Computación y Sistemas Vol. 3 No.1 pp. 58-61
© 1999, CIC - IPN. *ISSN 1405-5546* Impreso en México

RESUMEN DE TESIS DOCTORAL

Identification of Non Stationary ARMA Models Based on Matrix Forgetting

Medel - Juárez J. J[1]
Advisor: Poznyak., A. S. [2]

Abstract

To identify time-varying matrix parameter participating in ARMAX-model description, a new recursive procedure is suggested in this thesis. This algorithm presents a combination of recursive version of Instrumental Variable procedure together with Matrix Forgetting Factor. The asymptotic value of the identification error "in average" is shown to have a bound which turns out to be dependent on the rate of parameter changing as well as on the variance of noise to be applied. By Monte-Carlo method it was shown that identification performance index has a minimum within the set of matrix forgetting with a norm less then 1. The optimum value as well as the corresponding optimal matrix forgetting are dependent on unknown parameters of a given ARMAX model and also on statistic characteristics of the applied noises. They can not be calculated a priory before the estimating procedure start to work. An adaptive approach seems to be needed. To derive an adaptation procedure for adjusting the matrix forgetting factor, stochastic gradient approach is suggested to be in use. Direct calculation of the stochastic gradient realization corresponding to the given performance index is impossible because it needs the use of some sort of sensitivity vector and also requires the formation about real values of unknown parameters that is in contradiction with the original problem setting. In addition the original identification criterion (norm of difference between the outputs of real process and its model) turns out to be multimodal function of matrix forgetting, so the gradient-like technique could lead to local minima - this is the crucial point of the direct stochastic gradient approach if it would be applied to the problem under consideration. To avoid these problems we suggest to consider a smoothed functional which is close (in some sense) to the original identification criterion, but has several advantages: first, it is quasi-convex (practically, unimodal) and, second, its gradient can be easily calculated on-line. The convergence and rate convergence analysis is presented based on several new optimization results as well as on martingale technique applied to discrete-time random sequences. The rate of convergence of adaptive forgetting to the optimal one is shown to be equal to harmonic function .The optimal parameters of the matrix forgetting procedure with adaptation were calculated. They turn out to be dependent only on dimensions of the vector and matrices participating in the description of a given ARMAX model, it means, this information is available a priory before the estimation process. So, this optimal adaptive procedure is realizable. The effectiveness of the suggested approach is illustrated by several numerical examples including DC-induction motor model.

Each chapter of this thesis present the next struc-

[1] C.I.C - I.P.N. AP. 07738 México D.F, email jjmedel@pollux.cic.ipn.mx
[2] CINVESTAV - I.P.N, México D.F., email apoznyak@ctrl.cinvestav.mx

ture, without loss the objective; that is, to identify of ARMA models.

In chapter 1, introduces the general concepts of identification, filtering, predicting and smoothing and also presents a brief review of publication concerning the identification problem of linear discrete-time stochastic models under color noises of moving average type. It is shown that in this situation the least square method (LSM) approach turns, but to be unworkable and some other ones are needed to solve the problem. One of them is the instrumental variable (IV) approach. Different techniques are discussed for the case of parameters varying in time. General thesis structure concludes this section.

In chapter 2, we suggest a new approach to provide time-varying parameter estimates in ARMA (Auto Regression Moving Average) - models of stochastic nature based on the use of the recursive version of Instrumental Variable Method (IVM) with a Matrix Forgetting Factor (MFF). This combination is a tool for estimating the entries of a nonstationary parameter matrix involved in the ARMA model. An asymptotic analysis of the error matrix is presented. Simulation results demonstrate the effectiveness of the suggested approach.

In chapter 3, we suggest an approach to provide time-varying parameter estimates in ARMA (Auto Regression Moving Average) - models of stochastic nature based on the use of the recursive version of Instrumental Variable Method (IVM) with a Matrix Forgetting Factor (MFF). We demonstrate that there exists the best selection of MFF minimizing the error strip bound. This optimal MFF is depending on a complex manner of a group of unknown parameters. An adaptation procedure is suggested to obtain in asymptotic this optimal value using only the available measurements. The adaptation procedure is based on Gaussian smoothing technique. The combination of IVM with adaptive MFF is a tool for estimating the entries of a nonstationary parameter matrix involved in the ARMA model. An asymptotic analysis of the error matrix is presented. Simulation results demonstrate the effectiveness of the suggested approach.

In chapter 4, we presents the new results concerning the adaptive controller design for a direct current induction motor. The mathematical model of this DC induction motor, which we are dealing with, is a MIMO linear system, containing unknown time varying parameters. To estimate them we apply the new identification procedure based on Adaptive Matrix Forgetting Factor (AMFF) involved into Instrumental Variables (IV) numerical scheme. Using these estimates we construct an adaptive controller, realizing one-step minimal variance (locally optimal) control law, so that a good tracking for a nominal vector process of pulse function type is obtained.. The these results are very competitive with respect to PID controller traditionally applied for such sort of plants.

1 Model Description and the Statement of the Probelm

Let us consider a matrixc ARMA-model of fixed order n_a and noises ζ_t on the moving average type of order n_d, disturbing the state vector $x_t \in \mathcal{R}^N$ of the system:

$$\begin{aligned} x_t &= A_{1,t}x_{t-1} + \cdots + A_{n_a,t}x_{t-n_a} + \zeta_t \quad (1) \\ \zeta_t &: = D_{0,t}\xi_t + \cdots + D_{n_d,t}\xi_{n_d} \end{aligned}$$

where $\xi_t \in \mathcal{R}^N$ is a white noise vector of centred random variables with uniform (on t) 4-th bounded moments and where $D_{j,t} \in \mathcal{R}^{N\times N}$ $(j = 0, \ldots, n_d)$ are unkown deterministic bounded matrices. All bounded sequenses are suggested to be given on a probability space $(\Omega, \mathcal{F}, P)$ (see the thesis apendix about of the stochastic processes).

Let also assume that the output model is given by the linear algebraic relation which also contains a white noise vector $\xi_t^{'} \in \mathcal{R}^M$, disturbing the measured output signal vector $y_t \in \mathcal{R}^M$:

$$y_t = C_t x_t + \xi_t^{'}. \quad (2)$$

Hereafter we accept that $C_t \in \mathcal{R}^{M\times M}$ is known full rank matrix $(C_t^T C_t > 0)$. But considerign that the output system in the standard vector-form presentation is depicted by:

$$y_t = A_t z_t + \nu_t, \quad (3a)$$

where the vector $z_t \in \mathcal{R}^{Mn_a}$ of *"genralized inputs"* and the extended matrix $A_t \in \mathcal{R}^{M \times Mn_a}$ of nonstationary parameters as defined as follow:

$$\begin{aligned} z_t \quad &: = \left(C_{t-1}^{+} y_{t-1}, \cdots, C_{t-n_a}^{+} y_{t-n_a}\right), \\ A_t \quad &: = \left(C_t A_{1,t}, \cdots, C_{n_a} A_{1,t-n_a}\right). \end{aligned}$$

To deal with nonstationary models (3a) containing a nonstationary unknown matrix A_t as well as nonstationary random disturbances ν_t we have to introduce an exponential discouting procedure into the estimating procedure. Torealize that let us consider some sort of combination of instrumental variable method (IVM) with the vector $\vartheta_t \in \mathcal{R}^{Mn_a}$ of the instrumental variables and matrix forgetting factor (MFF) denoted later by $0 < R = R^T \in \mathcal{R}^{M \times Mn_a}$.

Finally, the identification procedure in recurrent form is depicted by the next algebraic expression:

$$\begin{aligned} \widehat{A}_n &= \widehat{A}_{n-1} + \left[y_n - \widehat{A}_{n-1} z_n\right] \vartheta_n^T \Gamma_n, \quad n \geq n_0, \\ \Gamma_n &= R^{-1}\Gamma_{n-1} - \frac{R^{-1}\Gamma_{n-1} z_n \vartheta_n^T R^{-1}\Gamma_{n-1}}{1 + \vartheta_n^T R^{-1}\Gamma_{n-1} z_n}. \end{aligned}$$

The aim fo this study was to investigate the properties of this algorithms in the general case (IVM with MFF) and we demonstrated analitically and iluatrativelly that this algortihms that the boundless of the estimated trajectoires the estimation tracking error will remain bounded and these bounds are consistent with the classical results when $A_n \to A$ for the ordinary LSM that corresponds to the (IV-MFF) procedure presented into the thesis with $R = \rho I$ and $\rho \to 1$.

In general how to select the best MFF ?

Let us considered the adaptive version of the identification algorithm with time-varying forgetting factor R_n which was changed in time according to the following adaptive procedure:

$$r_{n+1} = \pi_{\mathcal{D}_{\varepsilon_n}} \left\{ r_n - \gamma_n \left(\det \Theta_n\right)^{(Mn_a)^2} \Theta_n^{-\frac{1}{2}} \varkappa_n q_n \right\}$$

where

$$q_n := \left\| y_n - \widehat{y}_n \left(col^{-1} \left(r_n + \Theta_n^{-\frac{1}{2}} \varkappa_n \right) \right) \right\|^2,$$

$$r_n := col\left(R_n\right), \quad R_n = col^{-1}\left(r_n\right),$$

γ_n is a positive time-varying gain step, $\Theta_n = \Theta_n^T \in \mathcal{R}^{Mn_a \times Mn_a}$ is a positive time-varying matrix and $\varkappa_n$ is the idependent Gaussian vector with standard distribution $\mathcal{N}(0, I)$. $\pi_{\mathcal{D}_{\varepsilon_n}}\{\cdot\}$ is the projection operator acting from $\mathcal{R}^{Mn_a \times Mn_a}$ to the set

$$\mathcal{D}_{\varepsilon_n} := \left\{ r \in \mathcal{R}^{Mn_a \times Mn_a} : col^{-1}(r) \in \mathcal{R}_{\varepsilon_n} \right\}$$

and satifaying for any $r \in \mathcal{R}^{Mn_a \cdot Mn_a}$ and nay $r^* \in \mathcal{D}_{\varepsilon_n}$ to the following inequality

$$\left\| \pi_{\mathcal{D}_{\varepsilon_n}}\{r\} - r^* \right\| \leq \left\| r - r^* \right\|.$$

In this algorithm the vector $\widehat{y}_n \left(col^{-1} \left(r_n + \Theta_n^{-\frac{1}{2}} \varkappa_n \right) \right)$ is calculated in acordance to

$$\widehat{y}_\tau(R) = \widehat{A}_\tau z_\tau.$$

The simulation results were depicted into the thesis presntation.

2 General Conclusions

- To identify time-varying matrix parameter participating in ARMAX-model' description, a new recursive procedure is suggested in this thesis.
- The suggested algorithm presents a combination of recursive version of Instrumental Variable procedure together with Matrix Forgetting Factor.
- It is shown that the asymptotic value of the identification error "in average" has a bound which turns out to be dependent on the rate of parameter changing as well as on the variance of noise to be applied.
- By Monte-Carlo method is was shown that identification performance index has a minimum within the set of matrix forgetting are dependent on unknown parameters of agiven ARMAX-model and also on statistic characteristics of the applied noises. They can not be calculated a priory before the estimating procedure start to work. An adaptation procedure seems to be needed.

- To derive an adaptatio procedure for adjusting the matrix forgetting factor, stochastic gradient approach is suggested to be in use. Direct calculation of the stochastic gradient realization conrresponding to the gieven perfomrance index is possible because it needs the use of some sort of sensitivity vector and also requires theinformation about real values of unknown parameters that is in contradiction with the original problem setting. In addition the original identification criterion (norm of difference between the outputs of real process and its model) turns out to be multimodal function of matrix forgetting, so the gradient like techique could lead to local minima - this is the crucial point of the direct stochastic gradient approach if it would be applied to the problem under consideration.

- To avoid these problems suggest to consider a smoothed functional which is close (in some sense) to the original identification criterion, but has several adavantages: first, it is quasi convex (practically, unimodal) and, second, its gradient can be easily calculated on-line.

- The convergence and rate convergence analysis si presented based on several new optimization results as well as on matingale technique applied to discrete time random sequences.

- The order of convergence of adaptive forgetting to the optimal one is shown to be equal to $\mathcal{O}(n^{-1})$.

- The optimal parameters of the matrix foregetting factor procedure with adaptation were calculated. They trn out to be dependent only on dimensions of the vector and matrices participating in the description o fa given ARMAX model, it means, this information is available a priory before the estimation process. So, this optimal adaptive procedure is realizable.

- The effectiveness fo the suggestes aproach is illustrated by several numerical examples including DC-indcution motor model.

- Futher investigation can be concected with the generalization of this approach to the case of recursive algorithms containing nonlinear resifual trasnformations that can provide the improvement of the identification error behaivor under different noise distribution (no only Gaussian).

3 References

Anderson, P., 1985, Adaptive forgetting in recursive identification through multiple models, *Int. J. Control,* **45**(5), 1175 - 1193.

Bittanti, S., Campi, M., 1991, Tuning the forgetting factor in recursive least Square identification algorithms, *IEEE, Conference on Decision and Control,* **2,** 1688-1689.

Goodwin, G. C., and **Payne, R. L.**, 1977, Dynamic system identification, Experiment Design and Data Analysis, Mathematics in Science and Engineering, **vol. 136**.

Jazwinski, A. H., 1970, Stochastic Process and Filtering Theory, Academic Press, A Subsidiary of Harcourt Brace Jovanovich, Publishers.

Ljung, L., 1987, *System identification theory for the user* (Prentice Hall Information and System Sciences).

Medel - Juárez, J.J., and **Poznyak, A. S.**, 1998, Adaptive tracking for DC-induction motor based on matrix forgetting. *Control Engineering Practice* (sended for publication).

Poznyak, A. S., and **Medel - Juárez, J. J.**, 1998a, (accepted for published), Matrix Forgetting Factor, International Journal of Systems Science.

Poznyak, A. S., and **Medel-Juárez,J.J.**, 1998b, (accepted for publication), Adaptive Matrix Forgetting Factor, International Journal of Systems Science.

Sastry, S., and **Bodson, M.**, 1989, Adaptive Control Stability, Convergence, and Robustness,

Shiryaev, A. N; 1984, Probability, *Nauka,* Moscú.

Wasan, M. T., 1969, *Stochastic Approximation*, Cambrige University Press.