Generalized treatment for diffusion waves

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Intended for teaching purposes, the phenomenon of diffusion in the presence of periodical sources is described, taking into account a characteristic operator, $\hat{F}(t)$, leading to a generalized hyperbolic equation. The essential features of the accompanying harmonic flux are presented. For this purpose the solution to the problem is interpreted in terms of diffusion waves, a peculiar class of waves with complex wave numbers whose generation, propagation and detection constitute the basis of modern analytical techniques able to measure optical and transport properties of materials in the condensed or gaseous phase. A generalized mathematical equation describing this kind of waves is shown and the existence of critical modulation frequencies, at which the diffusive fluxes change their behaviour, is demonstrated for different physical phenomena involving diffusion waves. The dispersion equation for diffusion waves is given, and different particular cases in modulation frequency "spectrum" are discussed.

Keywords: Diffusion; periodical sources; dispersion equation.

Con propósitos de enseñanza se describe el fenómeno de difusión en presencia de fuentes periódicas teniendo en cuenta un operador característico, $\hat{F}(t)$, que conduce a una ecuación de difusión hiperbólica generalizada. Se presentan las características fundamentales del flujo de calor harmónico asociado a ella. Para ello se interpreta la solución del problema en términos de ondas de difusión, un tipo particular de ondas con números de onda complejos y cuya generación, propagación y detección constituyen las bases de técnicas analíticas modernas capaces de medir propiedades ópticas y de transporte de materiales en la fase condensada o gaseosa. Se presenta una ecuación matemática generalizada para describir esta clase de ondas y se demuestra para diferentes fenómenos que involucran las ondas de difusión la existencia de frecuencias de modulación características a las cuales el flujo difusivo cambia su carácter. Se presenta la ecuación de dispersión y se discuten diferentes casos particulares en el "espectro" de frecuencia de modulación.

Descriptores: Difusión; fuentes periódicas; ecuación de dispersión.

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1. Introduction

In the last 30 years, the concept of Diffusion Waves [1] has been increasingly used for the description of several physical phenomena [2-9] for which the presence of a periodically varying source is common. Therefore, many authors have adopted the concepts of wave physics that were used successfully in the explanation of other periodic phenomena, to interpret their experimental results.

The concept of waves is involved in many fields of science, and is becoming an integral part of the physics curricula at different levels. The increasing use of concepts related to wave propagation in the teaching of physics argues in favor of their introduction whenever possible. As the analysis of transport problems presented in standard textbooks does not make systematic use of the wave treatment, it is the purpose of this work to discuss some aspects of the so-called diffusion waves, *i.e.*, solutions of the diffusion equation in the case of periodic modulated sources. A generalized equation is presented describing diffusion wave fields of a general nature. For time-varying harmonic sources, and based on the dispersion relation, we shall emphasize some of the physical aspects related to the nature of the diffusion fields and their frequency dependence, attempting to present these questions in a unified style so that their use for educational aims will be favored.

2. Theory

Consider a sample where oscillatory wave fields $\varphi(\vec{r},t)$ are generated by a source with periodically modulated intensity, or driving force [1], of the form $Q(\vec{r})e^{i\omega t}$, where ω is the angular modulation frequency, $\vec{r}$ is the spatial coordinate, and t is time. The function $\varphi\vec{r},t)$ can be a thermal or temperature wave, $T(\vec{r},t)$ [10]; or a charge carrier density wave, $N(\vec{r},t)$ [11], in photothermal [3] experiments; a diffuse photon density wave in a turbid medium excited by periodically infrared light, $u(\vec{r},t)$ [2], among others. These phenomena will be used as illustrative examples for the discussion. For example, diffusion waves were used in the past in the analysis of compound migration in stratified media [4]; in the study of molecular diffusion processes by means of pressure oscillations in vacuum chambers [5]; as well as in applications related to mass transport in metals [6], electrolytes [7] and dialysis membranes [8]. Early works concerning the diffusion of a periodic flux of neutrons were also reported elsewhere [9].

We shall assume, as is often encountered in the praxis, that the sample is homogeneous and isotropic, and its properties are constant throughout the changes in temperature involved. The results achieved here can be extended by the interested reader, with suitable modifications, to a more general situation.