CONCEPTS FROM MATHEMATICS EDUCATION RESEARCH AS A TRIGGER FOR MATHEMATICS TEACHERS' REFLECTIONS

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This article reports part of a research project that attempts to identify the elements of an online course that promote the emergence of teachers' reflections. A definition of reflection that is helpful to identify instances of reflections that appear spontaneously in an online course is used in the study. Elements of the documentational approach (Gueudet & Trouche, 2009) are applied to try to establish connections between components of the design of an online course and the emergence of teachers' reflections. The main finding presented is that concepts from mathematics education research can help teachers to see their own teaching practice from a different perspective and thus stimulate reflexions in them.

Keywords: Reflection, online mathematics teacher education, research literature as a tool for teacher development.

INTRODUCTION

This paper reports part of a research project focused on identifying some of the elements in the design of an online course that promote the emergence of reflections in in-service mathematics teachers (see Sánchez, 2010a). In particular, this article addresses the following research question:

Which non-human elements of an online course promote the emergence of mathematics teachers' reflections?

The previous research question is located at the intersection of two sub-areas of research within the field of mathematics teacher education research, namely, *reflective thinking* and *online mathematics teacher education*. Its scientific relevance lies in trying to identify components of an online course that have the potential to encourage the emergence of teachers' reflections, which in turn are considered as an important element for the development of mathematics teachers (see for example Ticha & Hospesova, 2006).

This research was developed in an online mathematics teacher education program, aimed at in-service mathematics teachers working at different educational levels, and coming from all over Latin America[1].

CONCEPTUAL FRAMEWORK

In this section I clarify the key terms that are involved in the research question, but I also refer to the theoretical constructs that I used to address such question.

What is an online course?

An online course is a course that is based on the use of the Internet. This means that the content and the activities of the course are delivered via the Internet. The participants in this type of course do not meet physically to interact and discuss. All the interaction and communication within the course are carried out by using the Internet and related communication tools such as email, discussion forums, audio and video conferencing.

Human and non-human elements

Another key term is *non-human elements of an online course*. I perceive the structure and content of an online course as an amalgam of human elements and non-human elements. I use the term human elements to refer to the people who participate in an online course. In the context of this study, the human elements are the mathematics teachers and the teacher educators who are participating in an online course. When I use the term non-human elements I refer to the resources that a participant in an online course interact with, but which are intentionally provided by the teacher educator. These are resources that are part of the design of an online course. The resources can be of different nature: software, video, activities, articles, audio files, web pages. The two main characteristics of the non-human elements of an online course are: (1) they are elements that are intentionally provided by the course designer. The designer has control over them in the sense that he/she decides when and how they will appear within the course; and (2) they are elements that serve to represent and communicate mathematical and/or didactical ideas that are considered relevant to mathematics teachers' development.

I find relevant to differentiate between human elements and non-human elements of an online course, because the latter are more likely to be controlled by the designer of an online course. That is, although it could be possible to identify some of the human elements in an online course that favour the emergence of reflections (for example, attitudes or types of human interactions), such elements cannot be easily controlled and regulated within an online course.

Reflection

A central theoretical construct used in this study is reflection. I think of *reflection* as a mental process by which our actions, beliefs, knowledge or feelings are consciously considered and examined. To reflect involves more than just recalling or considering something consciously. A process of reflection provides enlightenment about the actions or ideas that are being considered. A process of reflection involves a kind of "Aha! moment" in which something is discovered or revealed.

Comparing my definition of the concept of reflection

A fundamental similarity between my definition of the concept of reflection and other definitions that can be found in the specialized literature is that reflection is

interpreted as a mental process in which *something* is considered or examined in a conscious way. I wrote "something" using italics because many researchers in mathematics teacher education usually interpret such "something" as the act of teaching. In other words, researchers in mathematics teacher education lay particular emphasis on the kind of reflections that are anchored in teaching practice. The widespread use of video recordings in reflection research, through which teachers are asked to analyse classroom episodes, can be considered as an evidence of the emphasis on reflection on teaching practice (see for example Stockero, 2008). The extensive use of theoretical concepts such as reflection-for action, reflection-in-action and reflection-on-action is another kind of evidence of this emphasis on teaching practice (see for example Scherer & Steinbring, 2007). However, in my interpretation of the concept of reflection not only the teaching practice can be the focus of a reflection. You can also reflect on your mathematical knowledge, on the role and application of mathematics in non-mathematical contexts or even on your own feelings and values.

An important difference between my definition of the concept of reflection and other definitions that can be found in the literature is that, in my definition, emphasis is placed on the stage of discovery or revelation (the "Aha! moment") that a reflection can produce. I decided to include the Aha! moment in the definition of reflection on methodological grounds. This point is discussed in the methodology section of the article.

Documentational approach

In order to answer the research question it was necessary to investigate the possible connections between the components of an online course and the emergence of mathematics teachers' reflections. I used the documentational approach (Gueudet & Trouche, 2009) to investigate such connections. This theoretical approach is adequate to address the research question because it helps to study the "effects" that the different resources that a teacher interact with (books, webpages, notes, discussions with colleagues, etc.), produce in his/her practice and schemas. Thus, I used the documentational approach to try to identify the non-human elements of an online course that produced teachers' reflections. The concepts of *instrumentation process* and *documentational orchestration* were particularly useful for the study.

In the documentational approach it is claimed that the professional development of mathematics teachers can be tracked by focusing our attention on the activities that mathematics teachers develop outside the classroom, but that influence their work within the classroom. The focus is particularly centred on *teachers' documentation work*. That is, the interaction between the teachers and a set of elements that allows them to shape and define their work in the classroom. Expressions of such interaction are for example: to extract examples and exercises from a textbook in order to include them in their lesson plans; to analyse their students' mathematical productions; to listen to the suggestions, ideas and experiences from colleagues; to

review the contents of a website that contains educational materials; to study a curriculum reform to be applied in their own school, etc. The set of elements with which a teacher interacts during her documentation work is called *resources*.

In the documentational approach it is argued that, when an interaction between a teacher and a set of resources takes place, *a documentational genesis* (DG) may appear. The concept of DG can be interpreted as an analogy of the concept of instrumental genesis (Trouche 2005) applied to the field of mathematics teacher education. Like the instrumental genesis, the DG is a two-way process in which the teacher appropriates and/or modify the set of resources with which she interacts (this part of the process is called *instrumentalization*), but the set of resources also shapes and influences teacher's activity and way of thinking (this part of the process is called *instrumentation*). The latter concept was used to try to establish links between the non-human elements of an online course and the emergence of reflections.

Finally, a *documentational orchestration* (DO) can be defined as the selection and arrangement of resources that a teacher educator (or a group of teacher educators) carry out with the intention of facilitating teachers' documentation work. Such documentation work is aimed at contributing to the development of teachers' professional knowledge.

METHODOLOGY

In order to answer the research question, I designed an online course which had the scientific aim of promoting the emergence of teachers' reflections, and thus help me to study the influence of the non-human elements of the course on the emergence of such reflections. Three methodological challenges were identified at this stage: (1) to determine what non-human elements were likely to stimulate teachers' reflections (in order to include them as part of the course design); (2) if reflection is an entity that is not directly observable, how to detect a reflection in an online setting?; and (3) how to establish connections between the non-human elements of a course and the emergence of reflections?

To address the above-mentioned points (1) and (2), I conducted a literature review on the concept of reflection in mathematics teacher education research (see Sánchez, to appear). In this review I analysed, among other things, (a) what kind of methodological tools are used to detect a reflection, and (b) what type of elements or conditions have been identified as promoters of teachers' reflections. The information obtained in (a) and (b) was used as inspiration to devise a strategy to promote and identify teachers' reflections in an online setting. To try to establish the connections mentioned in point (3), I applied the concepts of documentational orchestration and instrumentation process. Next I illustrate these points.

Stimulating reflections in an online setting

Several elements were identified in the literature review as promoters of reflections, but only three of them were considered in the design of the course because of their applicability in an online setting. Here I refer to the act of writing, the availability of time, and the reading of mathematics education publications.

Several researchers claim that the *act of writing* is a vehicle for reflection. For example Ponte & Santos (2005) assert: "[W]riting is a powerful way of reflecting, helping teachers to clarify ideas, to look at them from different angles, to come back and revise; the steadiness of the written word also seems to provide more depth to the ideas" (p. 123). I also found that *the relevance of time* in the emergency and the depth of a reflection has been highlighted by several researchers: For instance Sowder (2007) underlines: "[T]ime is needed for developing the ability and habit of reflection. Reflection rarely occurs when time is not a resource available to teachers" (p. 198). These two elements, the act of writing and the availability of time, were considered in the design of the course through the inclusion of asynchronous discussion forums. In this kind of forum people interact through the exchange of written messages. Here the feedback or responses to your written messages and comments are not received immediately. You can post a question in a discussion forum and get an answer some hours or even days later. The asynchronous interactions usually last several days, allowing the participants to have more time to formulate their opinions and to consider the comments and opinions expressed by the other participants. The comments and discussions expressed in the asynchronous discussion forums were the main empirical evidence used in this investigation

Researchers like Shari L. Stockero suggest that the *reading of mathematics education publications* is another activity that improves the level of reflection: "Course readings, for example, exposed the PTs [prospective teachers] to alternative ideas that allowed them to begin to think about learning mathematics in ways other than how they had learned as students. Without these readings to draw upon, the PTs may not have had the tools necessary to reflect at higher levels" (Stockero, 2008, p. 391). Thus, I decided that the structure of the online course should include some sort of writing produced within the community of mathematics education research.

Detecting reflections in an online setting

In the literature review that I conducted it was found that sometimes researchers explicitly ask teachers to produce reflections. This is usually done through the application of questionnaires or through some sort of written assignment. Let me present the following quotation as an illustration of this practice:

> "[T]he PTs [prospective teachers] were required to write a paper in which they reflected on their experience by analyzing how they as the teacher helped or hindered the development of students' mathematical understanding of the problem" (Stockero, 2008, p. 378).

I think that this way of identifying reflections is somewhat artificial. I was interested in identifying reflections that could appear more spontaneously. This was one of the reasons why I decided to include the "Aha! moment" as part of my definition of reflection. My intention was to use the "Aha! moment" as an indicator that the teacher had experienced a reflection. Another reason for using the "Aha! moment" was to avoid confusing instances of reflection with instances of remembering or recalling. The "Aha! moment" indicated to me that the teacher had done more than just remembering. It indicated me that the teacher had discovered or learned something based on the explicit consideration of his/her actions or values.

Establishing connections between non-human elements and reflections

To try to detect the possible connections between the emergence of reflections and the non-human elements of the online course that I designed, I did the following: Firstly, I ordered the set of non-human elements (which in terms of the theory can be called "resources") of the course into stages. Each stage had a particular purpose and comprised a particular subset of resources. I explicitly defined the resources that each stage should contain, and the function and location of the stages within the course. I have called this sort of arrangement *documentational orchestration* (see Sánchez, 2010b).

When the course was being implemented, the concept of reflection was applied to identify teachers' reflections within the asynchronous discussion forums. It was necessary to read and reread several times each utterance within a forum in order to become familiar with its contents. While I was trying to get familiar with the contents of a specific discussion, I also focused on locating the moments of an interaction that could be labelled as reflections, according to my own definition of the concept.

After having these two sets (the set of ordered resources and the set of teachers' reflections), I focused on observing the instrumentation and instrumentalization processes that appeared between these two sets (Gueudet & Trouche, 2009). That is, it was studied how teachers used the resources (instrumentalization processes), but the kinds of effects that the resources produced on teachers (instrumentation processes) were also observed. When the effect produced by an instrumentation process was a reflection, then the development of such process was analysed "backwards" in order to identify the particular resource that produced it.

DATA ANALYSIS

The course that was designed for this study was an in-service course on the use of technology in mathematics teaching. The didactical aim of the course was to make teachers aware of the potential changes that may occur in the mathematics classroom when the use of CAS technology is introduced.

During the course, teachers were solving different mathematical tasks (algebraic factorisations, for example), and comparing techniques based on the use of CAS and techniques based on the use of paper-and-pencil. Then they discussed their experiences obtained through these comparisons in asynchronous discussion forums.

Due to space limitations, it is not possible to describe each of the stages that constituted the course. I will only refer to the latter stage of the course in which the teachers read and discussed the paper by Lagrange (2005). The aim of this stage of the course was to make teachers to compare the experiences that they obtained when using CAS and paper-and-pencil techniques during the course, with the ideas and concepts included in the article.

Two of the main theoretical concepts included in Lagrange (2005) are the pragmatic and the epistemic value of a technique that is based on the use of technology. The *pragmatic value* of a technique refers to the efficiency and economy (of time, of effort) with which a technique helps to solve a mathematical task. For example, the pragmatic value of any CAS software may be related to the speed and efficiency with which the software performs algebraic factorisations. The *epistemic value* of a technique refers to its potential to serve as a means to understand the mathematical objects involved in the application of the technique. For instance, the epistemic value of CAS-based techniques may be related to the fact that such techniques allow a more experimental approach to elemental algebra, where students can explore several particular cases of the factorisation of an algebraic expression ($x^n - 1$, for example) and produce conjectures about the general factorisation of the expression.

After analysing the teachers' asynchronous discussions produced during the initial stages of the course, it became clear that many of them only acknowledged the pragmatic value of CAS techniques. In other words, teachers perceived CAS software as a tool that facilitates the execution and verification of algorithms, but not as a tool that can serve as a means for mathematical inquiry and the construction of mathematical knowledge. See for instance the following comment expressed by a teacher called Francisco[3]:

> I agree with Rosa on the usefulness of the calculator in the sense that it saves a lot of work [...] In general, when there is a discussion on this topic I always conclude that it is important for students to first learn the methods by hand, let us say pencil and paper. [...]

However, eight days later, and after reading Lagrange's paper, this same teacher expressed the following reflection in an asynchronous forum:

> Until I read Lagrange's article I only applied it [the technology], using the terminology of the article, in a pragmatic way. I even felt that without a prior knowledge the use of tools such as CAS and/or calculators did not help to generate learning, i.e., I did support the use of these tools but apparently only attaching value to their pragmatic aspect. In integral calculus I encouraged the use of these tools in all the required calculations up to derivation. In differential equations I incentivise its application in the calculation of

integrals and so on. So I was very surprised that the article emphasises the epistemic aspect of these applications. Partly he was right, because the epistemic application apparently requires planning and construction of new specific activities that do not arise naturally from the teaching with paper and pencil. I would like to conclude this contribution leaving the reflection and concern of how a methodology for applying the epistemic value should be.

RESULTS AND FINAL DISCUSSION

My interpretation of the reflection mentioned in the last section is that such reflection was triggered by the interaction between Francisco and the contents of the article Lagrange (2005). The theoretical concepts contained in Lagrange's paper were the only non-human resource identified in the study as trigger for mathematics teachers' reflections. Thus, a possible answer to the research question posed at the beginning of this paper is that theoretical concepts from mathematics education research can promote the emergence of mathematics teachers' reflections.

I try to be cautious and say that it is a "possible" answer because I did not obtain more empirical evidence to confirm that the theoretical concepts contained in mathematics education articles are non-human elements that promote the emergence of reflections. The lack of more instances of reflections to support this conclusion can be caused by the definition of reflection applied in the study. Such definition is restrictive in the sense that requires the appearance of an Aha! moment. The definition for example is not appropriate for detecting reflections that are internally experienced by the individual, but which are not expressed externally by an Aha! moment.

I however claim that the answer to the research question is likely to be a result with some degree of generality. I claim this because there are other studies where it is also argued that the study of concepts and theories from mathematics education research promotes critical reflection on our own beliefs and practices as mathematics educators (see for example Even, 1999 and Tsamir, 2008). If one accepts that the theoretical concepts from mathematics education research have the potential to encourage the emergence of teachers' reflections, then a question naturally arises: what kind of theoretical concepts must be used for this purpose? Tsamir (2008) raises similar questions, without providing a specific answer. Of course these questions deserve further investigation, however, it is possible to formulate a hypothesis: I believe that the type of theoretical concepts that can help teachers to reflect on their own practice and values, must be concepts that seem applicable to them. In other words, teachers need to find some relationship or application between such concepts and their own teaching practice. Thus, it is likely that theoretical concepts with little or no relation to teachers' practice will not serve for this purpose.

NOTES

1. More information about this educational program can be found at www.matedu.cicata.ipn.mx (in Spanish).

2. All teachers' names are pseudonyms.

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