A Stackelberg security game with random strategies based on the extraproximal theoretic approach

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ABSTRACT

In this paper we present a novel approach for representing a real-world attacker–defender Stackelberg security game-theoretic model based on the extraproximal method. We focus on a class of ergodic controlled finite Markov chain games. The extraproximal problem formulation is considered as a nonlinear programming problem with respect to stationary distributions. The Lagrange principle and Tikhonov’s regularization method are employed to ensure the convergence of the cost functions. We transform the problem into a system of equations in a proximal format, and a two-step (prediction and basic) iterated procedure is applied to solve the formulated problem. In particular, the extraproximal method is employed for computing mixed strategies, providing a strong optimization formulation to compute the Stackelberg/Nash equilibrium. Mixed strategies are especially found when the resources are available for both the defender and the attacker are limited. In this sense, each equation in this system is an optimization problem for which the minimum is found using a quadratic programming approach. The model supports a defender and N attackers. In order to address the dynamic execution uncertainty in security patrolling, we provide a game-theoretic based method for scheduling randomized patrols. Simulation results provide a validation of our approach.

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1. Introduction

1.1. Stackelberg game solution

The classical Nash original equilibrium solution concept (Nash, 1951) provides a reasonable non-cooperative equilibrium solution when the strategy profiles of the players are symmetric (no single player dominates the decision process). Nevertheless, there are other types of non-cooperative decision problems that introduce a hierarchical equilibrium solution concept that differs in many aspects from the classical Nash’s approach. In these games, the player who has the ability to enforce his strategy on the other player(s) and knows the rational reaction set of his/her opponent is called the leader. The other players are called the followers. The equilibrium solution concept is the Stackelberg equilibrium solution and the corresponding game is called Stackelberg game, named after Heinrich von Stackelberg in recognition of his pioneering work on static games (von Stackelberg, 1934). The main problem arising in this concept is that, for each fixed strategy of the leader, there may exist several (or a set of) Nash equilibria (Clemmer and Poznyak, 2011, 2013) for the followers.

The Stackelberg strategy for dynamic games was introduced in the works of Chen and Cruz (1972), and of Simaan and Cruz (1973a, 1973b), who also introduced the concept of a feedback Stackelberg solution, with the restriction that the follower’s response was unique for each strategy of a leader. Its complement in the context of infinite games was first studied in Basar and Olsder (1995). Such a strategy exhibits an information bias between players leading to establishing a hierarchy between them. In addition, more recently, the scope of the applicability of the Stackelberg equilibrium concept in the field of game-theoretic modeling of preferences in hierarchical systems can be found in the work of Kołodziej and Xhafla (2011).

The extraproximal approach (Antipin, 2005) can be considered as a natural extension of the proximal and the gradient optimization methods used for solving the more difficult problems for finding an equilibrium point in game theory. The simplest and most natural approach for solving the proximal method is to use a simple iteration by omitting the prediction step. However, as shown in Antipin (2005), this approach fails. A more versatile procedure would be to perform an "extraproximal step," i.e., gathering certain information on the future development of the process. Using this information, it is possible to execute the