Solving the mean–variance customer portfolio in Markov chains using iterated quadratic/Lagrange programming: A credit-card customer limits approach

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A B S T R A C T

In this paper we present a new mean–variance customer portfolio optimization algorithm for a class of ergodic finite controllable Markov chains. In order to have a realistic result we propose an iterated two-step method for solving the given portfolio constraint problem: (a) the first step is designed to optimize the nonlinear problem using a quadratic programming method for finding the long run fraction of the time that the system is in a given state (segment) and an action (promotion) is chosen and, (b) the second step is designed to find the optimal number of customers using a Lagrange programming approach. Both steps are based on the -variable method to make the problem computationally tractable and obtain the optimal solution for the customer portfolio. The Tikhonov’s regularization method is used to ensure the convergence of the objective function to a single optimal portfolio solution. We prove that the proposed method converges by the Weierstrass theorem: the objective function of the mean–variance customer portfolio problem decreases, it is monotonically non-decreasing and bounded from above. In addition, for solving the customer portfolio problem we consider both, a constant risk-aversion restriction and budget limitations. The constraints imposed by the system produce mixed strategies. Effectiveness of the proposed method is successfully demonstrated theoretically and by a simulated experiment related with credit-card and customer-credit limits approach for a bank.

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1. Introduction

1.1. Brief review

Mean–variance analysis for optimal asset allocation is one of the classical results of financial economics (Markowitz, 1952). However, it is well-known by financial managers that firms often perform the portfolio selection in a suboptimal manner. Then, the mix together of huge investment of money and inefficient management of the budget motivates the interest in better understanding of optimal asset allocation. We are focus on optimal asset allocation from the monopoly point of view considering the analysis of customer portfolios selection from a risk management perspective (Ryals, 2003) (we leave the individual customer relationships (Rae and Kuniar, 2000) point of view of the optimal customer portfolio out of the scope of this paper).

The estimation of the risk is a fundamental topic for (risk-averse) financial managers. Risk management based on diversified portfolio makes it possible to reduce the risk of suffering a large loss and, at the same time, securing a certain level of profitability (Cornuejols & Tutuncu, 2007). Portfolio selection accordingly plays an important role in financial decision making. The risk contribution of each customer to the customer portfolio is taken into account in a customer portfolio valuation applying the Markowitz’s theory portfolio selection (Markowitz, 1952). The goal of the customer portfolio is to determine the optimal number of customer types from a value-based risk management perspective. In order to better predict individual customer behavior customers are grouped into segments (Ho, Thomas, P. P. O. R., & Scherer, 2004). These segments are created by trying to group together customers having similar behavior (Wedel & Kamakura, 2000). Finally, the customer portfolio determines how these segments will be addressed.

In this paper, we consider the single-period budgeted campaign optimization problem (Dar, Mansur, Mirrokh, Muthukrishnan, & Nadav, 2009; Feldman, Muthukrishnan, Pal, & Stein, 2007;...