Generalized Coherent States and Non-Linear Schrödinger Equation

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The quantum coherent states were established under the bases of the quantum laws of the nature. These coherent states play an important role in science and technology, for example in optics, where they are used in the theoretical and experimental study of lasers. Here, some of the properties of these quantum states and some of their applications to many-body classic problems of modern physics related to the well-known non-linear Schrödinger equation are analyzed.

**Key Words:** Coherent states, Lattice model, Non-linear schrödinger equation, Solitons

Introduction

In 1926, Schrödinger constructed the Coherent States (CS) for the quantum harmonic oscillator. These quantum states were retaken in 1963 by Glauber, Klauder and Sudarshan among others for studies in quantum optics (Klauder and Sudarshan, 1968). The main focus is on laser. The basic understanding of the laser beams is due to Glauber, which led to the development of quantum theory of lasers; after these CS were discovered. Glauber gave the official name of coherent states to the quantum states of lasers (Glauber, 1963). The fluctuations in their amplitude and phases, for example, vanish simultaneously. For a general formulation of the theory of the electro-magnetic field and the radiation, it is but advisable to use the method of the second quantization. This method consists of the potential and the corresponding fields and like operators (q-numbers) that satisfy certain relations of commutation. These operators operate on the vector of state that describes the state of the electro-magnetic field as a generalized system quantum. The vector of state is defined in the space of numbers of particles (photons) and do not depend on the co-ordinates. It is also possible to define in such a way that it does not depend on the time. A field operator in the Heisenberg representation is defined. In this form, the states that satisfy the equations (Ajiezer and Berestetsky, 1981).

\[ E^+(r,t)\left| \psi E^-(r,t)\right| = \epsilon(r,t) \left| \psi \right|, \quad \left< E^+(r,t) \right| = \left< E^-(r,t) \right| \epsilon(r,t) \]

are called coherent states.

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