PHYSICAL REVIEW D **79,** 124016 (2009)

Class of Einstein-Maxwell-dilaton-axion space-times

Tonatiuh Matos[*,†]
Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, Apartado Postal 14-740,
07000 Distrito Federal, México

Galaxia Miranda[*,‡]
Departamento de Física, Escuela Superior de Física y Matemáticas del IPN, Edificio 9, 07738 Distrito Federal, México

Rubén Sánchez-Sánchez[§]
Centro de Investigacion en Ciencia Aplicada y Tecnologia Avanzada del IPN, Legaria 694, 11500 Distrito Federal, México

Petra Wiederhold[||]
Departamento de Control Automático, Centro de Investigación y de Estudios Avanzados del IPN, Apartado Postal 14-740,
07000 Distrito Federal, México

(Received 21 May 2009; published 11 June 2009)

We use the harmonic maps ansatz to find exact solutions of the Einstein-Maxwell-dilaton-axion (EMDA) equations. The solutions are harmonic maps invariant to the symplectic real group in four dimensions $Sp(4,\mathbb{R}) \sim O(5)$. We find solutions of the EMDA field equations for the one- and two-dimensional subspaces of the symplectic group. Specially, for illustration of the method, we find space-times that generalize the Schwarzschild solution with dilaton, axion, and electromagnetic fields.

DOI: 10.1103/PhysRevD.79.124016 PACS numbers: 04.20.Jb, 04.20.−q

I. INTRODUCTION

The new discoveries of the last years have changed our perspective and understanding of the Universe. Specially, the discovery of the dark matter and the dark energy have opened new big questions about the nature of the matter in cosmos. Doubtless, it is time to propose new paradigms in order to give some light to these questions. One of the most accepted candidates to be the nature of the dark energy is a scalar field [1], and maybe it is less known that scalar fields are also very good candidates to be the nature of the dark matter [2].

At the same time, theories like superstrings propose the existence of several scalar fields. In particular, at low energy the superstrings theory contains at least two scalar fields called the dilaton and the axion. There are some attempts to compare these two scalar fields with the dark matter and dark energy [3,4], but the main problem for this is to go from the higher dimension theory to the four-dimensional one [5]. In some cases, it seems that this theory could explain the Universe, but this question is still open.

In this work we study the Einstein-Maxwell-dilaton-axion (EMDA) system, from the effective point of view, i.e., we start from the corresponding Lagrangian and derive the field equations. Later we use the harmonic maps ansatz to solve the system of six coupled, nonlinear differential equations for the axial symmetric stationary case.

The method of harmonic maps to find exact solutions of the Einstein, Einstein-Maxwell, and Einstein-Maxwell-dilaton fields has been used with great success. This ansatz was first used by Neugebauer and Kramer to find exact solutions to Einstein-Maxwell equations [6], and in [7] this ansatz was generalized to the Einstein-Maxwell-dilaton system with a coupling constant α between the dilaton and the Maxwell fields given by $\alpha = \sqrt{3}$. Later on this ansatz was generalized in [8] for an arbitrary α. The ansatz has been used also for solving the Einstein-Maxwell-phantom system with arbitrary α [9]. Here we apply the harmonic maps ansatz to solve the equations of motion for the Einstein-Maxwell-dilaton-axion theory in the target space.

This work is organized as follows. In Sec. II, we introduce the fields of the potential space we are working with. In Sec. III, we write the field equations as a nonlinear σ model to be used in Sec. IV, where we use the harmonic maps ansatz to solve the system. In Sec. V, we solve the field equations for the one-dimensional subgroups of $Sp(4,\mathbb{R}) \sim O(5)$, and, in Sec. VI, for the subgroup $SO(2,1)$. In Sec. VII, some conclusions and perspectives are given. In the Appendix we review the use of the harmonic maps ansatz for the chiral equations, the nonlinear σ models.

II. THE EFFECTIVE ACTION FOR EMDA

Gravity with two scalar fields, the dilaton and the axion and a $U(1)$ vector field can be described with the action

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[*] Part of the Instituto Avanzado de Cosmología (IAC) collaboration.
http://www.iac.edu.mx/

[†] tmatos@fis.cinvestav.mx
[‡] galaxia@fis.cinvestav.mx
[§] rsanchezs@ipn.mx
[||] biene@ctrl.cinvestav.mx