work gives a better performance under time delay uncertainty.

Now, consider the control strategy proposed by Normey-Rico [20] applied to system (19). The tuning parameters are set as recommended, this is, $T_r = L_n = 1.5$. Assuming also a dead-time estimation error of 5%, $T_0 = 1.575$ is chosen. The above parameters gives as a result the following controllers (see [20] for details on the control structure),

$$C(s) = \frac{0.3889(5.25s+1)}{5.25s}, \quad F(s) = \frac{1.5s+1}{5.25s+1}$$

and

$$F_r(s) = \frac{(1.5s+1)^2(28.71s+1)}{(5.25s+1)(1.575s+1)^2}.$$

In Fig. 8, it is shown the output response of the process by considering an initial condition in the plant with a magnitude of 0.01. As mentioned in the Introduction, it is verified that the control strategy proposed in [20] and [19] it is not able to handle the specified problem due to the minimal initial condition error between the plant and the compensator.

On the other hand, it is important to note that when the time delay is large enough (i.e. near of the limit $\frac{2}{a}$), the stability region of the closed loop system becomes more limited. In this case, the computation of the controller parameters is more involved. In order to illustrate this fact, let us consider again the system given by (19) together with different values of τ. The

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Table I. Parameter values.

τ	k_1k_2	k_2	M_g
1.1	0.2	0.21	1.06
1.5	0.6	0.27	1.02
1.8	0.86	0.311	1.006

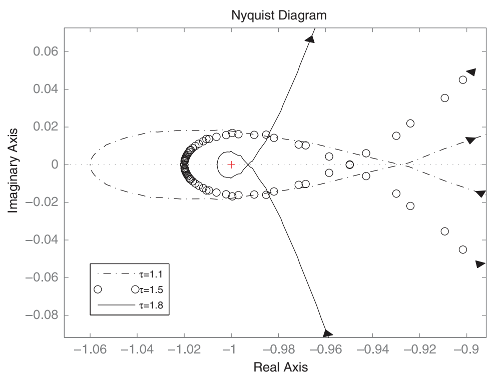

Fig. 9. Nyquist diagram for different values of τ.

parameters of the controller for each τ are provided in Table I.

From the closed loop characteristic equation,

$$(s-1)+\beta(s+\alpha)6e^{-\tau s}=0, \quad (20)$$

with, $\beta=\frac{k_1k2}{6}$ and $\alpha=-a+\frac{6}{k_1}$, the corresponding Nyquist diagram for different values of τ is shown in Fig. 9 where from the Nyquist criterion the conditions that assures stability are satisfied. In Table I is also presented the gain margin M_g of the closed loop system for each case, making evident the fact that when the time delay increase its value, the stability region of the system decrease in size.

V. CONCLUSIONS

Unstable processes with significant time delays are commonly a challenging control problem. In fact, the existence of a large input delay represents the worst case scenario of the regulation problem due to the instability problems associated with this phenomenon. This work presents necessary and sufficient conditions for the stabilization of unstable first order systems with large time delays at the input-output path. In fact, the stabilization conditions presented in this work allows to stabilize First Order Unstable systems with a large time delay in a simpler manner that the tuning methods presented in recent works in the literature. The problem is solved by proposing a two degrees of freedom feedback that considers two simple static gains and adds a time delay effect. The stability conditions are obtained by considering an alternative discrete time approach that allows us to derive the continuous time case by taking the sampling time T tending to zero. The effectiveness of the proposed strategy is evaluated by simulations on an academic example.

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B.3 On the Control of Unstable First Order with Large Time lag

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On the Control of Unstable First Order Linear Systems with Large Time lag: Observer Based Approach

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Abstract

This work considers the problem of stabilization and control of a class of unstable first order linear systems subject to a relatively large input-output delay. As a first step, the conditions to ensure the stability of the system in closed loop with an output injection strategy are presented. In a second step, the stability conditions are used to design an observer-based scheme that provides a forward output estimation together with a feedback compensation to guarantee prediction convergence. The robustness of the overall observer-based strategy is analyzed when considering uncertainties on the magnitude of the time delay associated with the plant and the one considered on the design of the observer. A stability region as a function of these two time delays is obtained. The proposed prediction scheme is complemented by the use of a PI compensator to track step reference signals and to reject step disturbances.

Keyword: Time delay, stabilization, state prediction.

1 Introduction

Time delays, appearing in the modeling of different classes of systems (chemical processes, manufacturing chains, economy, etc.) are originated by several mechanisms like material transport, recycling loops or even by the approximation of a high order system by means of a lower dimension one [22, 6]. From a control viewpoint, time delays become a challenging situation that should be affronted to yield acceptable closed-loop stability and performance. Several control strategies have been developed to deal with time delays. The simplest approach consists in ignoring the effects of the time delay, designing a compensator for the delay-free process and applying the obtained controller to the delayed system. It is clear that this method works only in the case of processes with sufficiently small delay. When the control law is implemented in a digital computer, the discretization of an input delay process, for a rational time delay τ, produces a rational transfer function in the complex variable "z" free of delays (in this case the sampling period T, satisfies $T = \tau/n$, for some integer n). When the continuous case is considered, the delay operator can be approximated by means of a Taylor or Padé series expansions which could leads to non-minimum-phase process with rational transfer function representation

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[3]. Some works have considered the well known Rekasius substitution approach in order to obtain stability results, see for instance [13].

A second class of compensation strategies consist in counteracting the time delay effects by means of strategies intended to predict the effects of current inputs over future outputs. The Smith Predictor Compensator (SPC) [23] has been the most used prediction strategy providing a future output estimation by means of a type of open-loop observer scheme (see Figure 1). The main limitation of the original SPC is the fact that the prediction scheme has not a stabilization step that restricts its application to open-loop stable plants. To alleviate this problem, some extensions to deal with processes with an integrator and large time delay have been reported [2, 7, 18]. Also, some modifications of the original structure that allow to consider the non-stable case has been studied [8, 19, 16, 17]. For instance, Seshagiri et al., [20] present an efficient modification to the SPC in order to control unstable first order plus time delay plants. Their methodology is restricted to systems of the type $\tau < 1.5\tau_{un}$ where τ is the process time delay and τ_{un} the unstable time-constant.

The case of first order plus dead time unstable processes has been analyzed in [14], where a stability analysis is done in order to calculate all the stabilizing values of a proportional controller for such systems. In [5] the well known D-partition technique to estimate the stabilization limits of PID controllers is used. For unstable processes with dominant unstable time-constant and under the action of a PID control, the analysis leads to a constraint of the type $\tau < \tau_{un}$. Based on an extension of the Hermite-Bielhler Theorem, a complete set of PID-controllers for time-delay systems has been analyzed by [21] where different bounds of stabilization for first order dead time systems were provided. In [12], upper bounds on the delay size are provided when using linear time invariant controllers on the stabilization of strictly proper delayed real rational plants. It is important to note that in general, the bounds provided in [12] are not tight and that for case of a plant with one unstable pole it is obtained an exact bound. The authors prevent that the developed controllers are not intended as practical solutions and are only used as a tool to compute the achievable delay margin for some particular cases. A continuous pole placement algorithm is presented in [11] for the stabilization of a class of linear time-delay systems of order n. In the case that only the output is available for measurement it is considered the stabilization of the system by means of an observer-based approach.

In a more classical perspective, Normey-Rico et al., [16] proposes a modification to the original Smith Structure in order to deal with unstable first order delayed systems. Using a similar structure, the result is extended to delayed high order systems [17]. In both works, a robustness analysis is done concluding that for unstable dead time dominant systems, the obtained closed-loop can be destabilized with an infinitesimal value of the modeling error, i.e., that robustness is strongly dependent on the relationship τ/τ_{un}. For the control scheme proposed in this two latter works, it can be easily proven that in the case of unstable plants, the internal stability is not guaranteed. In fact, it is obtained an unstable estimation error and, as a result, a minimal initial condition mismatch, between the original plant and the model produces an internal unbounded signal. Notice that in a practical situation it is no possible to exactly measure the initial condition of the plant to assign the same value to the model considered on the modified Smith compensator.

This work focuses precisely on this latter problem, let say, the consideration of a continuous first order linear unstable processes subject to large input delays, with special interest in the case $\tau > \tau_{un}$. To solve the described problem, in this work, an observer-based strategy is proposed to obtain the prediction of the signal located between the plant and the delay operator. As a preliminary result, the standard case of systems restricted to a time delay τ smaller than the system unstable time constant ($\tau < \tau_{un}$) is analyzed. Then, following a similar procedure, the stabilization problem for first order unstable processes with significant large time delay at the direct path ($\tau > \tau_{un}$) is addressed. In both cases, necessary and sufficient conditions are stated in terms of τ and τ_{un}. The closed-loop system is analyzed under the assumption of time-delay uncertainties that yield as a consequence the consideration of different time delay for the original plant and the designed observer. It is shown that the system is robust under these circumstances and a region of stability as a function of both time-delays is provided. To complement the strategy proposed in this work, a PI control action that makes use of the predicted signal is designed to induce step tracking and asymptotic rejection under external step disturbances.

The paper is organized as follows. After a brief introduction, in Section 2 it is presented the considered class of systems together with a recall of the original SPC. In addition, as a preamble to the main result, it is presented an initial estimator where its convergence error is analyzed restricted to the case of systems with time-delay smaller than the unstable time constant. After this, in Section 3 it is addressed the case of first order unstable systems with significant large time-delay at the direct path and, with a particular distribution of the time delay inspired by [11], a control strategy is proposed achieving an improvement in the stability condition. In Section 4, robustness analysis with respect to time delay uncertainty is included. The regulation problem is solved in Section 5 and a step disturbance strategy is also implemented. The performance of the overall control strategy is evaluated by means of numerical simulations in Section 6. Section 7 presents the final conclusions of the work.

2 Class of Systems

Consider the class of single-input single-output (SISO) linear systems with delay at the input:

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} \tag{1}$$

where $U(s)$ and $Y(s)$ are the input and output signals respectively, $\tau \geq 0$ is the time delay and $G(s)$ is the delay-free transfer function. Notice that with respect to the class of systems (1) a traditional control strategy based on an output feedback of the form

$$U(s) = [R(s) - Y(s)]Q(s) \tag{2}$$

produces a closed loop system given by

$$\frac{Y(s)}{R(s)} = \frac{Q(s)G(s)e^{-\tau s}}{1 + Q(s)G(s)e^{-\tau s}} \tag{3}$$

where the term $e^{-\tau s}$ located at the denominator of the transfer function (3) leads to a system with an infinite number of poles and where the closed loop stability properties should be carefully stated. From the classical structure of the SPC depicted in Figure 1, the closed loop transfer function is given by,

$$\frac{Y(s)}{R(s)} = \frac{Q(s)G(s)}{1 + Q(s)G_1(s) + T(s)}e^{-\tau s},$$

with,

$$T(s) = Q(s)G_1(s)e^{-\tau_1 s} - Q(s)G(s)e^{-\tau s}.$$

It is easily seen that under ideal conditions, this is, under the exact knowledge of the plant parameters ($G(s) = G_1(s)$ and $e^{-\tau s} = e^{-\tau_1 s}$), the transfer function $Y(s)/R(s)$ of the closed loop system is obtained as,

$$\frac{Y(s)}{R(s)} = \frac{Q(s)G(s)}{1 + Q(s)G(s)}e^{-\tau s}. \tag{4}$$

The SPC provides a future estimation, τ units of time ahead, of the signal $y(t)$ that could be used in a specific feedback scheme. However, if $G(s)$ is not a stable plant then $T(s)$ implies an unstable cancellation. Then, the classical SPC is restricted to the case of stable plants [23] and as mentioned earlier, in order to deal with the unstable case, some authors have proposed several modifications to the original compensator that results on adequate solutions for some particular cases [2, 10, 20, 16]. In the following section, we will propose, instead of a modification of the SPC, an observer based scheme for the unstable case that yields stable closed-loop operation. The proposed scheme is designed based on the traditional observer theory. Hence, the plant model and two static gains are enough in order to get an adequate estimation of the output delay free signal. The main idea is to propose a prediction scheme with a simpler structure and stronger stability properties face to large delays, when compared to controllers proposed in recent literature [19, 16, 17, 20, 11, 12].

2.1 Estimation Strategy Restricted to $\tau < \frac{1}{a}$

Consider the unstable input-output delay system,

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} = \frac{b}{s-a}e^{-\tau s} \tag{5}$$

where $a > 0$. Notice that $\tau_{un} = a^{-1}$ can be seen as the unstable time-constant of the process. In what follows the existence conditions of a static output feedback that assures the stability of the closed loop system is stated. With this purpose, consider a proportional output feedback,

$$U(s) = R(s) - kY(s). \tag{6}$$

If the time delay τ is small relatively to the time constant τ_{un}, then, there exists a gain k such that the closed loop system (5)-(6)

$$\frac{Y(s)}{R(s)} = \frac{be^{-\tau s}}{s - a + kbe^{-\tau s}} \tag{7}$$

is stable. More precisely, the following result can be stated.

Lemma 1 *Consider the delayed system (5) and the proportional output feedback (6). Then, there exist a proportional gain k such that the closed loop system (7), is stable if and only if $\tau < \frac{1}{a}$.*

The stability of time delay (7) has been widely studied in the literature [6, 21, 15] and the proof of Lemma 1 can be easily obtained by considering different approaches as a classical frequency domain; D-decomposition or even by the classical Pontryagin Method.

Assuming that conditions in Lemma 1 holds, it is possible to design a predictor compensator that provides the estimation of the system state, i.e. estimate the signal $w(t)$ located between the plant and the delay operator, that could be used on the solution of the more general path tracking or disturbance decoupling problem and not only for regulation purposes.

Consider the observer-based scheme described in Figure 2 where, as in the case of a SPC, it is also depicted the estimation module (dotted line) as well as a possible controller compensator $Q(s)$. In what follows necessary and sufficient conditions for the existence of a future output estimator for unstable processes are presented.

Lemma 2 *Consider the observer based scheme given in Figure 2. There exists a proportional gain k such that $\lim_{t\to\infty}[\widehat{w}(t) - w(t)] = 0$ if and only if $\tau < \frac{1}{a}$.*

Proof. From Figure 2, the complete dynamics of the prediction scheme can be written as,

$$\begin{bmatrix} \dot{w}(t) \\ \dot{\widehat{w}}(t) \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} w(t) \\ \widehat{w}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ bk & -bk \end{bmatrix} \begin{bmatrix} y(t) \\ \widehat{y}(t) \end{bmatrix} + \begin{bmatrix} b \\ b \end{bmatrix} u(t)$$

$$\begin{bmatrix} y(t+\tau) \\ \widehat{y}(t+\tau) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w(t) \\ \widehat{w}(t) \end{bmatrix}.$$

Defining the prediction error as $e_w(t) = \widehat{w}(t) - w(t)$ it is easy to obtain,

$$\dot{e}_w(t) = ae_w(t) - kbe_w(t-\tau).$$

Therefore, using Lemma 1 the result is stated. ■

Essentially, Lemma 2 states that an observer based control strategy, as the one presented on Figure 2, can be implemented for the unstable first order system as long as the time delay is not larger in magnitude than the unstable time constant $\tau_{un} = a^{-1}$.

3 Estimation Strategy for Large Time Delays

An observer based scheme is proposed in order to consider the case when a significant large time delay $\tau > \frac{1}{a}$ is present at the direct path. This scheme is depicted in Figure 3. In what follows, necessary and sufficient conditions are obtained for the existence of a future output estimator for unstable plants that improves the conditions given in the previous section. In addition, it is proposed a simple and effective methodology in order to explicitly obtain the mentioned estimator.

3.1 Time-delay $\tau < \frac{2}{a}$

As a preliminary result, the stability conditions for the auxiliary closed loop system shown in Figure 4 are stated. These conditions will be used later in the proof of the main result of this work.

Lemma 3 *Consider the delayed system (5) and the static output injection scheme shown in Figure 4. There exist constants g_1 and g_2 such that the closed loop system*

$$\frac{Y(s)}{U(s)} = \frac{be^{-\tau s}}{(s-a)(1+g_1 e^{-\tau s}) + g_2 b e^{-\tau s}} \tag{8}$$

is stable if and only if $\tau < \frac{2}{a}$.

Proof. The proof of this lemma is provided in Appendix A ■

As a consequence of the previous results we can state now the main result of this work.

Theorem 4 *Consider the observer based scheme shown in Figure 3. Then there exist constants g_1 and g_2 such that $\lim_{t\to\infty} [\hat{w}(t) - w(t)] = 0$ if and only if $\tau < \frac{2}{a}$.*

Proof. The proof can be easily done by taking into account the stability conditions given in Lemma 3. With this aim, consider the dynamic of the prediction scheme shown in Figure 3 that can be written in state space form as,

$$\begin{bmatrix} \dot{w}(t) \\ \dot{\widehat{w}}(t) \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} w(t) \\ \widehat{w}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ bg_2 & -bg_2 \end{bmatrix} \begin{bmatrix} y(t) \\ \widehat{y}(t) \end{bmatrix} + \begin{bmatrix} b \\ b \end{bmatrix} u(t) \tag{9}$$

$$\begin{bmatrix} y(t+\tau) \\ \widehat{y}(t+\tau) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w(t) \\ \widehat{w}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ g_1 & -g_1 \end{bmatrix} \begin{bmatrix} y(t) \\ \widehat{y}(t) \end{bmatrix} \tag{10}$$

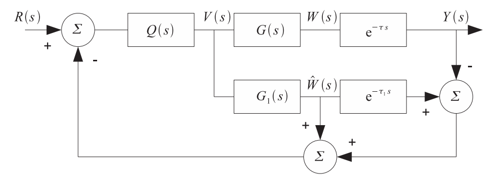

Figure 1: Classical Smith predictor compensator scheme.

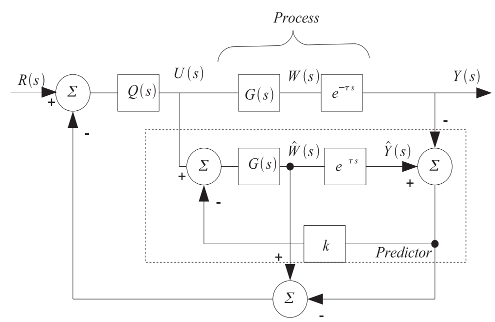

Figure 2: Proposed control scheme for $\tau < \frac{1}{a}$.

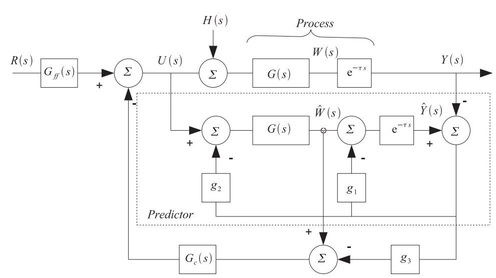

Figure 3: Proposed control scheme for $\tau < \frac{2}{a}$.

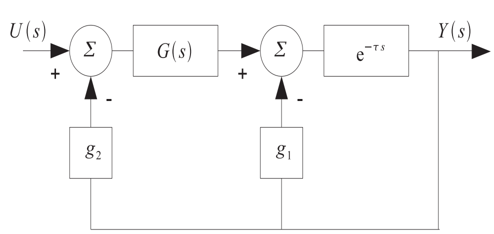

Figure 4: Observer base predictor scheme for $\tau < \frac{2}{a}$.

with $\widehat{w}(t)$ the estimation of $w(t)$. Defining first the state prediction error $e_w(t) = \widehat{w}(t) - w(t)$ and the output estimation error $e_y(t) = \widehat{y}(t) - y(t)$ it is possible to describe the behavior of the error signal as:

$$\begin{bmatrix} \dot{e}_w(t) \\ e_y(t+\tau) \end{bmatrix} = \begin{bmatrix} a & -bg_2 \\ 1 & -g_1 \end{bmatrix} \begin{bmatrix} e_w(t) \\ e_y(t) \end{bmatrix}. \tag{11}$$

Consider now a state space realization of system (8) (described in Figure 4) that can be written as,

$$\begin{bmatrix} \dot{x}(t) \\ y(t+\tau) \end{bmatrix} = \begin{bmatrix} a & -bg_2 \\ 1 & -g_1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} u(t). \tag{12}$$

It is clear now that the stability conditions of systems (12), given in Lemma 3, are equivalent to the ones of system (11), from where, the result of the theorem follows. ■

It is not an easy task to get the stability region associated with gains g_1 and g_2, however a useful and practical result in order to compute the parameters involved on the predictor scheme is the following one.

Corollary 5 *Consider the observer based scheme shown in Figure 3. If $\tau < \frac{2}{a}$, then the parameters g_1 and g_2 such that $\lim_{t\to\infty} [\hat{w}(t) - w(t)] = 0$ can be computed by considering the inequalities $a\tau - 1 < g_1 \leq a\tau - 1 + \varepsilon$, $\frac{a}{b}(g_1 + 1) < g_2 \leq \frac{a}{b}(g_1 + 1) + \bar{\varepsilon}$, for some constants $\varepsilon, \bar{\varepsilon} > 0$.*

Proof. The proof of this result is given in Appendix A ■

Remark 6 *It should be pointed out that the closed loop stability of equation (8) can also be stated following the results presented in ([9]) where the conditions are stated by considering the set of finite poles of the equivalent transfer function of the free delay system ($\tau = 0$) and their behavior when the input delay is not null. However, the results are significantly different, in ([9]) the stability property of the system is obtained for fixed g_1 and g_2. In our case, the conditions are given explicitly for the system parameters τ and $1/a$ and a practical and easy way is proposed to obtain the stabilizing controller parameters g_1 and g_2 as stated in Corollary 5.*

From Corollary 5 it is now possible to give a recursive algorithm in order to obtain stabilizing parameters g_1 and g_2. This procedure can be given as follows.

Algorithm 7 *Step 0:*

1. *Define $\varepsilon_0 = 0.6(2/a - \tau)$ and $\bar{\varepsilon}_0 = 0.02(2/a - \tau)$.*

Step i:

1. *Define $\varepsilon_i = \varepsilon_{i-1}/2$ and $\bar{\varepsilon}_i = \bar{\varepsilon}_{i-1}/2$.*
2. *Obtain g_2 and g_1 as,*

$$g_2 = \frac{a}{b}(a\tau + \varepsilon_i) + \bar{\varepsilon}_i,$$
$$g_1 = a\tau - 1 + \varepsilon_i.$$

If at the step i it is obtained an unstable closed loop system for the obtained g_1 and g_2, proceeds to step $i+1$. The algorithm ends when the obtained closed loop system is stable.

Notice that even if the resulting closed loop dynamic for the obtained g_1 and g_2 results stable it is possible to continue the algorithm without breaking the stability properties of the system in order to improve the general closed-loop response.

3.2 Time-delay $\tau < \frac{3}{a}$

In Michels et al. [11] it is considered the stabilization of time-delay systems of order n based on a numerical stabilization approach. The methodology consists in shifting the unstable eigenvalues to the left half plane by static state feedback by applying small changes to the feedback gain. Special attention is devoted to the scalar case, which coincides with the class of the systems addressed in the present work. In terms of stability, it is shown that for an observed state feedback, the problem can be solved for $\tau < 2\tau_{un}$.

Although the methodology proposed in [11] is completely different to our strategy, inspired by the time-delay splitting strategy of [11], it is possible to improve the result presented previously as follows.

Consider the unstable input-output delay system (5), rewritten as,

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} = e^{-\tau_2 s}\frac{b}{s-a}e^{-\tau_1 s},$$

where $\tau_1 = \tau/3$ and $\tau_2 = 2\tau/3$. The compensation scheme given in Figure 3 can be modified as the one presented in Figure 5. Then, we can state the following result.

Corollary 8 *Consider the stabilizing scheme shown in Figure 5. Then there exists constants g_1, g_2 and k such that the closed loop system $Y(s)/R(s)$ is stable if and only if $\tau < \frac{3}{a}$.*

Sketch of the proof. From the results presented previously and from Figure 5 just note that Theorem 4 can be applied directly for a system with a delayed input $u(t - \tau_1)$ since the convergence of the observer is not affected, i.e., there exist constants g_1 and g_2 such that $\lim_{t\to\infty}[\hat{w}(t) - w(t)] = 0$ if and only if $\tau_2 < \frac{2}{a}$. Based on the convergence of $\hat{w}(t)$, Lemma 1 can now be applied by considering the estimated signal $\hat{w}(t)$, i.e., there exists a constant k such that the closed-loop system $Y(s)/R(s)$ is stable if and only if $\tau_1 < \frac{1}{a}$. Then, it can be concluded that the bound for the original time-delay τ results $\tau = \tau_1 + \tau_2 < \frac{3}{a}$. ■

Remark 9 *It is important to note that the splitting strategy introduced in [11] combined with the control scheme described in Figure 5 produces a stable closed-loop system but not allow the implementation of a PI controller to achieve step tracking or step disturbance rejection.*

4 Robustness with respect to time-delay uncertainty

In the preceding developments, a control strategy has been presented under the assumption of a complete knowledge of the actual process. In practice, it is desired to get a control strategy that provides stability conditions with respect to model uncertainties, in particular, due to the observer based strategy considered in this work, the observer time-delay may be different from one associated with the plant. In what follows, it will be shown that the results presented in [4], can be used in the case presented in this work in order to analyze the robustness properties of the control strategy addressed in this work with respect to the time-delay. With this aim, consider a characteristic quasipolynomial of the form,

$$p(s) = p_0(s) + p_1(s)e^{-\tau s} + p_2(s)e^{-\tau_0 s} = 0 \tag{13}$$

where its stability properties will be established as a function of the time-delays τ and τ_0.

Following [4] it is possible to give a general framework for our particular case. Let T denote the set of all points $(\tau, \tau_0) \in \mathbb{R}^2_+$ such that $p(s)$ has at least one zero on the imaginary axis. Any

$(\tau, \tau_0) \in T$ is known as a crossing point and T is the collection of all stability crossing curves. Consider now system (5) and the predictor scheme shown in Figure 3 with $G_{ff} = 1$, $g_3 = 0$ and $G_c = \overline{k}$ which leads to the feedback law $U(s) = \overline{k}\widehat{W}(s)$. After straightforward computations, considering τ as the delay in the observer and τ_0 as the one in the process, the closed-loop characteristic equation is given by,

$$p_A(s) = p_a(s) + p_b(s)e^{-\tau s} + p_c(s)e^{-\tau_0 s} = 0, \tag{14}$$

with,

$$\begin{aligned} p_a(s) &= s^2 + (b\overline{k} - 2a)s + a(a - b\overline{k}) \\ p_b(s) &= g_1 s^2 + \left[b(g_2 + \overline{k}g_1) - 2ag_1\right] s + \left[a^2 g_1 - ab(g_2 + \overline{k}g_1)\right] \\ p_c(s) &= b^2\overline{k}g_2. \end{aligned}$$

It is clear that the characteristic equation (14) has the form of (13), therefore it is possible to identify the regions of (τ, τ_0) in $\mathbb{R}^2_+$ such that $p_A(s)$ is stable.

Following [4], Figure 6 shows the region (τ, τ_0) for the characteristic equation (14). This figure illustrates the range of values $[\tau_{0\,\min}, \tau_{0\,\max}]$ such that the proposed observer-based controller with a nominal delay τ is able to stabilize the closed-loop system, i.e., such that the characteristic equation (14) remains stable.

5 Regulation and disturbance rejection problem

Once the prediction scheme has been established, the proposed control structure will be complemented with a proportional-integral action and a simple and effective step disturbance rejection strategy. It should be noticed that the control strategy can be implemented independent of the estimation strategy and therefore we are not forced to use a PI control structure. In Subsection 5.1 the proposed controller is designed as if the delay free output signal $w(t)$ were available. Obviously, at the implementation, the estimated signal $\hat{w}(t)$ is used according to Figure 3. In order to achieve step disturbance rejection an additional gain g_3 is required. This is formalized in Subsection 5.2.

5.1 Proportional-Integral Action

The traditional tuning methods of PI/PID controllers induce a zero on the closed loop system that produces an undesirable overshoot. To improve the tracking properties of the system together with an adequate overshoot response and set time reduction, it has been proposed in the literature a two degree of freedom control scheme [1], also known as PI-setpoint weighting tuning. Following this approach, the proposed PI-controller is given by,

$$u(t) = K\left[(e_p(t) + \frac{1}{T_i}\int_0^t e(s)ds\right] \tag{15}$$

with a modified proportional error given by, $e_p(t) = \sigma r(t) - y(t)$ and an integral error of the form $e(t) = r(t) - y(t)$. Under these conditions, feedback (15) can be rewritten in a two-degree-of-freedom structure as,

$$U(s) = R(s)G_{ff}(s) - Y(s)G_c(s) \tag{16}$$

where,

$$G_{ff}(s) = K(\sigma + \frac{1}{sT_i}) \text{ and } G_c(s) = K(1 + \frac{1}{sT_i}). \tag{17}$$

According with the Smith prediction idea, the tuning of the PI controller is based on the delay free model of the plant and in this case, it is based on a pole placement strategy that attempts to find a controller that gives a desired closed-loop behavior. For doing this, consider the free-delay transfer function $G(s)$ given in equation (5) in closed loop with the PI compensator (15). It is obtained the characteristic equation,

$$s^2 + (bK - a)\, s + \frac{bK}{T_i} = 0. \tag{18}$$

By considering the general characterization of a second order system in terms of the relative damping parameter ζ and the natural frequency ω_0, equation (18) takes the standard form, $s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$, from where it is possible to obtain $K = \frac{2\zeta\omega_0 + a}{b}$ and $T_i = \frac{2\zeta\omega_0 + a}{\omega_0^2}$.

Notice that the transfer function from the setpoint input to the process output has a zero at $s = -\frac{1}{\sigma T_i}$. Following Astrom et al. [1], to avoid excessive overshoot in the response; parameter σ has to be chosen so that the zero is located to the left of the dominant closed-loop poles. A reasonable value is $\sigma = \frac{1}{\omega_0 T_i}$, which places the zero at $s = -\,\omega_0$. Also, the integral time T_i can be approximated for a sufficiently large ω_0 as $T_i = \frac{2\zeta}{\omega_0}$, and thus, independent of the process dynamics.

5.2 Step disturbance rejection

A step disturbance rejection property is easily added to the observer-based control scheme given in Figure 3.

Lemma 10 *Consider the proposed observer scheme shown in Figure 3. Then, there exist a PI controller with two degree of freedom given by (16) able to reject the step disturbance H(s) if* $g_3 = g_1 + 1$.

Proof. Consider the transfer function $Y(s)/H(s)$ of the control structure given in Figure 3,

$$\frac{Y(s)}{H(s)} = \frac{G(s)e^{-\tau s}[T(s) + G_c(s)G(s)(g_1 e^{-\tau s} - g_3 e^{-\tau s})]}{T(s) + G_c(s)G(s)[g_1 e^{-\tau s} - G(s)g_2 e^{-\tau s}]}, \tag{19}$$

where, $T(s) = 1 + g_1 e^{-\tau s} + G(s)g_2 e^{-\tau s} + G_c(s)G(s)$ and $G(s)$, $G_c(s)$ are defined previously in equations (5) and (17) respectively. Consider the application of the final value theorem to equation (19) with $H(s) = \frac{1}{s}$ and $g_3 = g_1 + 1$, then it is an easy task to verify that under the assumption of the lemma, it is obtained,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 0.$$

Hence, the control strategy is able to reject step disturbance. ■

The proposed methodology, intended to stabilize and at the same time improve the overall response of the system, can be summarized as follows:

1. Fulfillment of the conditions of Theorem 4 ($\tau < \frac{2}{a}$). This fact states the existence of a prediction scheme.
2. Predictor stabilization. This can be achieved by tuning the parameters g_1 and g_2, using the results in Corollary 5.
3. Compute g_3, in order to reject step disturbance.
4. Design of a PI-controller with a “set point weighting” strategy (refer to Figure 3), or any other desired controller stabilizing the delay free plant $G(s)$.

6 Simulation Results

The effectiveness of the proposed methodology will be now evaluated by means of three academic examples. The results will be compared with alternative strategies taken from the recent related literature.

Example 1. Consider the control concentration of the unstable reactor addressed in [17]. The open-loop system is given by,

$$\frac{Y(s)}{U(s)} = \frac{3.433}{103.1s - 1}e^{-20s}. \tag{20}$$

The control structure proposed in this work (depicted in Figure 3) is implemented by considering,

$$g_1 = a\tau - 1 + \varepsilon,\ g_2 = \frac{a}{b}(1 + g_1) + \bar{\varepsilon} \text{ and } g_3 = 1 + g_1$$

with $\varepsilon = 0.8060$ and $\bar{\varepsilon} = 0.4087$, obtaining $g_1 = 0$, $g_2 = 0.7$ and $g_3 = 1$. The PI controller parameters, given in equation (17), are set to $K = 22.6$, $T_i = 1$ and $\sigma = 0.5$.

For process (20), Normey-Rico et. al [17] proposed the following controllers (for the considered control structure see [17]),

$$C(s) = \frac{3.29(43.87 + 1)}{43.87s},\ F(s) = \frac{20s + 1}{43.87s + 1}$$

and

$$F_r(s) = \frac{(20s + 1)^2(93.16s + 1)}{(43.87s + 1)(26s + 1)^2}.$$

The performance of the two schemes is compared by considering a positive unit step input and a step disturbance $H(s)$ acting at $t = 300$. Figure 7, shows the closed-loop responses when considering an exact knowledge of the model parameters. Notice that the proposed methodology produces a better disturbance rejection result than the one obtained by the method addressed in [17]. The initial conditions problems mentioned in the introduction for the methodology proposed in [17] are evident in Figure 8 where a minimal initial conditions error $y(t)-\widehat{y}(t) = 0.01$ shows the unstable error dynamics.

Example 2. Consider the unstable delay system,

$$\frac{Y(s)}{U(s)} = \frac{6}{s - 1}e^{-1.5s}. \tag{21}$$

Let us consider the particular control strategy presented by Seshagiri et al., [20] and implemented here by considering the parameters design: $\lambda = 0.7$, $\theta_m = 1.05$, $k_c = 0.4841$, $\tau_i = 3.2021$, $\epsilon = 0.35$ and $k_d = 0.1701$. For the methodology proposed in the present work and depicted in Figure 3, it was considered the controller parameters $\varepsilon = 0.1$, $\bar{\varepsilon} = 0.0033$ producing as a consequence $g_1 = 0.6$, $g_2 = 0.27$ and $g_3 = 1.6$. The PI compensator, given in equation (17), was tuned by consider $K = 0.4841$, $T_i = 3.2021$ and $\sigma = 0.35$ in order to achieve a similar set-point tracking speed as that one of [20]. To evaluate the output signals evolution on both schemes it was considered a positive unit step input and a positive step disturbance $h(t) = 0.003$ acting at $t = 20$. Figure 9 shows the obtained responses of both cases when it is considered the exact knowledge of the model parameters. The method proposed in this work is observed to give a better response. In Figure 10 it is shown the responses obtained for the two control structure when the input time-delay is increased by 5%. In this case, the structure proposed in [20] becomes unstable while the method proposed here remains stable. It can be shown from Section 4 that for a 5% disturbance on the time-delay, our strategy remains on the stability

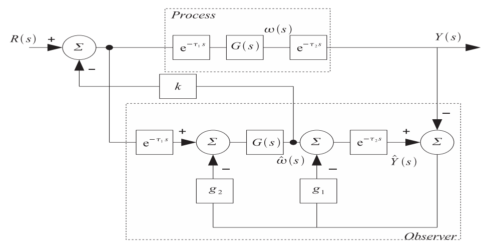

Figure 5: Proposed control scheme for $\tau < 3/a$

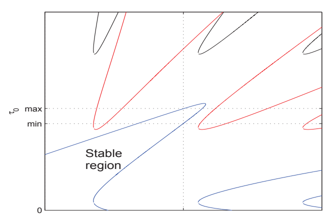

Figure 6: Stability region for τ and τ_0 for the characteristic equation (14)

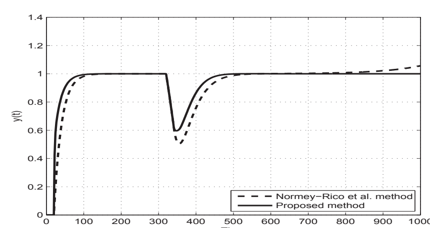

Figure 7: Output time evolution when considering exact knowledge of parameters on Example 1.

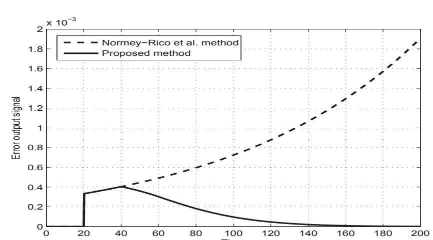

Figure 8: Error output signals under initial state conditions.

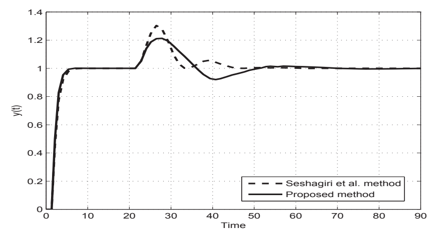

Figure 9: Output time evolution when considering exact knowledge of parameters on Example 2.

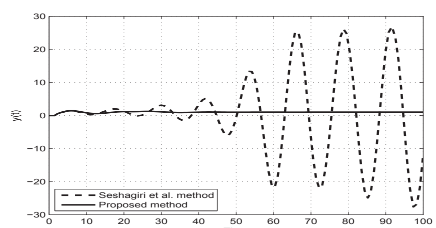

Figure 10: Output time evolution for a parametric variation of +20% on Example 2.

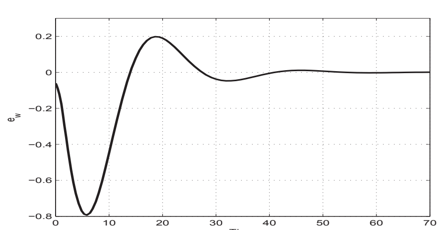

Figure 11: Estimation error $e_w(t) = \widehat{w}(t) - w(t)$ on Example 2.

Table 1: Range of time delay uncertainties.

a	b	τ	g_1	g_2	$\bar{k}$	$\tau_{0\,\min}$	$\tau_{0\,\max}$
1	6	1.5	0.6	0.27	1.1	1.313	1.5367
1	6	1.5	0.75	0.292	1.1	1.3904	1.6198
1	2	1.8	0.86	0.934	1.1	1.7834	1.8097
1	2	1.8	0.87	0.94	1.1	1.7875	1.8105

region described by τ and τ_0. Figure 11 presents the estimation error $e_w(t) = \widehat{w}(t) - w(t)$ of the scheme given in Figure 3.

The next example, that cannot be treated with the strategy reported in [17] or in [20] is presented in order to show the effectiveness of our control scheme.

Example 3. Consider the unstable delayed system,

$$\frac{Y(s)}{U(s)} = \frac{2}{s-1}e^{-1.8s}. \tag{22}$$

It is clear that condition $\tau < \frac{2}{a}$ of Theorem 4 is satisfied. Notice that the delay term is almost equal to the relation $\frac{2}{a}$. The control strategy presented in this work and depicted in Figure 3 is computed by using Corollary 5 producing the gains $g_1 = 0.86$ and $g_2 = 0.934$ for $\varepsilon = 0.06$ and $\bar{\varepsilon} = 0.001$. The PI controller parameters (equation 17) are $k = 3$, $T_i = 3$ and $\sigma = 0.3$. The observer based structure proposed in [11] is considered with $k = 0.53$ and $l = 0.51$, see [11] for details about its implementation. Consider also the PID controller proposed in [21] with a stabilizing proportional gain region given by $-0.5078 < k_p < -0.5$ and the set of PID parameters picked as $k_p = -0.503$, $k_i = -0.0002$ and $k_d = -0.46$. Figure 12 shows the stability region (k_i, k_d) for different values of k_p, as we can see, the stable region (inside the quadrilateral areas) becomes difficult to find due to the size of the time delay. It is important to note that the transfer function in [21] is defined with a negative gain when compared with (22), then the parameters (k_p, k_i and k_d) in the implementation must be inverted.

Considering an exact knowledge of the plant parameters and a small error between the initial conditions of the plant and its model, Figure 13 shows the comparison of the output signal evolution with respect to the control strategy proposed by [11], note how our strategy provide a slightly better performance that the one proposed by [11]. Notice also that for the strategy proposed by [11], due to its numerical nature, it is not evident to include a PI or PID controller in order to get step tracking reference since the closed loop stability is compromised. Figure 14 shows the robustness of the strategy by considering a process time-delay variation of $\tau = 1.8043$ and from where the advantages of our strategy are evident. To end the comparison with [11], Figure 15 shows the rejection of a step disturbance $h(t)$ acting at 100 *sec.*

Under ideal conditions, the PID controller performance, designed in [21] only for the stabilization problem, is presented in Figure 16, where the output signal response is depicted. It is clear the excessive overshoot in the output response as well as the large setting time, even when a unitary step reference is considered. This result is a consequence of the challenging delay consideration $\tau = 1.8 < 2/a$ that restrict the stabilization conditions of the closed-loop system as is described in the following remark.

Remark 11 *To make emphasis on the problems when dealing with large time delays as in the Example 3 ($\tau \to 2/a$), the analysis developed in Section 4 can be used to show the range of time delay that guarantee closed-loop stability for several values of parameters a, b, τ, g_1, g_2 and $\bar{k}$. This case is shown in Table 1 from where it is possible to see how the robustness can be improved*

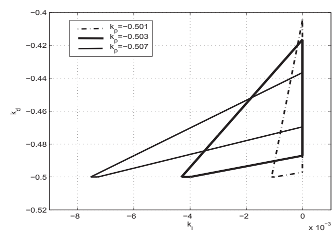

Figure 12: Stable regions (k_i, k_d), for different values of k_p

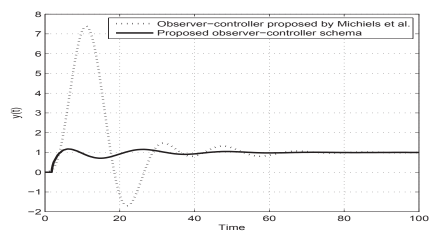

Figure 13: Output signal in Example 3.

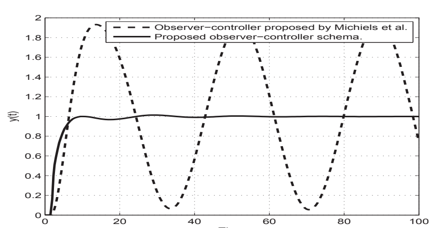

Figure 14: Output signal under parametric variations.

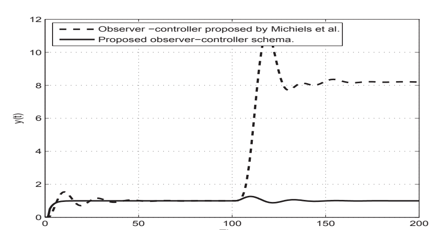

Figure 15: Output signal under step disturbance.

by changing the observer parameters g_1 and g_2, leading to a compromise between performance and robustness.

7 Conclusions

Unstable processes with significant time delays are commonly a challenging control problem. In fact, the existence of large input delays represents the worst scenario in the case of any closed loop-strategy control due to the instability problems associated with this phenomenon. This work presents the regulation problem of continuous first order linear unstable processes with large direct path delays. It is proposed an observer based prediction strategy that under explicit conditions provides convergent error estimation. The delay free output estimation obtained by the observer strategy is used together with a PI controller to reduce the output overshoot and reject step disturbances. The main advantage of the proposed scheme is not only their simplicity but also that it allow to stabilize delayed systems with considerable large time delay when compared with the unstable time constant, i.e., $\tau < 2\tau_{un}$ and $\tau < 3\tau_{un}$. The effectiveness of the propose strategy is evaluated by numerical simulations and compared with some previous works, verifying that this scheme allows a nonzero internal initial conditions error in opposition to the work reported in [17]. In addition, the proposed scheme gives a better response when compared with the scheme proposed in [20] under delay uncertainties where the maximum considered time-delay is $\tau < 1.5\tau_{un}$. In a more challenging situation $\tau \to 2\tau_{un}$, our strategy was compared with the results given in [11] and [21] showing clearly the advantages of our approach under parametric uncertainties. Derived from the main results of this work as well as a particular distribution of the time delay inspired by the methodology in [11], a new control scheme is proposed, achieving an improvement in the stability conditions.

A Complementary proofs

A.1 Proof of Lemma 3

In order to state the stability conditions for the system (8), consider a discrete time version of the original plant (5) together with the output injection scheme given in Figure 4. To carry out this task, it is assumed that there exist a sampling period T that satisfies the condition $T = \frac{\tau}{n}$ for an integer n and that a zero order hold is located at the input of the system. Under these conditions, the discrete time closed-loop transfer function is,

$$\frac{Y(z)}{U(z)} = \frac{(b/a)(e^{aT}-1)}{(z-e^{aT})(z^n+g_1)+g_2(b/a)(e^{aT}-1)}, \tag{23}$$

with the characteristic polynomial given by,

$$p_1(z) = (z-e^{aT})(z^n+g_1)+g_2(b/a)(e^{aT}-1). \tag{24}$$

The proof of the theorem is based on demonstrate that all roots in (24) lie inside the unit circle when $\lim_{n\to\infty} \frac{\tau}{n}$, i.e., when the sampling period T tend to zero (the continuous case), if and only if, $\tau < \frac{2}{a}$.

To begin with, consider first the simple case when $g_1 = 0$ in (23), this produces the characteristic equation,

$$(z-e^{aT})z^n - g_2(b/a)(e^{aT}-1) = 0. \tag{25}$$

The root locus diagram associated to (25) shows that the open loop system has n poles at the origin and one at $z = e^{aT}$. Then, there exist $n+1$ branches to infinity, $n-1$ of them starting at the origin and going directly to infinity. The two remaining branches starting at a breaking point z_1 located over the real axis between the origin and $z = e^{aT}$ (this situation is illustrated in Figure 17 for the case $n = 5$). z_1 can be found by considering the equation,

$$\frac{dg_2}{dz} = \frac{d}{dz}\left[-\frac{z^n(z - e^{aT})}{\frac{b}{a}(1 - e^{aT})}\right] = 0,$$

that produces,

$$(n + 1)z^n - nz^{n-1}e^{aT} = 0,$$

which has $n - 1$ roots at the origin and one at,

$$z_1 = \frac{n}{n+1}e^{a\frac{\tau}{n}}.$$

If the breaking point z_1 over the real axis is located inside the unit circle, the closed loop system could have a region of stability, otherwise will be unstable for any g_2. The stability properties of the continuous system (8) are obtained by considering the limit as $n \to \infty$, or equivalently, when $T \to 0$, this is,

$$\lim_{n\to\infty} z_1 = \lim_{n\to\infty} \frac{n}{n+1}e^{a\frac{\tau}{n}} = 1. \tag{26}$$

It is important to note that any point $s = \theta$, over the real axis on the complex plane s is mapped to $z = e^{\theta T}$ on the z plane and as a consequence this point converges to $z = 1$ when T tends to zero. Notice also that any real point $s = \theta$ on the left half side of the complex plane ($\theta < 0$) is mapped to a point $e^{\theta T}$ that tends to one over the stable region of the z plane. On the contrary, if θ is on the right side of the complex plane over the real axis ($\theta > 0$), the point $e^{\theta T}$ tends to one over the unstable region. Then, from (25), it is not difficult to see that if $a\tau < 1$ (i.e., $\tau < 1/a$) there exists a gain g_2 that stabilizes the closed loop system (i.e., the limit tends to one from the left). In the case that $a\tau \geq 1$ (always considering $g_1 = 0$) it is not possible to get g_2 that stabilize the system.

Consider now the case $g_1 \neq 0$. Applying again a root locus analysis for system (23) and its characteristic equation (24), as g_1 grows from zero, the breaking point over the real axis moves in the root locus diagram (indeed, goes to the left). This point can be found by taking into account the equation,

$$\frac{dg_2}{dz} = \frac{d}{dz}\left[-\frac{(z - e^{aT})(z^n + g_1)}{(b/a)(e^{aT} - 1)}\right] = 0, \tag{27}$$

yielding,

$$(n + 1)z^n - nz^{n-1}e^{aT} + g_1 = 0. \tag{28}$$

Expression (28) corresponds to the characteristic equation of a fictitious system of the form,

$$\begin{aligned}\tfrac{Y(z)}{V(z)} &= G(z)\\ &= \frac{1/(n+1)}{z^n - z^{n-1}e^{aT}n/(n+1)}\\ &= \frac{1/(n+1)}{z^{n-1}(z - e^{aT}n/(n+1))}\end{aligned} \tag{29}$$

in closed loop with the feedback,

$$V(z) = U(z) - g_1 Y(z). \tag{30}$$

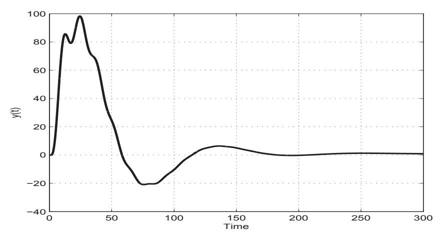

Figure 16: Output signal controlled by a PID structure [21].

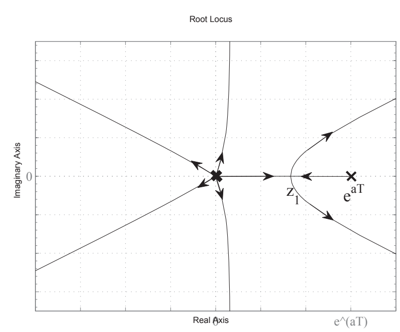

Figure 17: Root locus of equation (25) for $n = 5$.

The open loop system (29) has $n-1$ root at the origin and one at

$$z = \frac{n}{n+1} e^{a\frac{\tau}{n}}.$$

If the breaking point over the real axis is located inside the unit circle, the closed loop system (29)-(30) could have a region of stability (once proved that the others $n-2$ poles are inside the unitary circle), otherwise the system will be unstable for any g_1. This point can be found by considering,

$$\frac{dg_1}{dz} = \frac{d}{dz}\left[-\frac{z^{n-1}\{z - e^{aT}n/(n+1)\}}{1/(n+1)}\right] = 0, \tag{31}$$

that produces,

$$z^{n-2}(z - \frac{n-1}{n+1}e^{aT}) = 0,$$

which has $n-2$ roots at the origin and one at,

$$z = \frac{n-1}{n+1} e^{a\frac{\tau}{n}}.$$

As previously, the stability properties of the equivalent continuous system (8) are obtained by considering the limit as $n \to \infty$, or equivalently, when $T \to 0$. That is,

$$\lim_{n\to\infty} z = \lim_{n\to\infty} \frac{n-1}{n+1} e^{a\frac{\tau}{n}} = 1.$$

Again, since this limit point is located on the stability boundary, in this case it is possible to see that if $a\tau \leq 2$ (i.e., the limit tends to one from the left) there exists a gain g_1 that places the breaking point (two poles) inside the unit circle in the original discrete Root Locus diagram. Then, if the remaining $n-1$ roots are into the unit circle, the closed loop system is stable. In the case that $a\tau > 2$ it is not possible to stabilize the system by static output injection (i.e., the limit goes to one from the right). Let us now prove that the remaining $n-1$ roots are into the unitary circle if and only if $a\tau < 2$. Assume that $a\tau \leq 2$ and to take into account the continuous case, the characteristic equation (24) it is modified as,

$$\begin{aligned}\lim_{n\to\infty} p_1(z) = \lim_{n\to\infty} [&(z - e^{a\frac{\tau}{n}})(z^n + g_1) \\ &+ g_2(b/a)(e^{a\frac{\tau}{n}} - 1)] \\ &= (z-1)\lim_{n\to\infty}(z^n + g_1) = 0\end{aligned}$$

from where it is stated that while one pole is on the neighborhood of $z = 1$, the remaining poles are in a neighborhood of the points $(-g_1)^{1/n}$, inside the unit circle producing a stable closed loop system if, as it was previously stated, it is satisfied, $g_1 < 1$. From equation (31),

$$g_1 = -\frac{z^n\{z - e^{aT}n/(n+1)\}}{1/(n+1)},$$

then if $z = 1$,

$$\begin{aligned} g_1 &= -\frac{\{1 - e^{aT}n/(n+1)\}}{1/(n+1)} \\ &= -(n + 1 - ne^{aT}).\end{aligned}$$

Taking into account the continuous case as previously done, it is obtained,

$$\begin{aligned}\lim_{n\to\infty} g_1 &= \lim_{n\to\infty} -(n + 1 - ne^{a\tau/n}) \\ &= a\tau - 1.\end{aligned} \tag{32}$$

As $g_1 < 1$ is a necessary condition for the stability, $a\tau - 1 < 1$, then $a\tau < 2$.

A.2 Proof of Corollary 5

From equation (32) in the proof of Lemma 3 we have:

$$\lim_{n\to\infty} g_1 = a\tau - 1.$$

Therefore if $\tau < \frac{2}{a}$, there exist g_2 that stabilizes the closed loop system (8), with $a\tau - 1 < g_1 \leq a\tau - 1 + \varepsilon$ for $\varepsilon > 0$.

Now, from equation (27),

$$g_2 = -\frac{(z - e^{aT})(z^n + g_1)}{(b/a)(e^{aT} - 1)},$$

then, if $z = 1$,

$$g_2 = \frac{g_1 + 1}{(b/a)} = (a/b)(g_1 + 1).$$

Therefore, the gain g_2 can be obtained by considering the condition $\frac{a}{b}(g_1+1) < g_2 < \frac{a}{b}(g_1+1)+\bar{\varepsilon}$, for some $\bar{\varepsilon} > 0$.

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B.4 Observer-PID for Unstable First Order with Large Time Delay

Observer-PID Control for Unstable First Order Linear Systems with Large Time Delay. J.F. Marquez-Rubio, B. del Muro-Cuéllar and M. Velasco-Villa. Submitted to the journal Industrial & Engineering Chemistry Research.

Observer–PID Control for Unstable First Order Linear Systems with Large Time Delay

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Abstract

This work considers the problem of stabilization and control of a class of unstable first order linear systems subject to a relatively large input-output delay. As a first step, a particular observer schema is proposed in order to predict an specific internal signal in the process. The conditions to ensure the adequate prediction convergence of the signals are formally stated. In a second step, this internal predicted signal is used to implement classical P, PI and PID controllers providing the stability conditions of the resulting closed-loop system. This proposed control startegy allows to deal with time-delays as large as four times the unstable time constant of the open loop system. The proposed observer-based structure considers also the tracking of step reference signals and the rejection of input step disturbances.

Keyword: Time delay, stabilization, state prediction.

1 Introduction

Time delays, appearing in the modeling of different classes of systems (chemical processes, manufacturing chains, economy, etc.), become a challenging situation from a control viewpoint that should be affronted to yield acceptable closed-loop stability and performance. Several control strategies have been developed to deal with time delays. When the continuous case is considered, the delay operator can be approximated by means of a Taylor or Padé series expansions which could leads to a non-minimum-phase process with rational transfer function representation [4]. With the same stability purpose analysis, some works have applied the Rekasius substitution; see for instance [13].

A second class of compensation strategies consist in counteracting the time delay effects by means of strategies intended to predict the effects of current inputs over future outputs. The Smith Predictor Compensator (SPC) [22] has been the most used prediction strategy providing a future output estimation by means of a type of open-loop observer scheme. The main limitation of the original SPC is the fact that the prediction scheme has not a stabilization step, restricting

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its application to open-loop stable plants. To alleviate this problem, some extensions to deal with processes with an integrator and large time-delay have been reported [2, 7, 17]. Also, some modifications of the original structure that allow to consider the non-stable case has been studied [8, 15, 16, 18]. For instance, Seshagiri *et al.*, [19] present an efficient modification to the SPC in order to control unstable first order plus time-delay. Their methodology is restricted to systems satisfying $\tau < 1.5\tau_{un}$ where τ is the process time-delay and τ_{un} the unstable time-constant.

With a different perspective, in [12] upper bounds on the delay size ($\tau < 2\tau_{un}$) are provided when using linear time invariant controllers on the stabilization of strictly proper delayed real rational plants. It is important to note that in general, the provided bounds are not tight and the authors prevent that the developed controllers are not intended as practical solutions and are only used as a tool to compute the achievable delay margin for some particular cases. In [11], based in a numerical method, it is considered the stabilization of linear time-delay systems of order n. However, stability conditions with respect to time-delay and time constant of the process are not provided. The proposed method consists in shifting the unstable eigenvalues to the left half plane by static state feedback by applying small changes to the feedback gain, the same approach is implemented by considering an observer-based strategy. Furthermore, special attention is devoted in [11] to the first order unstable case, where the problem of stabilization of systems satisfying $\tau < 2\tau_{un}$ is solved by the observer-based approach. It is important to point out that for the proposed observer-based scheme in [11] it is not evident how to implement a proportional-integral (PI) or proportional-integral-derivative (PID) controller to get step reference tracking and step disturbance rejection.

The case of first order plus dead-time unstable processes has been analyzed in [14], where a stability analysis is done in order to calculate all the stabilizing values of proportional controllers for such systems. In [6], the well-known D-partition technique is used to estimate the stabilization limits of PID controllers. For unstable processes with dominant unstable time-constant and under the action of a PID control, the analysis leads to a constraint of the type $\tau < \tau_{un}$. Based on an extension of the Hermite-Bielhler Theorem, a complete set of PID-controllers for time-delay systems has been analyzed in [20, 21]. Different bounds for the stabilization of first order dead-time unstable systems are provided as well as a complete parameterization of the stabilizing P and PI controller in the case $\tau < \tau_{un}$ and the stabilizing PID controllers for the case $\tau < 2\tau_{un}$. Under a different perspective, in [5] it is presented a complete analysis that includes also the case of neutral systems.

It should be notice that in the works cited above, the stabilization problem of a first order dead-time system is restricted to the condition $\tau < 2\tau_{un}$. This stabilization upper bound is precisely the main topic of this work, i.e., the consideration of continuous first order linear unstable processes subject to large input time-delays, with special interest in the case $\tau > 2\tau_{un}$. The proposed strategy is based on an observer-controller design inspired in the methodology reported in [11], using the PID stabilizing parameterization in [20, 21]. As main results, this paper presents an observer-based stabilization structure with a P or PI controller that provides a new larger stabilization bound $\tau < 3\tau_{un}$. With the same stabilization scheme and considering as a controller a PID action it is proved that the closed-loop system can be stabilized for the improved condition $\tau < 4\tau_{un}$. Up to our best knowledge, until now, it has not been reported in the literature a control structure stabilizing a system under the condition $\tau > 2\tau_{un}$.

To complete our stabilization strategy it is shown that a particular modification of our control scheme, based on an additional static internal loop, allows to reject input step disturbances when a PI or PID control is used..

The paper is organized as follows. After a brief introduction, Section 2 presents the considered class of systems as well as some preliminary results. After this, in Section 3, the proposed observer strategy is developed. The regulation problem is solved in Section 4 and the step disturbance strategy is also implemented. The performance of the overall control strategy is evaluated by means of numerical simulations in Section 5. Finally, in Section 6 some conclusions are presented.

2 Preliminary results

Consider the linear, unstable, input-output delay system,

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} \tag{1}$$

where $U(s)$ and $Y(s)$ are the input and output signals respectively, $\tau \geq 0$ is the input time-delay and

$$G(s) = \frac{b}{s-a}$$

is the delay-free transfer function with $a, b > 0$. Notice that $\tau_{un} = a^{-1}$ can be seen as the unstable time-constant of the process. With respect to the class of systems (1) a traditional control strategy based on an output feedback of the form,

$$U(s) = C(s)[R(s) - Y(s)], \tag{2}$$

produces a closed loop system given by,

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)e^{-\tau s}}{1 + C(s)G(s)e^{-\tau s}} \tag{3}$$

where the term $e^{-\tau s}$ located at the denominator of the transfer function (3) leads to a system with an infinite number of poles and where the closed loop stability properties should be carefully stated. In this work, it is proposed an observer-based scheme for the class of unstable systems (1) that yields stable closed-loop operation, based on the traditional observer theory. It is show how two static gains are enough in order to get an adequate estimation of a specific internal signal. The main idea is to propose a prediction scheme with a simpler structure and stronger stability margin face to large time-delays, when compared to controllers proposed in recent literature [11, 12, 15, 16, 18, 19], that in addition, can not handle the case $\tau > 2\tau_{un}$ treated in this work. The strategy is completed by incorporating P, PI, PID controllers.

In what follows, taking into account the results presented in [20], the existence conditions for the stabilizing feedback given by (2) are stated, for the following three types of $C(s)$ compensators.

i) P controller, $C(s) = k_p$

ii) PI controller, $C(s) = k_p + k_i/s$

iii) PID controller, $C(s) = k_p + k_i/s + k_d s$.

The following results are recalled, for the sake of completeness, and will be used later in the main result of this work.

Theorem 1 *[20] Consider the transfer function (1) and the control feedback (2), a necessary condition for a proportional controller P to simultaneously stabilize the delay-free plant and the plant with delay is $\tau < \frac{1}{a}$. If this necessary condition is satisfied, then the set of all stabilizing gains k_p for a given open-loop unstable plant with transfer function as in (1) is given by,*

$$\frac{a}{b} < k_p < \frac{1}{b\tau}\sqrt{z_1^2 + a^2\tau^2}$$

where z_1 is the solution of the equation, $\tan(z) = \frac{1}{a\tau}z$, in the interval $(0, \frac{\pi}{2})$.

Theorem 2 *[20] Consider the transfer function (1) and the control feedback (2), a necessary condition for a PI controller to simultaneously stabilize the delay-free plant and the plant with delay is $\tau < \frac{1}{a}$. If this necessary condition is satisfied, then the range of k_p values for which a solution exists to the PI stabilization problem of a given open-loop unstable plant with transfer function as in (1) is given by,*

$$\frac{a}{b} < k_p < \frac{1}{b\tau}\sqrt{\alpha_1^2 + a^2\tau^2}$$

where α_1 is the solution of the equation, $\tan(\alpha) = \frac{1}{a\tau}\alpha$, in the interval $(0, \frac{\pi}{2})$.

Remark 3 *It is important to note that Theorem 1 (Theorem 2) is stated (as in [20]) as a necessary condition. However, it is easy to see that the condition $\tau < \frac{1}{a}$ is necessary and sufficient for the existence of an stabilizing proportional (proportional-integral) feedback.*

Remark 4 *Once the range of k_p has been obtained from Theorem 2, one should choose and fix a value of the parameter k_p inside such range. For this value of k_p, the range of k_i is given by,*

$$0 < k_i < -\frac{az_1}{b\tau}[\sin(z_1) - \frac{1}{a\tau}z_1\cos(z_1)],$$

where z_1 is the first positive real root of,

$$-\frac{b}{a}k_p + \cos(z) + \frac{z}{a\tau}\sin(z) = 0.$$

Theorem 5 *[20] A necessary and sufficient condition for the existence of a stabilizing PID controller for the open-loop unstable plant (1) is $\tau < \frac{2}{a}$. If this condition is satisfied, then the range of k_p values for which a given open-loop unstable plant, with transfer function as in (1), can be stabilized using a PID controller is given by,*

$$\frac{a}{b} < k_p < \frac{a}{b}[\frac{\alpha_1}{a\tau}\sin(\alpha_1) + \cos(\alpha_1)]$$

where α_1 is the solution of the equation

$$\tan(\alpha) = \frac{1}{a\tau - 1}\alpha$$

in the interval $(0, \pi)$. In the special case of $\tau = \frac{1}{a}$, we have $\alpha_1 = \frac{\pi}{2}$. For k_p values outside this range, there are no stabilizing PID controllers. Moreover, the complete stabilizing region is given by Figure 1. For each $k_p \in (k_l := \frac{a}{b}, \frac{a}{b}[\frac{\alpha_1}{a\tau}\sin(\alpha_1) + \cos(\alpha_1)])$, the cross-section of the stabilizing region in the (k_i, k_d) space is the quadrilateral Q.

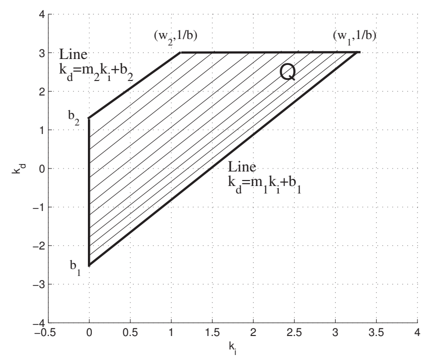

Figure 1: Stability region (k_i, k_d)

Remark 6 *The parameters involved in the stability quadrilateral Q depicted in Figure 1 are given by,*

$$m_j = \frac{\tau^2}{z_j^2}$$
$$b_j = \frac{a\tau}{bz_j}[\sin(z_j) - \frac{1}{a\tau}z_j\cos(z_j)]$$
$$w_j = -\frac{az_j}{b\tau}[\sin(z_j) - \frac{1}{a\tau}z_j(\cos(z_j)+1)]$$

for $j = 1, 2$*, where* z_1 *and* z_2 *are the first and second positive real roots of,*

$$-\frac{b}{a}k_p + \cos(z) + \frac{z}{a\tau}\sin(z) = 0,$$

respectively.

3 Observer Strategy

Consider now the unstable, input-output delayed system (1), rewritten by splitting the input time-delay in the form,

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} = e^{-\tau_2 s}\frac{b}{s-a}e^{-\tau_1 s} \tag{4}$$

where $a, b > 0$, and $\tau = \tau_1 + \tau_2$.

In what follows, taking into account, the new delay-spplited representation (4), a novel control structure will be presented in order to stabilize the original system (1) and at the same time solve the regulation and step disturbance problems. This new strategy considers an observer-based scheme together with a P or PI compensator defined by observed states that as a consequence allows to get the new stabilization bound $\tau < \frac{3}{a}$. Also, it can be shown that when it is considered a PID compensator, the stabilization bound is improved to $\tau < \frac{4}{a}$. In

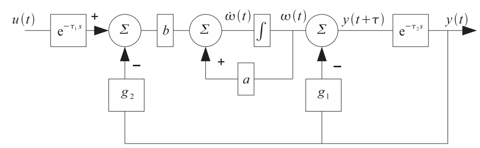

Figure 2: Static output injection.

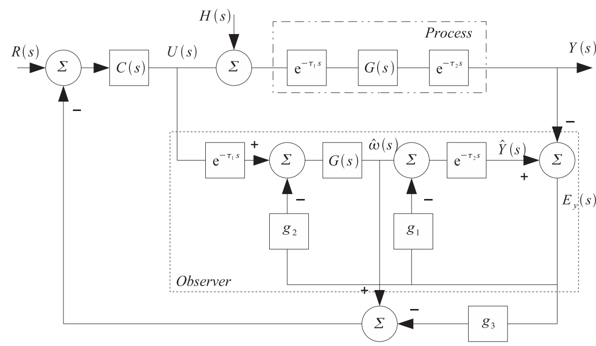

Figure 3: Proposed control satrategy.

particular, the general strategy presented here, considers the notation and parameterization for the P, PI, PID compensators analyzed in [20, 21]. In order to present the main result of our work, as a preliminary step, it will be presented the stabilizing conditions for the static output injection scheme shown in Figure 2.

Lemma 7 *Consider the stabilizing scheme shown in Figure 2. Then there exist constants* g_1 *and* g_2 *such that the closed-loop system,*

$$\frac{Y(s)}{U(s)} = \frac{be^{-\tau s}}{(s-a)(1+g_1 e^{-\tau_2 s}) + g_2 b e^{-\tau_2 s}} \tag{5}$$

is stable, if and only if, $\tau_2 < \frac{2}{a}$.

Proof. The proof of this lemma is provided in Appendix A and it is carried out as a slight modification of the one presented in [10] where system (1) is analyzed without considering the splitting strategy (4). ■

As a consequence of Lemma 7 it is possible to state the following result.

Theorem 8 *Consider the observer scheme shown in Figure 3. Then, there exist constants* g_1 *and* g_2 *such that* $\lim_{t\to\infty} [\omega(t) - \widehat{\omega}(t)] = 0$ *if and only if* $\tau_2 < \frac{2}{a}$.

Proof. Consider the original system (1) where the input delay has splitted as given in (4). Under this new representation, it is possible to give a state space representation for the internal signal $\omega(t)$ as,

$$\begin{array}{rcl} \dot{\omega}(t) & = & a\omega(t) + bu(t-\tau_1) \\ y_w(t+\tau_2) & = & \omega(t) \end{array} \tag{6}$$

It is now possible to consider a Luenverger-type observer for system (6) in the form,

$$\begin{array}{r} \dot{\hat{\omega}}(t) = a\hat{\omega}(t) + bg_2(y(t)-\hat{y}(t)) + bu(t-\tau_1) \\ \hat{y}_w(t+\tau_2) = \hat{\omega}(t) + g_1(y(t)-\hat{y}(t)) \end{array}$$

Systems (6)-(7) can be rewritten as,

$$\begin{array}{rcl} \begin{bmatrix} \dot{\omega}(t) \\ \dot{\widehat{\omega}}(t) \end{bmatrix} & = & \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} \omega(t) \\ \widehat{\omega}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ bg_2 & -bg_2 \end{bmatrix} \begin{bmatrix} y(t) \\ \widehat{y}(t) \end{bmatrix} + \begin{bmatrix} b \\ b \end{bmatrix} u(t-\tau_1) \\ \begin{bmatrix} y(t+\tau_2) \\ \widehat{y}(t+\tau_2) \end{bmatrix} & = & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \omega(t) \\ \widehat{\omega}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ g_1 & -g_1 \end{bmatrix} \begin{bmatrix} y(t) \\ \widehat{y}(t) \end{bmatrix} \end{array} \tag{8}$$

with $\widehat{\omega}(t)$ the estimation of $\omega(t)$. Defining first the state prediction error $e_\omega(t) = \omega(t) - \widehat{\omega}(t)$ and the output estimation error $e_y(t) = y(t) - \widehat{y}(t)$ it is possible to describe the behavior of the error signal as:

$$\begin{bmatrix} \dot{e}_\omega(t) \\ e_y(t+\tau_2) \end{bmatrix} = \begin{bmatrix} a & -bg_2 \\ 1 & -g_1 \end{bmatrix} \begin{bmatrix} e_\omega(t) \\ e_y(t) \end{bmatrix}. \tag{9}$$

Consider now a state space realization of system (5) (described in Figure 2) written as,

$$\begin{bmatrix} \dot{x}(t) \\ y(t+\tau_2) \end{bmatrix} = \begin{bmatrix} a & -bg_2 \\ 1 & -g_1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} u(t-\tau_1). \tag{10}$$

It is clear now that the stability conditions of systems (10), given in Lemma 7, are equivalent to the ones of system (9), from where, the result of the theorem follows. ■

Notice that with the observer structure presented above, it is possible to estimate the internal signal $\omega(t)$ that correspond to the prediction of the output signal $y(t)$, τ_2 units of time ahead.

It should be pointed out that the discrete dynamics in (9),

$$e_y(t+\tau_2) = -g_1 e_y(t) + e_\omega(t)$$

imposes the initial restriction, $|g_1| < 1$. It is not an easy task to get the stability region associated with gains g_1 and g_2, however a useful and practical result in order to compute the parameters involved on the predictor scheme is the following one.

Corollary 9 *Consider the observer based scheme shown in Figure 3. If $\tau_2 < \frac{2}{a}$, then the parameters g_1 and g_2 such that $\lim_{t\to\infty}[\omega(t) - \hat{\omega}(t)] = 0$ can be computed by considering the inequalities $a\tau_2 - 1 < g_1 \leq a\tau_2 - 1 + \epsilon$, $\frac{a}{b}(g_1+1) < g_2 \leq \frac{a}{b}(g_1+1) + \bar{\epsilon}$, for some constants $\epsilon, \bar{\epsilon} > 0$.*

Proof. The proof of this result is given in Appendix B ■

Remark 10 *It should be pointed out that the closed loop stability of equation (5) can also be stated by considering a continuous time approach. This can be done by following the results presented in ([9]) where the conditions are stated by considering the set of finite poles of the equivalent transfer function of the free delay system ($\tau = 0$) and theirs behavior when the input delay is not null. However, the results are significantly different, in ([9]) the stability property of the system is obtained for fixed g_1 and g_2. In our case, the conditions are given explicitly for the system parameters τ and $1/a$ and a practical and easy way to obtain the stabilizing controller parameters g_1 and g_2 is stated in Corollary 9.*

From Corollary 9 it is now possible to state a recursive algorithm in order to obtain stabilizing parameters g_1 and g_2. This procedure can be given as follows.

Algorithm 11
Step 0:

1. *Define* $\epsilon_0 = 0.6(2/a - \tau_2)$ *and* $\bar{\epsilon}_0 = 0.02(2/a - \tau_2)$.

Step i:

1. *Define* $\epsilon_i = \epsilon_{i-1}/2$ *and* $\bar{\epsilon}_i = \bar{\epsilon}_{i-1}/2$.
2. *Obtain* g_2 *and* g_1 *as,*

$$g_2 = \frac{a}{b}(a\tau_2 + \epsilon_i) + \bar{\epsilon}_i \quad \text{and } g_1 = a\tau_2 - 1 + \epsilon_i.$$

If at the step i *it is obtained an unstable closed loop system for the obtained* g_1 *and* g_2*, proceeds to step* $i+1$*. The algorithm ends when the obtained closed loop system is stable.*

Notice that even if the resulting closed-loop dynamic for the obtained g_1 and g_2 results stable it is possible to continue the algorithm without breaking the stability properties of the system in order to improve the general closed-loop response. A practical tuning of parameters g_1 and g_2 can also be done by considering the stability properties of the closed-loop system shown in Figure 2.

In [11] it is shown by means of an observer-based strategy that the delayed system (1) is stabilizable if and only if $\tau < \frac{2}{a}$. The proposed methodology allows to stabilize the system but not to implement a PI controller to achieve step tracking or step disturbance rejection. In what follows, the conditions in order to improve this bound are stated.

Notice that from the conditions of Theorem 8, for the estimation of $\omega(t)$ it is required $\tau_2 < \frac{2}{a}$. If $\tau_1 \neq 0$ on the delays distributions of Figure 3, then the control strategy should try to compensate the effect of a total time-delay $\tau > \frac{2}{a}$. In what follows, the admissible value of τ_1 will be analyzed in order to get an overall stable closed-loop system depending on $\tau = \tau_1 + \tau_2$.

4 Stabilization, regulation and disturbance rejection problem

Once the prediction scheme has been established, the proposed control structure will be complemented with a proportional P, proportional-integral PI or proportional-integral-derivative PID action. For the PI and PID parameterization the stabilization results given in [20, 21] are considered. The main result provides an observer-based structure that when complemented with a P (PI) controller produces the necessary and sufficient stabilization condition $\tau < 3/a$. This necessary and sufficient stabilization condition is improved to $\tau < 4/a$ when a PID controller is alternatively considered.

4.1 Proportional Action

Theorem 12 *Consider the observer-based control scheme depicted in Figure 3, with a control law,*

$$U(s) = k_p[R(s) - \hat{\omega}(s)] \tag{11}$$

i.e., $C(s) = k_p$ *and* $g_3 = 0$*. Then, there exists a constant* k_p *such that the closed-loop system (1)-(7)-(11) is stable if, and only if,* $\tau < 3/a$.

Proof. Consider the observer scheme shown in Figure 3. From Theorem 8, an adequate estimation $\widehat{\omega}(s)$ of the signal $\omega(s)$ is assured if and only if $\tau_2 < \frac{2}{a}$. Therefore, by Theorem 1 (see also Remark 3), it is possible to find a proportional controller of the form (11), such that the closed loop system is stable if and only if $\tau_1 < \frac{1}{a}$. Then we can conclude that the closed loop system is stable if and only if $\tau < 3/a$. ∎

Remark 13 *If $\tau < 3/a$, then the proportional gain k_p can be computed by using Theorem 1.*

4.2 Proportional-Integral Action

Theorem 14 *Consider the observer-based control scheme depicted in Figure 3, with the control law,*

$$U(s) = (k_p + k_i/s)[R(s) - \hat{\omega}(s)] \tag{12}$$

i.e., $C(s) = k_p + k_i/s$ and $g_3 = 0$. Then, there exists constants k_p and k_i such that the closed-loop system (1)-(7)-(12) is stable if, $\tau < 3/a$.

Proof. The proof is similar to the one of Corollary 12, by using Theorem 2 instead of Theorem 1. ∎

Remark 15 *If $\tau < 3/a$, then set of stabilizing (k_p, k_i) can be determined by Theorem 2 and Remark 4.*

4.3 Proportional-Integral-Derivative Action

Theorem 16 *Consider the observer-based control scheme depicted in Figure 3, with the control law,*

$$U(s) = (k_p + k_i/s + k_d s)[R(s) - \widehat{\omega}(s)], \tag{13}$$

i.e., $C(s) = k_p + k_i/s + k_d s$ and $g_3 = 0$. Then, there exists constants k_p, k_i and k_d such that the closed-loop system (1)-(7)-(13) is stable if, and only if, $\tau < 4/a$.

Proof. Consider the observer scheme shown in Figure 3. From Theorem 8, an adequate estimation $\widehat{\omega}(s)$ of the signal $\omega(s)$ is assured if and only if $\tau_2 < \frac{2}{a}$. Therefore, by Theorem 5, it is possible to find a PID controller of the form (13), such that the closed loop system is stable if and only if $\tau_1 < \frac{2}{a}$. Then we can conclude that the closed loop system is stable if and only if $\tau < 4/a$. ∎

Remark 17 *If $\tau < 4/a$ then the gains k_p, k_i, k_d can be computed from Theorem 5.*

4.4 Step disturbance rejection

The observer-based control scheme presented previously can be improved by adding a step disturbance rejection property in the cases that an integral action is present in the compensator $C(s)$, i.e., for the cases given in equations (12)-(13). This result is stated under the following conditions.

Lemma 18 *Consider system (1) together with the observer-based control scheme depicted in Figure 3. Under these conditions, there exists a PI or PID controller able to reject input step disturbances ($H(s)$) if $g_3 = g_1 + 1$.*

Proof. Consider the transfer function $Y(s)/H(s)$ of the control structure given in Figure 3 with $G(s)$ defined in equation (1), and $C(s)$ being a PI or PID controller defined by equations (12) and (13), respectively.

To verify the assertion of the lemma, the classical "Final value theorem" can be applied to $Y(s)$ when the disturbance signal is given as $H(s) = \frac{1}{s}$. It is an easy task to verify that under the condition $g_3 = g_1 + 1$, it is obtained,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 0$$

proving the result. ■

5 Simulation Results

The effectiveness of the proposed methodology will be now evaluated by means of two numerical examples.

Example 1. An isothermal chemical reactor exhibiting multiple steady state solutions is considered. The mathematical model of the reactor is given as,

$$\frac{dC}{dt} = \frac{Q}{V}(C_f - C) - \frac{k_1 C}{(k_2 C + 1)^2}$$

where Q is the inlet flow rate and C_f is the inlet concentration. The values of the operating parameters are given as $Q = 0.03333$ L/s, $V = 1$ L, $k_1 = 10$ L/s, and $k_2 = 10$ L/mol. For the nominal value of $C_f = 3.288$ mol/L, the steady-state solution of the model equation gives the following two stable steady states at $C = 1.7673$ and 0.01424 mol/L. There is one unstable steady state at $C = 1.316$ mol/L. Feed concentration is considered as the manipulated variable. Linearization of the manipulated variable around this operating condition $C = 1.316$ gives the unstable transfer function model as $3.433/(103.1s - 1)$. In [19] and [16] a measurement time delay of 20 s is considered. For our particular case, time-delay is considered to be two times the time constant of the system, i.e., 206.2 s. In this way the unstable transfer function model is obtained as,

$$\frac{Y(s)}{U(s)} = \frac{3.433}{103.1s - 1} e^{-206.2s}$$

Since the time-delay satisfy $\tau < \frac{3}{a}$, from Theorem 14 there exist gains g_1 and g_2 that stabilize the closed-loop system depicted in Figure 3. For the observer design, $\tau = \tau_1 + \tau_2$ is considered, with $\tau_1 = 56.2$ and $\tau_2 = 150$. For the simulation experiments $g_1 = 0.6$ and $g_2 = 0.47$. Then, since $\tau_1 < \frac{1}{a}$, a PI controller is used as controller C(s), see Figure 3. From Theorem 2, the range of proportional gain is $0.29 < k_p < 0.66$; and $k_p = 0.46$ is chosen (see Remark 4). The k_i range is obtained as $0 < k_i < 0.0012$. The set of PI parameters is picked as $k_p = 0.46$ and $k_i = 0.0008$. In order to improve the performance of the response, a PI with two degree of freedom proposed by [1] is used. In this way, the control law is modified as,

$$U(s) = R(s)C_p(s) - C_c[\hat{\omega}(s) + g_3 E_y(s)] \tag{14}$$

where $g_3 = 1.6$, $C_p(s) = k_p\phi + \frac{k_i}{s}$ and $C_c(s) = k_p + \frac{k_i}{s}$. ϕ can be chosen from $0 < \phi < 1$, in this case $\phi = 0.001$. In Figure 4 it is depicted the behavior of the stabilized system by means of the evolution of the output signal $y(t)$. In this experiment the exact knowledge of the plant parameters is assumed and an initial condition in the plant with a magnitude of 0.1. Also a small step disturbance (of magnitude -0.005) is considered acting at 3500 s. Figure 5 presents the estimation error $e_\omega(t) = \omega(t) - \hat{\omega}(t)$ of the scheme given in Figure 3.

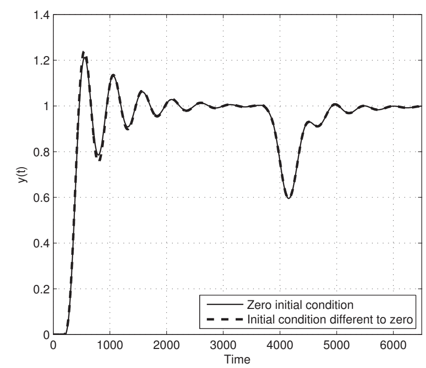

Figure 4: Output response $y(t)$, Example 1.

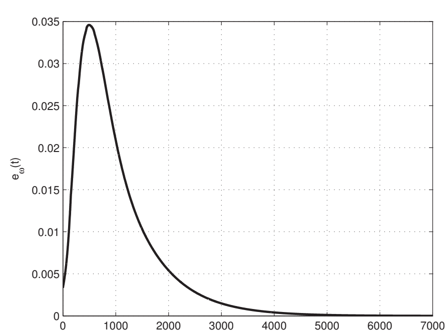

Figure 5: Estimation error $e_\omega(t) = \omega(t) - \hat{\omega}(t)$ on Example 1.

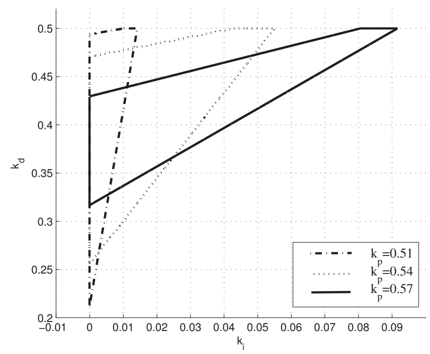

Figure 6: Region (k_i, k_d) for different values of k_p, Example 2.

Assumption 19 ***Example 2.*** *Consider the unstable first order system given by,*

$$\frac{Y(s)}{U(s)} = \frac{2}{s-1}e^{-\tau s}$$

with $\tau = 3.2$*. From Theorem 16, it is clear that there exist gains* g_1 *and* g_2 *that stabilize the closed-loop system depicted in Figure 3 since the time-delay satisfies* $\tau < \frac{4}{a}$*. For the observer design it is considered* $\tau = \tau_1 + \tau_2$*, where* $\tau_1 = 1.4$ *and* $\tau_2 = 1.8$*. For the simulation experiments it is considered* $\epsilon = 0.06$ *and* $\bar{\epsilon} = 0.004$ *and therefore it is obtained* $g_1 = 0.86$ *and* $g_2 = 0.934$*. Then, since* $\tau_1 < \frac{2}{a}$*, a* PID *controller is used as controller* $C(s)$*, see Figure 3. From Theorem 5, the range of proportional gain is* $0.5 < k_p < 0.5813$*; the stability region* (ki, kd) *for different values of* k_p *is shown in Figure 6, and the set of PID parameters is picked as* $k_p = 0.503$*,* $k_i = 0.0002$ *and* $k_d = 0.46$*. Also a* PID *with two degree of freedom as the one in (14) is used with* $g_3 = 1.86$*,* $C_p(s) = k_p\phi + \frac{k_i}{s} + k_d s$ *and* $C_c(s) = k_p + \frac{k_i}{s} + k_d s$*.* ϕ *can be chosen from* $0 < \phi < 1$*, in this case* $\phi = 0.001$ *is used. In Figure 7 it is depicted the behavior of the stabilized system by means of the evolution of the output signal* $y(t)$*. To carry out this experiment it was assumed the exact knowledge of the plant parameters and an initial condition in the plant with a magnitude of* 0.001*. Figure 8 presents the estimation error* $e_\omega(t) = \omega(t) - \hat{\omega}(t)$ *of the scheme given in Figure 3.*

6 Conclusions

In this work, it is presented a new stabilization bound for a class of unstable first order linear system with large time-delay at the input-output path. Considering an splitting strategy for the original time delay τ, i.e. $\tau = \tau_1 + \tau_2$, it is presented a double action methodology that is based on the estimation of the internal signal $\omega(t)$ of the original plant that represent the future value of the output, τ_2 units of time ahead. This estimation structure is used at the same time to counteract the effect of this τ_2 output delay. The predicted internal signal $\omega(t)$ is used also in an outer P, PI, PID loop that takes into account the effects of the remaining input delay τ_1.

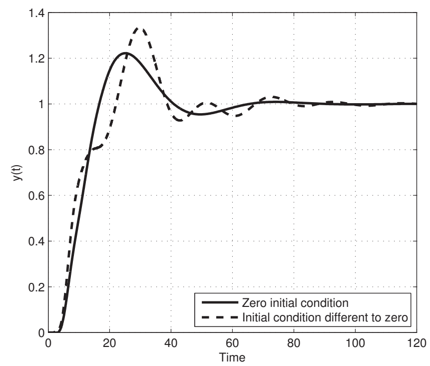

Figure 7: Output response $y(t)$, Example 2.

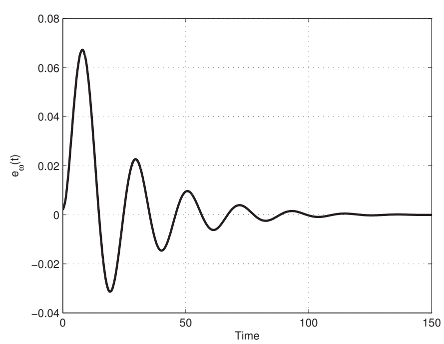

Figure 8: Estimation error $e_\omega(t) = \omega(t) - \hat{\omega}(t)$ on Example 2.

The novel strategy presented in this work allows to stabilize open loop plants up to the limit $\tau < 4\tau_{un}$ that up to our best knowledge has not been reported in the literature. The strategy is complemented with an input step disturbance rejection property for the overall closed-loop system. Numerical simulations show the effectiveness of the proposed control structure.

A Proof of Lemma 7

Based on the splitting strategy of τ, system (5) can be rewritten as,

$$\frac{Y(s)}{U(s)} = \frac{be^{-\tau_2 s}}{(s-a)(1+g_1e^{-\tau_2 s}) + g_2be^{-\tau_2 s}}e^{-\tau_1 s}$$

Note from Figure 2 that the delay term $e^{-\tau_1 s}$ does not affect the stability of the closed-loop system. Therefore, it is considered the expression,

$$\frac{Y(s)}{U(s)} = \frac{be^{-\tau_2 s}}{(s-a)(1+g_1e^{-\tau_2 s}) + g_2be^{-\tau_2 s}}$$

Consider now a discrete-time version of the original plant (1) together with the output injection scheme given in Figure 2. To carry out this task, it is assumed that there exist a sampling period T that satisfies the condition $T = \frac{\tau_2}{n}$ for an integer n and that a zero order hold is located at the input of the system. Under these conditions, the discrete-time closed-loop transfer function is,

$$\frac{Y(z)}{U(z)} = \frac{(b/a)(e^{aT}-1)}{(z-e^{aT})(z^n+g_1) + g_2(b/a)(e^{aT}-1)}, \tag{15}$$

with the characteristic polynomial given by,

$$p_1(z) = (z-e^{aT})(z^n+g_1) + g_2(b/a)(e^{aT}-1). \tag{16}$$

The proof of the theorem is based on demonstrate that all roots in (16) lie inside the unit circle when it is considered, $\lim_{n\to\infty} \frac{\tau_2}{n}$, if and only if, $\tau_2 < \frac{2}{a}$.

To begin with, consider first the simple case when $g_1 = 0$ in (15), this produces the characteristic equation,

$$(z-e^{aT})z^n - g_2(b/a)(e^{aT}-1) = 0. \tag{17}$$

The root locus diagram ([3]) associated to (17) shows that the open-loop system has n poles at the origin and one at $z = e^{aT}$. Then, there exist $n+1$ branches to infinity, $n-1$ of them starting at the origin and going directly to infinity. The two remaining branches starting at a breaking point z_1 located over the real axis between the origin and $z = e^{aT}$ (this situation is illustrated in Figure 9 for the case $n = 5$). z_1 can be found by considering the equation,

$$\frac{dg_2}{dz} = \frac{d}{dz}\left[-\frac{z^n(z-e^{aT})}{\frac{b}{a}(1-e^{aT})}\right] = 0,$$

that produces,

$$(n+1)z^n - nz^{n-1}e^{aT} = 0,$$

which has $n-1$ roots at the origin and one at,

$$z_1 = \frac{n}{n+1}e^{a\frac{\tau_2}{n}}.$$

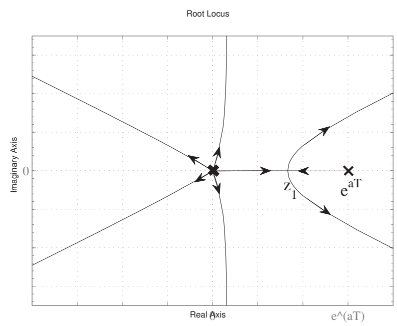

Figure 9: Root locus of equation (17) for $n = 5$.

If the breaking point z_1 over the real axis is located inside the unit circle, the closed loop system could have a region of stability, otherwise will be unstable for any g_2. The stability properties of the continuous system (5) are obtained by considering the limit as $n \to \infty$, or equivalently, when $T \to 0$, this is,

$$\lim_{n\to\infty} z_1 = \lim_{n\to\infty} \frac{n}{n+1} e^{a\frac{\tau_2}{n}} = 1. \tag{18}$$

It is important to note that any point $s = \theta$, over the real axis on the complex plane s is mapped to $z = e^{\theta T}$ on the z plane and as a consequence this point converges to $z = 1$ when T tends to zero. Notice also that any real point $s = \theta$ on the left half side of the complex plane ($\theta < 0$) is mapped to a point $e^{\theta T}$ that tends to one over the stable region of the z plane. On the contrary, if θ is on the right side of the complex plane over the real axis ($\theta > 0$), the point $e^{\theta T}$ tends to one over the unstable region. Then, from (17), it is not difficult to see that if $a\tau_2 < 1$ (i.e., $\tau_2 < 1/a$) there exists a gain g_2 that stabilizes the closed loop system (i.e., the limit tends to one from the left). In the case that $a\tau_2 \geq 1$ (always considering $g_1 = 0$) it is not possible to get g_2 that stabilize the system.

Consider now the case $g_1 \neq 0$. Applying again a root locus analysis for system (15) and its characteristic equation (16), as g_1 grows from zero, the breaking point over the real axis moves in the root locus diagram (indeed, goes to the left). This point can be found by taking into account the equation,

$$\frac{dg_2}{dz} = \frac{d}{dz}\left[-\frac{(z - e^{aT})(z^n + g_1)}{(b/a)(e^{aT} - 1)}\right] = 0, \tag{19}$$

yielding,

$$(n+1)z^n - nz^{n-1}e^{aT} + g_1 = 0. \tag{20}$$

Expression (20) corresponds to the characteristic equation of a fictitious system of the form,

$$\frac{Y(z)}{V(z)} = G(z) = \frac{1/(n+1)}{z^{n-1}(z - e^{aT}n/(n+1))} \tag{21}$$

in closed loop with the feedback,

$$V(z) = U(z) - g_1 Y(z). \tag{22}$$

The open loop system (21) has $n-1$ root at the origin and one at

$$z = \frac{n}{n+1} e^{a\frac{\tau_2}{n}}.$$

If the breaking point over the real axis is located inside the unit circle, the closed loop system (21)-(22) could have a region of stability (once proved that the others $n-2$ poles are inside the unitary circle), otherwise the system will be unstable for any g_1. This point can be found by considering,

$$\frac{dg_1}{dz} = \frac{d}{dz}\left[-\frac{z^{n-1}\{z - e^{aT} n/(n+1)\}}{1/(n+1)}\right] = 0, \tag{23}$$

that produces,

$$z^{n-2}(z - \frac{n-1}{n+1} e^{aT}) = 0,$$

which has $n-2$ roots at the origin and one at,

$$z = \frac{n-1}{n+1} e^{a\frac{\tau_2}{n}}.$$

As previously, the stability properties of the equivalent continuous system (5) are obtained by considering the limit as $n \to \infty$, or equivalently, when $T \to 0$. That is,

$$\lim_{n\to\infty} z = \lim_{n\to\infty} \frac{n-1}{n+1} e^{a\frac{\tau_2}{n}} = 1.$$

Again, since this limit point is located on the stability boundary, in this case it is possible to see that if $a\tau_2 \leq 2$ (i.e., the limit tends to one from the left) there exists a gain g_1 that places the breaking point (two poles) inside the unit circle in the original discrete Root Locus diagram. Then, if the remaining $n-1$ roots are into the unit circle, the closed loop system is stable. In the case that $a\tau_2 > 2$ it is not possible to stabilize the system by static output injection (i.e., the limit goes to one from the right). Let us now prove that the remaining $n-1$ roots are into the unitary circle if and only if $a\tau_2 < 2$. Assume that $a\tau_2 \leq 2$ and to take into account the continuous case, the characteristic equation (16) it is modified as,

$$\begin{aligned}\lim_{n\to\infty} p_1(z) &= \lim_{n\to\infty} [(z - e^{a\frac{\tau_2}{n}})(z^n + g_1) + g_2(b/a)(e^{a\frac{\tau_2}{n}} - 1)] \\ &= (z-1)\lim_{n\to\infty}(z^n + g_1) = 0\end{aligned}$$

from where it is stated that while one pole is on the neighborhood of $z = 1$, the remaining poles are in a neighborhood of the points $(-g_1)^{1/n}$, inside the unit circle producing a stable closed loop system if, as it was previously stated, it is satisfied, $g_1 < 1$. From equation (23),

$$g_1 = -\frac{z^n\{z - e^{aT} n/(n+1)\}}{1/(n+1)},$$

then if $z = 1$,

$$g_1 = -\frac{\{1 - e^{aT} n/(n+1)\}}{1/(n+1)} = -(n + 1 - ne^{aT}).$$

Taking into account the continuous case as previously done, it is obtained,

$$\lim_{n\to\infty} g_1 = \lim_{n\to\infty} -(n + 1 - ne^{a\tau_2/n}) = a\tau_2 - 1. \tag{24}$$

As $g_1 < 1$ is a necessary condition for the stability, $a\tau_2 - 1 < 1$, then $a\tau_2 < 2$.

B Proof of Corollary 9

From equation (24) in the proof of Lemma 7 we have:

$$\lim_{n\to\infty} g_1 = a\tau_2 - 1.$$

Therefore if $\tau_2 < \frac{2}{a}$, there exist g_2 that stabilizes the closed loop system (5), with $a\tau_2 - 1 < g_1 \leq a\tau_2 - 1 + \epsilon$ for $\epsilon > 0$.

Now, from equation (19),

$$g_2 = -\frac{(z - e^{aT})(z^n + g_1)}{(b/a)(e^{aT} - 1)},$$

then, if $z = 1$,

$$g_2 = \frac{g_1 + 1}{(b/a)} = (a/b)(g_1 + 1).$$

Therefore, the gain g_2 can be obtained by considering the condition $\frac{a}{b}(g_1+1) < g_2 < \frac{a}{b}(g_1+1)+\bar{\epsilon}$, for some $\bar{\epsilon} > 0$.

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B.5 Control of Recycling Systems with Unstable Forward Loop

Control of Delayed Recycling Systems with Unstable First Order Forward Loop. J.F. Marquez-Rubio, B. del Muro-Cuéllar, M. Velasco-Villa, D. Cortés and O. Sename. Submitted to Journal of Process Control

Control of Delayed Recycling Systems with Unstable First Order Forward Loop

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Abstract

Unstable time-delay systems and recycling systems are challenging problems for control analysis and design. When an unstable time-delay system has a recycle, its control problem becomes even more difficult. A control methodology for this class of systems is proposed in this paper. The considered strategy is based on the fact that if some internal system signals are available for measurement, then it will be possible to decouple the backward dynamics of the system and then a feedback controller could be designed for the forward dynamics. The key point for this strategy to be carried out is an asymptotic observer-predictor proposed to estimate these required internal signals. Necessary and sufficient conditions to assure convergence of this observer are given. After proving that the proposed control scheme tracks a step input signal and at the same time reject step disturbances, a procedure summarizing the methodology is provided. Robustness with respect to delay uncertainty and model parameters are also analyzed.

Keywords: Time-delay, recycling system, stabilization, observer.

1. Introduction

In recycling systems, the output of a process is partially fed-back to the input. Recycling processes reuse the energy and the partially processed matter increasing the efficiency of the overall process. They are commonly found in chemical industry, for instance, in order to implement a recycle stream for

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a continuous stirred tank reactor (CSTR), the output stream of the reactor is sent through a separation process (perhaps a centrifuge). Then, the unreacted reagents are returned into the CSTR by traveling through pipes. Because recycling reduces waste of reagents, and hence the cost of the reaction, it is widespread used in industry. As another example, recycling is often used in the manufacturing of nylon 66 or in the oxidation of cyclohexene to KA (a mixture of ketone and alcohol of cyclohexene), among many other reactions [1], in a typical plant formed by reactor/separator processes, where reactants are recycled back to the reactor [2], [3].

Recycling processes are systems with positive feedback that can give rise to some undesirable effects. In fact, instability can occur in recycle systems when the feedback gain is larger than unity, as an example, the recycle of the energy developed by an exothermic reaction in an adiabatic plug flow reactor for feed preheating. Instability could occur due to the exponential increase in the reaction rate with the temperature when this cannot be properly controlled [4]. Another example is the recycle of impurities in a plant with recycle, whose inventory cannot be kept at equilibrium by the separator system [5].

Also, the so-called snowball effect is observed in the operation of many chemical plants with recycle streams. Snowball means that a small change in a load variable causes a very large change in the flow rates around the recycle loop. Although snowballing is a steady state phenomenon and has nothing to do with dynamics, it depends on the control structure. Disadvantages of snowball effect has drawn the attention of some researchers proposing several control strategies to avoid the associated problems, [6], [7].

The control of time-delay systems has also been analyzed by the use of observer strategies mainly for systems without time-lag. In the case of systems with delayed-state in [8] it is considered a particular nonlinear triangular system and in [9] a predominant linear system perturbed by a delayed nonlinear term is considered while in [10] an observer adaptive control is proposed. A digital redesign of an analog Smith predictor compensator is developed in [11].

When significant transport delay is present in recycled systems, the control problem becomes more complex. For example, in the case of continuous stirred tank reactors (CSTR), a problem that arises on the recycling loop is that the process is modeled usually by assuming that there is not any time-delay on this path. While this assumption may produce a simpler theoretical analysis, it is highly unrealistic [1].

It is known that in a system with recycle loops and time-delays, exponential terms appears in forward and backward paths on its transfer function representation. In state space representation, recycled systems with time-delay correspond to systems with delays at the input and at the state. Model approximation has been proposed to remove the exponential terms from the transfer function denominator of a delayed system, such as the method of moments [12], and Pade-Taylor approximations [13], [14]. Other techniques, such as the seasonal time-series model [15], have also been proposed to obtain an approximate model. Del Muro et. al. [16] proposed an approximate model to represent recycle systems by using a discrete-time approach. In turn, such approximate models can be used for stability analysis or control design [17], [18], [19].

A system with time-delay and open-loop unstable poles is notably more difficult to control than a system with only open-loop stable poles. Introducing recycle in such a system would lead to a very difficult (although interesting) problem. To begin to overcome this situation, in this work, the problem of a recycled system composed of an unstable first order plant in the direct path and an stable system of order n in the recycle loop is addressed. The control strategy will be based on a particular observer-based structure. A preliminary analysis of this problem was presented at [20].

The rest of the paper is organized as follows: In Section 2 the problem to be tackled is formulated for the class of systems considered in this work and the general idea of its solution is outlined. At this point, the need of an observer-predictor arises. Section 3 presents the proposed control scheme. First, a preliminary result concerning the stability of a class of input-delay systems is presented, then a scheme to estimate some internal signals of the system is proposed. Based on the estimation of these necessary internal variables, the overall control scheme is presented in the last part of Section 3. As robustness of this kind of controllers is a fundamental issue, Section 4 is dedicated to its analysis. Some simulations results are provided in Section 5 in order to illustrate the performance of the proposed control strategy. Finally Section 6 presents some conclusions.

2. Problem formulation

Consider the class of recycling system shown in Figure 1, which can be described as,

$$Y(s) = \begin{bmatrix} G_d & G_dG_r \end{bmatrix} \begin{bmatrix} U(s) \\ Y(s) \end{bmatrix} \tag{1}$$

with,

$$G_d = G_1(s)e^{-\tau_1 s} = \frac{b}{s-a}e^{-\tau_1 s} \tag{2a}$$

$$G_r = G_2(s)e^{-\tau_2 s} = \frac{N_r(s)}{D_r(s)}e^{-\tau_2 s} \tag{2b}$$

where $G_d(s)$, and $G_r(s)$ are transfer functions of the forward (direct) and backward (recycle) paths, respectively; $\tau_1,\ \tau_2 \geq 0$ are the time-delays associated to $G_d(s)$, and $G_r(s)$. $a, b \in \mathbb{R}$, with $a > 0$, this is, G_d is considered unstable; $N_r(s)$ and $D_r(s)$ are polynomials on the complex variable s. $U(s)$ is the process input and $Y(s)$ is the process output.

The closed-loop transfer function of system (1) is given by

$$G_t(s) = \frac{D_r(s)be^{-\tau_1 s}}{(s-a)D_r(s) - bN_r(s)e^{-(\tau_1+\tau_2)s}} \tag{3}$$

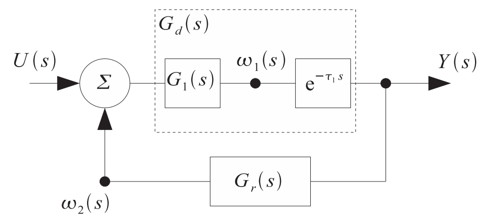

Figure 1: A process with recycle.

Note that exponential terms appear explicitly in numerator and denominator of $G_t(s)$. Stability of (3) is determined by the roots of its characteristic equation,

$$Q(s) = (s - a)D_r(s) - bN_r(s)e^{-(\tau_1+\tau_2)s} = 0 \tag{4}$$

More precisely, the overall path $U(s) \to Y(s)$ is stable if and only if all the roots of $Q(s)$ are contained in the open left-half complex plane. It is well known that the transcendental term in $Q(s)$ induces an infinite number of roots preventing the use of classical control design techniques and standard stability analysis methods.

Let us describe some ideas behind the proposed methodology. Consider an input reference $R(s)$ and a new control law $R_1(s)$. Then, with respect to Figure 1, if signal ω_2 is known, then we can set

$$U(s) = R_1(s) - \omega_2(s) \tag{5}$$

obtaining the system shown in Figure 2. At this point, it is possible to design $R_1(s)$ as $R_1(s) = J(s)\,(R(s) - \omega_1(s))$ as shown in Figure 3 and where $J(s)$ is a controller based on the delay free forward model. Since ω_1 and ω_2 are internal signals of the system, an observer-predictor scheme to estimate these variables is developed in the following section.

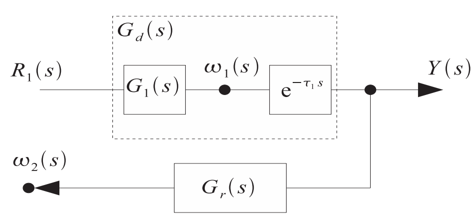

Figure 2: System of Figure 1 after applying U(s) given in (5).

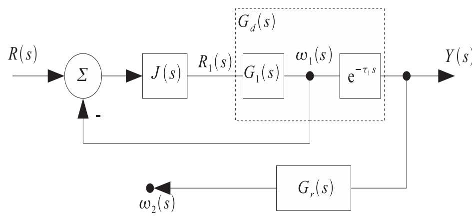

Figure 3: Control structure for the system of Figure 2.

3. Observer-predictor based control

3.1. ***Preliminary results***

In this section, a well known stability condition for an unstable first order plus time-delay system is recalled from [21] for the sake of completeness. This result will be used later in the proof of the observer-predictor convergence.

Lemma 1. *Consider the unstable input-output delay system*

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} = \frac{b}{s-a}e^{-\tau s}, \quad a > 0 \tag{6}$$

with a proportional output feedback

$$U(s) = R(s) - kY(s) \tag{7}$$

where $R(s)$ is the new reference input. There exist a proportional gain k such that the closed-loop system

$$\frac{Y(s)}{R(s)} = \frac{be^{-\tau s}}{s - a + kbe^{-\tau s}} \tag{8}$$

is stable if and only if $\tau < 1/a$.

Stability of (8) has been previously studied in the literature. Lemma 1 can be proved using a classical frequency domain; D-decomposition or even by the classical Pontryagin Method [22], [23], [24]. An alternative proof of Lemma 1 is provided in [21] by using a discrete-time approach and is included in Appendix A.

A useful practical result in order to compute the parameter k involved on the control scheme is as follows.

Corollary 1. *[21] Consider system given by* (6) *with $\tau < 1/a$. Then, there exists $k \in R^+$ that stabilizes the closed-loop system (8), satisfying $\alpha < k < \beta$, with $\alpha = a/b$ and some constant $\beta > a/b$.*

The proof of Corollary 1 is also presented in Appendix A.

Remark 1. *From a frequency domain analysis, it is not difficult to accurately determinate the value of* β *given in Corollary 1. In fact, such value is given by* $\beta = \frac{a}{b}\sqrt{1 + (\frac{\omega}{a})^2}$*, where* ω *satisfy* $\frac{\omega}{a} = \tan(\omega\tau)$ *for* $0 < \omega < \frac{\pi}{2\tau}$*. The utility of Corollary 1 comes from the fact that any* $k = \frac{a}{b} + \epsilon$*, with* $\epsilon > 0$ *stabilizes the closed-loop system (8) for* ϵ *sufficiently small. Notice that the upper bound* β *would depend directly on the values of the parameters* a*,* b *and* τ *and therefore it is determined by the considered plant parameters*

3.2. Prediction Strategy

In order to estimate signals ω_1 and ω_2 shown in Figure 1, the observer-predictor scheme depicted in Figure 4 is proposed. Its convergence is established in the following result.

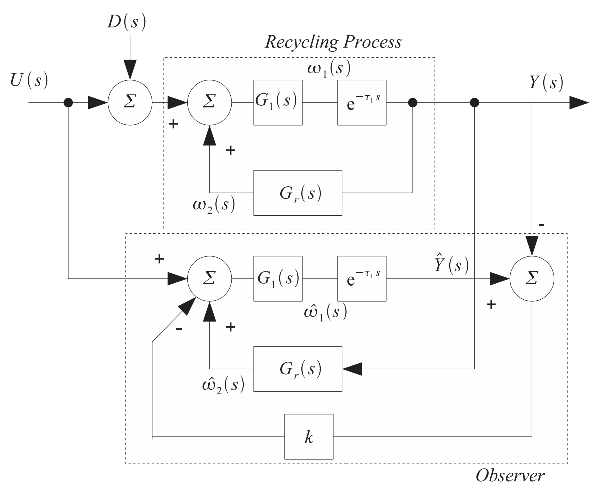

Figure 4: Proposed observer schema.

Theorem 1. *Consider the observer-predictor scheme shown in Figure 4, with* G_r *a delayed stable transfer function. Then, there exists a constant* k *such that*

$$\lim_{t\to\infty} [\omega_i - \widehat{\omega}_i] = 0, \text{ for } i = 1, 2, \tag{9}$$

if and only if $\tau_1 < \frac{1}{a}$*.*

Proof. A state space representation of the observer-predictor scheme shown in Figure 4 is

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + A_1\mathbf{x}(t-\tau_1) + A_2\mathbf{x}(t-\tau_2) + Bu(t) \tag{10a}$$

$$\mathbf{y}(t) = C_1\mathbf{x}(t-\tau_1) \tag{10b}$$

with,

$$\mathbf{x}(t) = \begin{bmatrix} x_d(t) & x_r(t) & \widehat{x}_d(t) & \widehat{x}_r(t) \end{bmatrix}^T$$
$$\mathbf{y}(t) = \begin{bmatrix} y(t) & \widehat{y}(t) \end{bmatrix}^T, B = \begin{bmatrix} B_d & 0 & B_d & 0 \end{bmatrix}^T$$
$$A = \begin{bmatrix} A_d & 0 & 0 & 0 \\ 0 & A_r & 0 & 0 \\ 0 & 0 & A_d & 0 \\ 0 & 0 & 0 & A_r \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ B_rC_d & 0 & 0 & 0 \\ B_rkC_d & 0 & -B_dkC_d & 0 \\ B_rC_d & 0 & 0 & 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0 & B_dC_r & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_dC_r \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} C_d & 0 & 0 & 0 \\ 0 & 0 & C_d & 0 \end{bmatrix}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input, $\mathbf{y} \in \mathbb{R}^2$ is the output, $\tau_1 \geq 0$ and $\tau_2 \geq 0$ are the time-delays present in the system. $A_d \in \mathbb{R}^{n\times n}$, $B_d \in \mathbb{R}^{n\times 1}$, and $C_d \in \mathbb{R}^{1\times n}$ are matrices and vectors parameters that corresponds to the forward loop in the process, and $A_r \in \mathbb{R}^{m\times m}$, $B_r \in \mathbb{R}^{m\times 1}$, and $C_r \in \mathbb{R}^{1\times m}$ are matrices and vectors parameters that corresponds to backward path in the process, $\widehat{x}(t)$ is the estimation of $x(t)$.

Defining the state prediction errors

$$e_{x_d}(t) = \widehat{x}_d(t) - x_d(t), \quad e_{x_r}(t) = \widehat{x}_r(t) - x_r(t), \tag{12}$$

and the output estimation

$$e_y(t) = \widehat{y}(t) - y(t), \tag{13}$$

it is possible to describe the behavior of the error signals as,

$$\begin{bmatrix} \dot{e}_{x_d}(t) \\ \dot{e}_{x_r}(t) \\ e_y(t+\tau_1) \\ e_{\omega_2}(t+\tau_2) \end{bmatrix} = \begin{bmatrix} A_d & 0 & -B_dk & B_d \\ 0 & A_r & 0 & 0 \\ C_d & 0 & 0 & 0 \\ 0 & C_r & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{x_d}(t) \\ e_{x_r}(t) \\ e_y(t) \\ e_{\omega_2}(t) \end{bmatrix} \tag{14}$$

Note that $e_y(t) = C_d e_{x_d}(t-\tau_1)$ and that $e_{\omega_2}(t) = C_r e_{x_r}(t-\tau_2)$. Then, system (14) can be rewritten as

$$\dot{e}_{x_d}(t) = A_d e_{x_d}(t) - B_dkC_d e_{x_d}(t-\tau_1) + B_dC_r e_{x_r}(t-\tau_2) \tag{15a}$$
$$\dot{e}_{x_r}(t) = A_r e_{x_r}(t) \tag{15b}$$

Since A_r is a Hurwitz matrix, the stability of system (15) can be analyzed by considering the partial dynamics

$$\dot{e}_{x_d}(t) = A_d e_{x_d}(t) - B_dkC_d e_{x_d}(t-\tau_1) \tag{16}$$

or equivalently,

$$\begin{bmatrix} \dot{e}_{x_d}(t) \\ e_y(t+\tau_1) \end{bmatrix} = \begin{bmatrix} A_d & -B_dk \\ C_d & 0 \end{bmatrix} \begin{bmatrix} e_{x_d}(t) \\ e_y(t) \end{bmatrix} \tag{17}$$

Consider now a state space realization of system (8). It is easy to see that this dynamics can be written in state space form as,

$$\begin{bmatrix} \dot{x}_d(t) \\ y(t+\tau_1) \end{bmatrix} = \begin{bmatrix} A_d & -B_d k \\ C_d & 0 \end{bmatrix} \begin{bmatrix} x_d(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} B_d \\ 0 \end{bmatrix} u(t) \tag{18}$$

Comparing (18) and (17) it is clear that Lemma 1 can be applied to system (17). Hence, the result of the theorem follows. ■

Remark 2. *A natural step toward a more general result is the extension of Lemma 1 to a higher order unstable system. In [25] it is presented the stabilization of delayed systems with one unstable pole and n stable poles. In the case that $n = 1$, the stability condition become $\tau_1 < \frac{1}{a} - \frac{1}{b}$, with b as the position of the stable pole. Notice that under these circumstances, it is possible to construct an observer-predictor for this more general system that will produce an error dynamics equivalent to the one described in equation (16). Notice that in the general case, this is, the consideration of an unstable transfer function on the forward path and a stable one on the recycle path together with a time-delay on the forward path it is also possible to construct an observer-predictor that will produce an error dynamics as in (16). The remaining problem consist in finding stabilization conditions in terms of the obtained A_d and $B_d k C_d$ that in the present case, can be accomplished due to the lower dimension of the problem.*

3.3. Proposed control scheme

Once the estimated internal signals $\widehat{\omega}_1$ and $\widehat{\omega}_2$ converge to their actual values, the ideas described in Section 2 can be implemented. The complete control strategy, proposed in this work, is depicted in Figure 5. In what follows, it will be shown that the proposed scheme achieves step input tracking and a particular disturbance rejection action by using a PI controller with two degree of freedom. With this aim, consider first the general control strategy depicted in Figure 5, given by,

$$U(s) = J(s)(R(s) - \hat{\omega}_1 + E_y(s)) - \hat{\omega}_2(s) \tag{19}$$

Then, in order to improve the output response performance, consider a PI controller with two degree of freedom provided in [26], instead of a simple controller $J(s)$. In such case, the control law can be implemented as,

$$U(s) = R(s)G_{ff}(s) - G_c(s)(\widehat{\omega}_1(s) - E_y(s)) - \widehat{\omega}_2(s) \tag{20}$$

where,

$$G_{ff}(s) = K_p(\sigma + \frac{1}{T_i s}) \tag{21a}$$

$$G_c(s) = K_p(1 + \frac{1}{T_i s}) \tag{21b}$$

The following results are concerned with the step tracking reference and the step disturbance rejection problem by considering a PI controller with two degree of freedom for the proposed observer strategy.

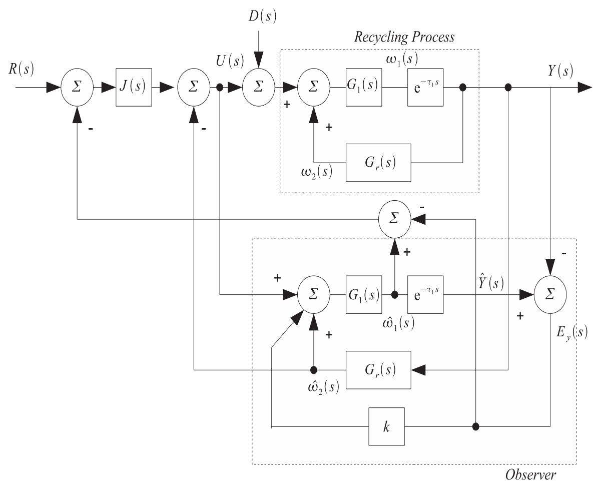

Figure 5: Proposed control schema.

Lemma 2. *Consider the observer scheme shown in Figure 5. Then, there exists a PI controller with two degree of freedom given by (20) such that* $\lim_{t\to\infty} y(t) = \alpha$*, where* $R(s) = \alpha/s$ *is the step input reference and* $D(s) = 0$*.*

Proof. Consider the observer scheme shown in Figure 5 and the control strategy given by (20). Then,

$$\frac{Y(s)}{R(s)} = \frac{G_d G_{ff}(1 + kG_1 e^{-s\tau_1})}{G_1 G_c + G_c G_d - G_1 G_c e^{-s\tau_1} + kG_1 e^{-s\tau_1} + kG_1 G_c G_d + 1} \tag{22}$$

Applying the Final Value Theorem with $R(s) = \alpha/s$, as input reference to the transfer function (22), it is produced,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\frac{N_1(s)}{D_1(s)}\frac{\alpha}{s} \tag{23}$$

with

$$N_1(s) = G_d G_{ff}(1 + kG_1 e^{-s\tau_1}) \tag{24}$$

$$D_1(s) = G_1 G_c + G_c G_d - G_1 G_c e^{-s\tau_1} + kG_1 e^{-s\tau_1} + kG_1 G_c G_d + 1 \tag{25}$$

Substituting G_d, G_1 given in (2a) and the controller G_{ff} and G_c given in (21), we have,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \alpha$$

■

Lemma 3. *Consider the proposed observer scheme shown in Figure 5. Then, there exist a PI controller with two degree of freedom provided by (20) such that* $\lim_{t\to\infty} y(t) = 0$*, where* $R(s) = 0$ *and* $D(s)$ *is a step input disturbance.*

Proof. According to Figure 5,

$$\frac{Y(s)}{D(s)} = \frac{G_d + G_1 G_c G_d + kG_1 G_d e^{-s\tau_1} - G_1 G_c G_d e^{-s\tau_1}}{G_1 G_c + G_c G_d - G_1 G_c e^{-s\tau_1} + kG_1 e^{-s\tau_1} + kG_1 G_c G_d + 1} \tag{26}$$

Applying the Final Value Theorem with $D(s) = \eta/s$, as input reference. From equation (22), we have,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\frac{N_2(s)}{D_2(s)}\frac{\eta}{s} \tag{27}$$

with

$$N_2(s) = G_d + G_1 G_c G_d + kG_1 G_d e^{-s\tau_1} - G_1 G_c G_d e^{-s\tau_1} \tag{28a}$$

$$D_2(s) = G_1 G_c + G_c G_d - G_1 G_c e^{-s\tau_1} + kG_1 e^{-s\tau_1} + kG_1 G_c G_d + 1. \tag{28b}$$

Substituting G_d, G_1 given in (2a) and the controller G_{ff} and G_c given in (21), we have,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 0 \tag{29}$$

■

From the previous discussions and results, the proposed methodology can be summarized as follows:

1. Make sure that the conditions of Theorem 1 are satisfied, that is, $G_r(s)$ a stable transfer function and $\tau_1 < \frac{1}{a}$ for the unstable first order delayed plant.
2. Tune the forward loop with the parameter k using Corollary 1.
3. Design of a controller $J(s)$ based on the delay free model of the forward path. A PI or PID control based strategy can be considered.
4. Finally, implement the general control structure as it is shown in Figure 5.

4. Robustness with respect to uncertainties

The control strategy presented in previous sections has been developed under the assumption that an accurate model of the process is available. In observer-based controllers for time-delay and recycled systems, it is important to analyze the robustness of the closed-loop system, under plant parameters and time-delays uncertainties. This analysis is carried out in this Section for the proposed controller.

Consider a state representation of the open-loop system with recycle in the nominal case (which can be obtained from the complete state representation of system-observer expressed in equation (10)),

$$\dot{x} = \bar{A}x + \bar{A}_1 x(t-\tau_1) + \bar{A}_2 x(t-\tau_2) + \bar{B}u(t) \tag{30a}$$
$$y = \bar{C}_1 x(t-\tau_1) \tag{30b}$$

where

$$x = \begin{bmatrix} x_d & x_r \end{bmatrix}^T, \quad \bar{A} = \begin{bmatrix} A_d & 0 \\ 0 & A_r \end{bmatrix}, \quad \bar{A}_1 = \begin{bmatrix} 0 & 0 \\ B_r C_d & 0 \end{bmatrix}, \tag{31a}$$
$$\bar{A}_2 = \begin{bmatrix} 0 & B_d C_r \\ 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_d \\ 0 \end{bmatrix}, \quad \bar{C}_1 = \begin{bmatrix} C_d & 0 \end{bmatrix} \tag{31b}$$

Before define perturbations for the recycling system, as an example, let the nominal system be,

$$\dot{x}(t) = Ax(t) + Bx(t-\tau),$$

and the actual system as above but with τ replaced by τ_0, then $p(x(t)) = B[x(t-\tau_0) - x(t-\tau)]$. In this way, with $\theta = \tau_0 - \tau$, we get,

$$P_1(s,\theta) = B[e^{-\tau_0 s} - e^{-\tau s}] = Be^{-\tau s}[e^{-\theta s} - 1].$$

Under the same idea, it is also possible to consider uncertainties on the matrix B. Let us consider the actual input matrix system as B_0, then $p(x(t)) = B[x(t-\tau_0) - x(t-\tau)] + [B_0 - B]x(t-\tau_0)$. Assuming $B_\delta = B_0 - B$, it is obtained,

$$P_2(s,\theta,\delta) = e^{-\tau s}[B(e^{-\theta s} - 1) + B_\delta e^{-\theta s}].$$

Now, let define perturbations on the time-delays for the recycling system (30). The nominal values of the time-delays are τ_1 and τ_2. Thus, the delay uncertainty is obtained as follows,

$$P(s,\theta) = \bar{A}_1 e^{-s\tau_1}(e^{-s\theta_1} - 1) + \bar{A}_2 e^{-s\tau_2}(e^{-s\theta_2} - 1), \tag{32a}$$
$$Q(s,\theta) = \bar{C}_1 e^{-s\tau_1}(e^{-s\theta_1} - 1). \tag{32b}$$

Notice that the uncertainties expressed in equations (32a)-(32b) allows to analyze the plant uncertainty in both, direct and recycle paths. Furthermore, the uncertainties can be acting with different proportion due to its independent characterization as θ_1 and θ_2. Also, independent robustness analysis for each time-delay τ_1 or τ_2 is possible (this can be done by removing τ_2 or τ_1 from equations (32a) or (32b), respectively).

In the case that uncertainties are present on both time-delays and matrices $\bar{A}$, $\bar{A}_1$ and $\bar{A}_2$, it is obtained,

$$\begin{aligned} P(s,\theta,\delta) =& \bar{A}_\delta + e^{-s\tau_1}[\bar{A}_1(e^{-s\theta_1} - 1) + \bar{A}_{1\delta_1} e^{-s\theta_1}] + \\ & e^{-s\tau_2}[\bar{A}_2(e^{-s\theta_2} - 1) + \bar{A}_{2\delta_2} e^{-s\theta_2}], \end{aligned} \tag{33a}$$
$$Q(s,\theta) = \bar{C}_1 e^{-s\tau_1}(e^{-s\theta_1} - 1). \tag{33b}$$

where, $\bar{A}$, $\bar{A}_1$ and $\bar{A}_2$ are the nominal matrices of the recycle system and the corresponding uncertainties are given as $\bar{A}_\delta$, $\bar{A}_{1\delta_1}$ and $\bar{A}_{2\delta_2}$.

A state space representation in the Laplace domain of the observer-based control structure shown in Figure 5 can be expressed as,

$$\mathrm{sX(s)} = \mathcal{A}\mathrm{X(s)} + \mathcal{A}_1 e^{-s\tau_1}\mathrm{X(s)} + \mathcal{A}_2 e^{-s\tau_2}\mathrm{X(s)} + \mathcal{B}R(s) \tag{34}$$

with $\mathrm{X(s)} = \begin{bmatrix} e_x(s) & X(s) \end{bmatrix}^T$, $e_x(s) = \widehat{X}(s) - X(s)$,

$$\mathcal{A} = \begin{bmatrix} \bar{A} & GQ - P \\ -\bar{B}J\bar{K} & \bar{A} - \bar{B}J\bar{K} + P - \bar{B}JQ \end{bmatrix},$$

$$\mathcal{A}_1 = \begin{bmatrix} \bar{A}_1 - G\bar{C}_1 & 0 \\ \bar{B}J\bar{C}_1 & \bar{A}_1 \end{bmatrix}, \mathcal{A}_2 = \begin{bmatrix} \bar{A}_2 & 0 \\ -\bar{B}L & \bar{A}_2 - \bar{B}L \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 0 \\ \bar{B}J \end{bmatrix}.$$

For simplicity of notation, in what follows, the simplest case where G_d and G_r are first order plants will be analyzed. This fact produces,

$$G = \begin{bmatrix} B_d k & B_r \end{bmatrix}^T, \quad \bar{K} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 0 & C_r \end{bmatrix}. \tag{36}$$

The characteristic equation of system (34), is given by,

$$\gamma = \det(sI - \mathcal{A} - \mathcal{A}_1 e^{-\tau_1 s} - \mathcal{A}_2 e^{-\tau_2 s})$$

this is,

$$\gamma = \det \begin{bmatrix} sI - F + G\bar{C}_1 e^{-\tau_1 s} & P - GQ \\ M - \bar{B}J\bar{C}_1 e^{-\tau_1 s} & sI - F + M + \bar{B}JQ - P \end{bmatrix} \tag{37a}$$

$$= \det(sI - F + G\bar{C}_1 e^{-\tau_1 s}) \det(sI - F + M) \det(I + \psi^{-1}\Theta(s,\theta)) = 0 \tag{37b}$$

where

$$F = \bar{A} + \bar{A}_1 e^{-\tau_1 s} + \bar{A}_2 e^{-\tau_2 s}, \tag{38}$$

$$M = \bar{B}J\bar{K} + \bar{B}Le^{-\tau_2 s}, \tag{39}$$

and

$$\psi = \begin{bmatrix} sI - F + G\bar{C}_1 e^{-\tau_1 s} & 0 \\ M - \bar{B}J\bar{C}_1 e^{-\tau_1 s} & sI - F + M \end{bmatrix} \tag{40}$$

is the matrix corresponding to the combined observer-controller for nominal system, and $\Theta(s;\theta)$ collects the plant uncertainties.

Considering that the closed-loop quasi polynomials $\det(sI - F + G\bar{C}_1 e^{-\tau_1 s})$ and $\det(sI - F + M)$ are stable for a proper choice of G, $\bar{K}$ and L, then they do not change sign when s sweeps the imaginary axis and the perturbed closed-loop system remains stable if $\det(I + \psi^{-1}\Theta(s;\theta))$ does not change sign for all $s = j\omega$.

Straightforward computations produce,

$$\begin{aligned} \det(I + \psi^{-1}\Theta(s,\theta)) &= \det \left[I + \begin{bmatrix} \widetilde{Q}_{pq} & \widetilde{Q}_p \end{bmatrix} \begin{bmatrix} P - GQ \\ \bar{B}JQ - P \end{bmatrix} \right] \\ &= \det \left[I + N_c(s) D_c(s) \right], \end{aligned} \tag{41}$$

where,

$$\widetilde{Q}_{pq} = -(sI - F + M)^{-1}(M - \bar{B}J\bar{C}_1 e^{-\tau_1 s})(sI - F + G\bar{C}_1 e^{-\tau_1 s})^{-1}, \quad (42)$$

$$\widetilde{Q}_p = (sI - F + M)^{-1}, \quad (43)$$

which only depends on the nominal system and observer/controller parameter. By using Rouche's theorem [27], it follows that the closed-loop stability condition for the perturbed system results,

$$\|N_c(s)D_c(s,\theta)\|_\infty < 1. \quad (44)$$

Therefore, the considered controller-observer strategy for the actual perturbed system (34) preserves the closed-loop stability for all uncertainties θ_1, θ_2, $\bar{A}_\delta$, $\bar{A}_{1\delta_1}$ and $\bar{A}_{2\delta_2}$ satisfying condition (44).

Remark 3. *The consideration of the Rouche's Theorem in the analysis of uncertain linear delayed systems has been previously reported in [28], [29], [30] in the case of time-delay uncertainties. A similar condition has also been proposed in the literature in terms of the singular values of the associated matrices [31], [32] in the analysis of structured uncertainties.*

5. Simulation results

In this section, the performance of the observer-based control strategy proposed previously is evaluated by means of two numerical examples.

Example 1

Let us consider a simple reaction $A \to B$ in a reactor-separator system with recycle of the type discussed in [33] and including time-delays at direct and forward loops. In this way, the open-loop linear model can be written in the form of (30) where the state x_d is the reactor temperature (T) and x_r is the component concentration (C_A). The manipulated input is the jacket reactor temperature (T_j). Also, the involved constant matrices are given by,

$$\bar{A} = \begin{bmatrix} \frac{F}{V}(1-\lambda) - \frac{F}{V} - \frac{UA}{V\rho c_p} + \frac{(-\Delta H)}{\rho c_p} C_{As} k_{ps} & 0 \\ 0 & \frac{F}{V}(1-\lambda) - \frac{F}{V} + k_s \end{bmatrix},$$

$$\bar{A}_1 = \begin{bmatrix} 0 & 0 \\ C_{As} k_{ps} & 0 \end{bmatrix}, \bar{A}_2 = \begin{bmatrix} 0 & 0 \\ C_{As} k_{ps} & 0 \end{bmatrix}, \ \bar{B} = \begin{bmatrix} \frac{UA}{V\rho c_p} \\ 0 \end{bmatrix}$$

where $k_s = k_0 \exp(-E_a/RT_s)$ and $k_{ps} = k_s(E_a/RT_s^2)$. It is assumed that τ_1 is the time-delay due to temperature measurement and τ_2 the time lag caused by physical transport. The output matrix is $\bar{C}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$, since temperature (first state) is the measured signal. The transfer functions of direct and recycle paths are given by,

$$G_d = \frac{\bar{B}(1,1)}{s + \bar{A}(1,1)} e^{-\tau_1 s}, \quad G_r = \frac{\bar{B}(1,1)^{-1}\bar{A}_1(2,1)\bar{A}_2(1,2)}{s + \bar{A}(2,2)} e^{-\tau_2 s} \quad (45)$$

Operating Volume (V)	500 ft^3
Operating Flowrate (F)	2000 ft^3/hr
Reactor Diameter (D_r)	7.5 ft
Overall heat-transfer coefficient (U)	492.3192 $Btu/(hr\ ft^2\ ^\circ F)$
Heat transfer area through reactor wall (A)	47.1238 ft^2
Preexponential factor (k_0)	$16.96x10^{12}\ hr^{-1}$
Activation energy (E_a)	32400 $Btu/lbmol$
Ideal gas constant (R)	1.987 $Btu/lbmol^\circ F$
Heat of reaction ($-\Delta H$)	39000 $Btu/lbmol\ PO$
Density of coolant (ρ)	53.25 lb/ft^3
Heat capacity of coolant (c_p)	1 $Btu/(lb^\circ F)$
Operating concentration (C_{As})	0.066 $lbmol/ft^3$
Operating temperature (T_s)	$560.77^\circ R$
Forward loop time-delay (τ_1)	0.1 hr
Backward loop time-delay (τ_2)	0.2 hr
Recirculation coefficient (λ)	0.5

Table 1: Constant parameters for Example 5

The proposed observer-based control strategy given in Figure 5, is implemented by considering the parameters values given in Table 1. Since conditions of Theorem 1 are satisfied, a set of proportional gains can be calculated as $8.18 < k < 13.19$ (see Corollary 1 and Remark 1), choosing in consequence $k = 9$ for all the experiments. The two degree of freedom PI controller is implemented by considering $K_p = 41$, $\sigma = 0.5$ and $T_i = 0.142$.

In order to analyze the robustness of the control strategy with respect to time delays, Figure 6 shows the stability condition given by equation (44), where the uncertainties in time-delays $\theta_1 = 0.012hr$ and $\theta_2 = 0.04hr$ are considered. In this case, such combination of uncertainties gives as result a stable closed-loop system since $\|N_c(s)D_c(s,\theta)\|_\infty = 0.9872 < 1$. Taking into account the time-delay uncertainty mentioned above, in Figure 6 it is also presented the stability condition (44) when all parameters of the model are different from the nominal. In this case, the following uncertainties are considered,

$$\bar{A}_\delta = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.08 \end{bmatrix}, \quad \bar{A}_{1\delta_1} = \begin{bmatrix} 0 & 0 \\ -0.11 & 0 \end{bmatrix}, \quad \bar{A}_{2\delta_2} = \begin{bmatrix} 0 & 0.025 \\ 0 & 0 \end{bmatrix}. \tag{46}$$

As it is seen from Figure 6, this set of uncertainties satisfies the stability condition (44) and therefore the stability of the closed-loop system is also assured.

Now, in order to evaluate the output signal evolution, it is considered a positive unit step input and initial conditions in the process and the observer of magnitude 0.1 and 0.2 units, respectively. In Figure 7, a continuous line shows the output response when it is considered the exact knowledge of the model parameters; a dashed line depicts the output signal when time-delays τ_1, τ_2, are increased by 10% and the nominal direct path unstable pole $s_1 = 7.1318$ is shifted to $s_1 = 7.3$ and the nominal stable recycle-path pole $s_1 = -6$ is considered as $s_2 = -6.2$. Additionally, a step disturbance $d(t)$ with a magnitude of 0.3 units acting at $3hr$ is considered. The corresponding control signals are also depicted in the lower part of the figure.

To conclude with this example, Figure 8 shows the corresponding output error $e_y(t)$, when it is considered exact knowledge of the model parameters; a positive unit step input and initial conditions in the process and the observer of magnitude 0.1 and 0.2 units, respectively.

It can be seen from Figures 7 and 8 how the observer-based control strategy produces an adequate response of the system.

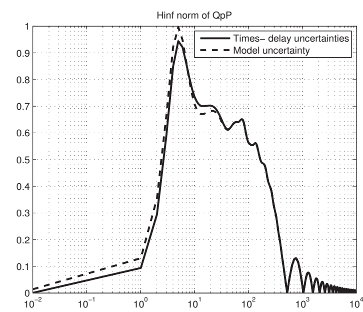

Figure 6: $\|N_c(s)D_c(s,\theta)\|_\infty$ for $\theta_1 = 0.012$ and $\theta_2 = 0.04$.

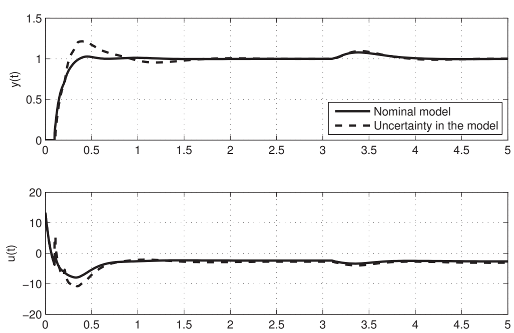

Figure 7: Output and control signals for different initial condition in process and observer, Example 5.

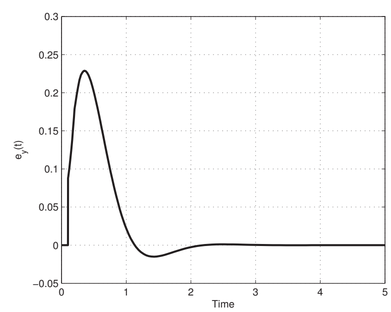

Figure 8: Estimation error $e_y(t)$ by considering different initial condition in process and observer, Example 5.

Example 2
Consider now a recycled system of the form (1) with,

$$G_d = \frac{1}{s-0.25}e^{-2s}, \quad G_r = \frac{10}{(s+1)(s+2)}e^{-2s}. \tag{47}$$

In this case, the proportional observer design gain is $k = 0.3$, chosen from $0.25 < k < 0.6342$, the control feedback (20) is obtained by considering,

$$G_{ff}(s) = 1.5\left(0.4 + \frac{1}{4s}\right) \text{ and } G_c(s) = 1.5\left(1 + \frac{1}{4s}\right) \tag{48}$$

Consider now time-delays uncertainties of $\theta_1 = 0.016$ and $\theta_2 = 0.05$. Thus, the stability condition given by (44) is shown in Figure 9. As it is seen, such combination of uncertainties produces a stable closed-loop operation since

$$\|N_c(s)D_c(s,\theta)\|_\infty = 0.9840 < 1.$$

In order to illustrate the robustness of the system when the observer parameter k is changed, in what follows, the stability condition (44) is analyzed. Consider time-delays uncertainties of $\theta_1 = 0.01$ and $\theta_2 = 0.04$. Under these conditions, Figure 10 shows condition (44) for different values of k. As it is seen, while the value of the parameter k increase, the stability condition (44) tends to its limit. Notice that for $k = 0.45$, condition (44) is not satisfied. From this analysis, it is possible to see that the robustness of the control strategy can be improved by properly choosing the observer parameter k. However, a compromise between performance and robustness is obtained.

As in the previous example, it is considered a step disturbance $D(s) = -0.05$ acting at $t = 40$ sec. Figure 11 shows the evolution of the output and control signals when considering a zero initial conditions (continuous line) and when the initial condition in the recycle path is set at *0.01* (dashed line). Figure 12 shows the error estimation of the recycle signal $e_{\omega_2}(t)$ when $D(s) = 0$ and a small initial condition of magnitude 0.07 is present in the backward path process.

Lemma 4 guarantee that the controller asymptotically reject step disturbances d(t), i.e., to make the sensitivity $S(s) = 0$ for $s = jw = 0$.

It could be interesting to have a "small" sensitivity $S(jw)$ for a wide range of w and not only for $w = 0$, but from expression (26) it is clear that the sensitivity depends on controller parameters k, K_p and T_i. Figure 13 illustrate $S(jw)$ for tree different values of k.

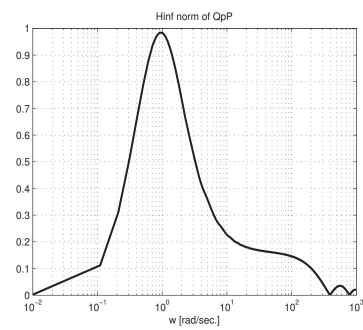

Figure 9: $\|N_c(s)D_c(s,\theta)\|_\infty$ for $\theta_1 = 0.016$ and $\theta_2 = 0.05$.

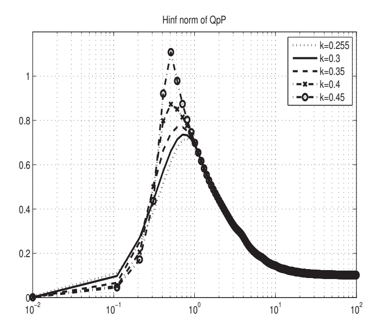

Figure 10: $\|N_c(s)D_c(s,\theta)\|_\infty$ for different values of k with $\theta_1 = 0.01$ and $\theta_2 = 0.04$.

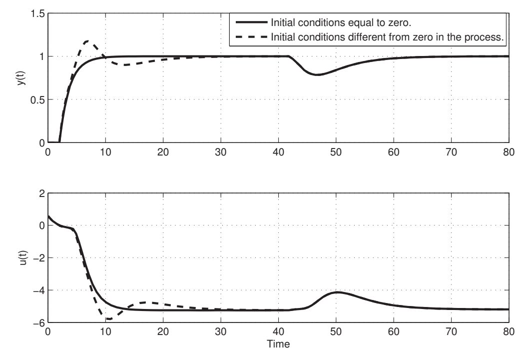

Figure 11: Output and control signals in Example 5.

6. Conclusions

Using recycle in unstable processes with significant time-delay leads to a challenging control problem. In this work this problem has been addressed for the particular case of one unstable pole with significant delay in the forward path. Explicit conditions for the construction of an stabilizing observer-based controller scheme for such class of systems are presented. The observer-prediction strategy is used to estimate some internal variables of the process required for: i) remove the dynamics of backward loop in the recycling process and ii) design a stabilizing control law for the free delay model in the forward path. The basis provided in this work could be useful for extending the class of systems for which the recycle can be used. Particularly, the case of one unstable pole with several stable poles plus significant time-delay in the forward path could be addressed. As it is well known, time-delay uncertainties can affect closed-loop stability of recycled systems, in order to prevent this possibility in the proposed controller, a robustness analysis has been developed. The proposed observer-based control scheme is evaluated by means of numerical simulations, showing an adequate response.

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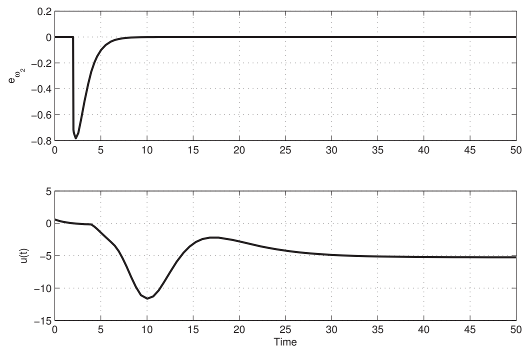

Figure 12: Estimation recycle error $e_\omega(t)$ and control signal $u(t)$, Example 5.

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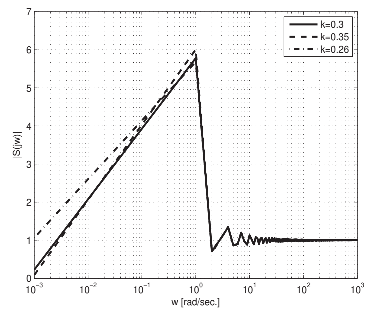

Figure 13: Sensitivity function in Example 5.

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Appendix A. Proof of Lemma 1 and Corollary 1

Proof of Lemma 1. The proof use the well known fact that a discrete time model derived from a continuous time system is equal to its continuous counterpart if the sampling period $T \to 0$ by considering a zero order hold device. It is carried out by discretizing the system and then showing that all the poles remain inside the unitary circle when the sampling period tends to zero iff $\tau < \frac{1}{a}$.

Discretizing model (6) using a zero order hold and a sampling period $T = \frac{\tau}{n}$ with $n \in \mathbb{N}$, it is obtained,

$$G(z) = \frac{b}{a} \frac{(e^{aT} - 1)}{z^n(z - e^{aT})} \tag{A.1}$$

Model (A.1) in closed-loop with the (discretized) output feedback (7) produces the characteristic equation,

$$p(z) = z^n(z - e^{aT}) + k\frac{b}{a}(e^{aT} - 1) = 0 \tag{A.2}$$

Let us to analyze the root locus of (A.2). Open loop system has n poles at the origin and one at $z = e^{aT}$. Then, there exist $n+1$ branches to infinity, $n-1$ of them starting at the origin and going directly to infinity. The two remaining branches starting at a breaking point z_1 located over the real axis between the origin and $z = e^{aT}$ (this situation is illustrated in Figure A.14 for the case $n = 5$). z_1 can be found by considering the equation,

$$\frac{dk}{dz} = \frac{d}{dz}\left[-\frac{z^n(z - e^{aT})}{\frac{b}{a}(1 - e^{aT})}\right] = 0,$$

producing the equation,

$$(n+1)z^n - nz^{n-1}e^{aT} = 0, \tag{A.3}$$

which has $n-1$ roots at the origin and one at,

$$z_1 = \frac{n}{n+1}e^{a\frac{\tau}{n}}.$$

If the breaking point z_1 over the real axis is located inside the unit circle, the closed-loop system could have a region of stability, otherwise the system is unstable for any k.

The stability properties of the continuous system (8) are alternately obtained by considering the limit as $n \to \infty$, or equivalently, when $T \to 0$ of equation (A.3), this is,

$$\lim_{n\to\infty} z_1 = \lim_{n\to\infty} \frac{n}{n+1}e^{a\frac{\tau}{n}} = 1. \tag{A.4}$$

It is important to note that any point $s = \theta$, over the real axis on the complex plane s is mapped to $z = e^{\theta T}$ on the z plane and as a consequence this point

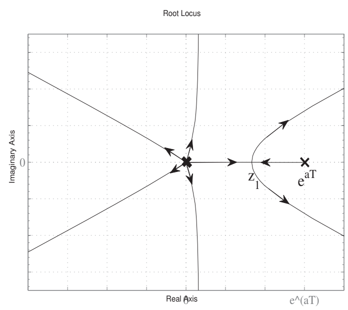

Figure A.14: Root locus of equation (A.2) for $n = 5$.

converges to $z = 1$ when T tends to zero. Notice also that any real point $s = \theta$ on the left half side of the complex plane ($\theta < 0$) is mapped to a point $e^{\theta T}$ that tends to one over the stable region of the z plane. On the contrary, if θ is on the right side of the complex plane over the real axis ($\theta > 0$), the point $e^{\theta T}$ tends to one over the unstable region. Then, from (A.2), it is not difficult to see that if $a\tau < 1$ (i.e., $\tau < 1/a$) there exists a gain k that stabilizes the closed-loop system (i.e., the limit tends to one from the left). In the case that $a\tau \geq 1$ it is not possible to get k that stabilize the system.

Then, if the remaining $n - 1$ roots are into the unit circle, the closed-loop is stable. Let us now prove that the remaining $n - 1$ roots are into the unitary circle if and only if $a\tau < 1$. Assume that $a\tau \leq 1$ and to take into account the continuous case, the characteristic equation (A.2) is modified as,

$$\begin{aligned}
\lim_{n\to\infty} p(z) &= \lim_{n\to\infty} \left[z^n(z - e^{aT}) + k\frac{b}{a}(e^{aT} - 1) \right] \\
&= \lim_{n\to\infty} \left[z^n(z - e^{a\frac{\tau}{n}}) + k\frac{b}{a}(e^{a\frac{\tau}{n}} - 1) \right] \\
&= (z - 1) \lim_{n\to\infty} z^n
\end{aligned}$$

from where it is stated that while one pole is on the neighborhood of $z = 1$, the remaining poles are in a neighborhood of the origin. Then, we can finally state that the system can be stabilized iff $a\tau < 1$.

Proof of Corollary 1.

Assume that $\tau < \frac{1}{a}$ and take into account the discretized system given by (A.1). Analyzing the root locus associated to such discrete system, it is possible to see that the open loop system has n poles at the origin and one at $z = e^{aT}$ without finite zeros. Then, there are $n - 1$ branches going to infinity and a pair converging to a point on the real axis located between the origin and

$z = 1$(stability region). Note that if $k = 0$ the system is unstable. The gain k that takes the systems to the border of the stability region ($z = 1$) is obtained by evaluating k for $z = 1$, this is,

$$k = -\left.\frac{z^n(z - e^{aT})}{\frac{b}{a}(1 - e^{aT})}\right|_{z=1} = \frac{a}{b} \tag{A.5}$$

Then by Lemma 1 the proof is concluded.