Linear relationships in heat transfer

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(Received 17 February 2009; accepted 24 April 2009)

Abstract
The use of linear relationships that can appear in heat transfer phenomena is described using a simple physical experimental situation in which the temperature evolution with time in a sample heated with low intensity continuous light is measured. These questions should be included in the introductory physics curricula of science and engineering studies to teach aspects from different branches of physics (for example thermodynamics) and mathematics (ranging from functional analysis to differential equations).

Keywords: Conduction heat transfer, Radiation, Convection.

Resumen
El uso de relaciones lineares que pueden aparecer en fenómenos de transferencia de calor es descrito utilizando una situación experimental sencilla en la cual la evolución de la temperatura con el tiempo es medida en una muestra calentada con luz de poca intensidad. Estas cuestiones podrían ser incluidas en el curriculum de Física de carreras de Ciencias e Ingeniería para enseñar aspectos de diferentes ramas de la Física (por ejemplo de Termodinámica) y de Matemática (desde análisis funcional hasta ecuaciones diferenciales).

Palabras clave: Transferencia de calor conducción, Radiación, Convección.

PACS: 44.25.+f, 47.20Bp, 44.10.+I; 66.30.Xj; 01.90.+g **ISSN 1870-9095**

There are several kinds of mathematical functional relationships in science between changing entities called variables, such as the exponential, the parabolic, the trigonometric, etc. One of the most common of them is the linear relationship, where an incremental change in one variable is matched by a proportional variation in the other. We can find several variables that depends linearly to one another, among others the velocity of a body and its displacement in which the former is constant, the applied voltage and the electrical resistance in a metal, the mass and the density of an object, the vapour pressure of a substance and its temperature, the gravitational and the electrostatic force between two charged objects and the inverse of the square of the distance between them, and so on. Linear equations can be written in the form of $y=mx+b$, in which x is the independent variable, y is the dependent variable, m is the slope, and b is the y-intercept. These equations appear to be straight lines in a xy-coordinate graph. Often the use of the logarithmic function allows the linearization of mathematical equations: It is well known that the exponential and the potential functions get linear in a semi-log and in the log-log plot of the dependent versus the independent variable respectively, a fact that is very often used in data processing to obtain typical parameters characterizing these functions. In this way we find, for example, that the logarithm of the Molar concentration versus time is a linear graph as well as the plot of the logarithm of the electrostatic force between two charged objects versus the logarithm of the distance between them.

But not always it is possible to find a linear relationship between the variables involved in a given problem, what makes sometimes difficult its solution, which must be found often numerically. Although the existence of powerful computational methods allows one today with relative ease to hand non-linear equations, a better physical insight in a studied problem can be obtained by means of analytical expressions, where particular limiting and asymptotic cases could be analyzed and, at the same time, could be easier programmed than complicated equations. Then, the use of linear relationships is always advantageous.

Here we will present a typical situation that can be encounter in thermal physics experiments, whose interpretation can involves non-linear equations. We will shown how a carefully analysis of the problem allow one to find the conditions for which these expressions become linear, what can make easier the look for an analytical solution of the problem.

Consider that a thin slab of a solid sample of thickness L is heated which a light beam that is uniformly focused onto one of its surfaces. On the opposite side, its