# Head Tracking by Glasses Detection 

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#### Abstract

One of the core components of virtual reality is tracking, therefore some of the image-based tracking techniques developed by the computer vision community are of potential value for this field.

In this work we describe a novel method for head tracking which is based on glasses detection and does not requiere camera calibration. The method provides five degrees of freedom while head tracking. It provides the relative position of the head from the camera (up to an unknown scale factor), and its rotation (line of sight).


Keywords: Computer Vision, Human-Computer Interaction, Pattern Recognition, Detection of Orientation.

[^0]
## 1 Introduction

Advances in model based vision, active vision and invariants are providing new solutions to the problems of object recognition and tracking [Blake \& Yuille, 1992; Mundy \& Zisserman, 1992; Rothe et al., 1996; Voss \& Suesse, 1997]. Some of this solutions form the basis for a new type of human-computer interaction based on gesture recognition. This technology is extending the known means of human-computer communication, since it allows the computer "to see" its user. For instance, it is now possible to some extent, to do face and emotions recognition from analysis of images of the human face [Schwartz, 1995]. Also, body, hand and eye tracking are being investigated as means of "visual" interaction with computers and "smart rooms" [Rehg \& Kanade, 1993; Pentland, 1996; Riba, 1996]. In some cases, this type of interfaces have been extended to interact with virtual environments, and it is possible to navigate and manipulate objects with them [Peña \& Rios, 1997].

The goal of this paper is to describe a method for head tracking and detection of the orientation of the view line based on the detection of glasses without calibration of the used camera and imaging system.

Although head tracking can be done to some extent
directly without wearing glasses, we are using a model of glasses which is useful for stereoscopic visualization as part of a virtual reality system. This particular model of glasses does not provide head tracking, but by using the method presented in this paper, we seek to provide this information. This means that we are presenting a computer vision based method for glasses tracking than can be used as part of a virtual reality system, for head tracking, for those systems that lack this resource.

The organization of the paper is as follows. The next section describes related works. Then, we describe the proposed method and show the results. Finally, we provide a conclusion.

## 2 Related Works

An important part of virtual reality systems are tracking technologies, since they keep track of body parts. The information they provide is used by the rendering subsystem to update the images for appropriate viewing [Vince, 1995].

The tracking technologies used in virtual reality systems can be classified among others in mechanical, optical, electromagnetic, ultrasonic or based on image analysis [Barfield \& Furnes, 1995]. The work on this paper is related to this last approach.

From the point of view of computer vision, tracking is a key problem and several techniques have been developed over the years [Blake \& Yuille, 1992; Faugeras, 1993]. Tracking is an important part of organisms living in a changing environment and this is also true for robots, if they are going to interact successfully with their surround [Blake \& Yuille, 1992].

Recently, several researchers have identified an interesting intersection between the fields of computer vision and human-computer interaction [Rehg \& Kanade, 1993; Pentland, 1996; Schwartz, 1995]. The purpose of this interdisciplinary work is to develop new forms of contactless human-computer communication, based on visual information. For instance, there is work going on face and emotion recognition [Pentland, 1996; Schwartz, 1995]. Also, some other people have used eye and hand tracking as an interface to computer systems [Riba, 1996; Peña \& Rios, 1997; Young et al., 1995; Starner \& Pentland, 1996].

The purpose of our paper is to develop a head tracking method based on the detection of glasses, such as those used for stereoscopic visualization in virtual reality systems.

Other works on head tracking exist in the literature, but they usually track a 2-D profile of the head, for applications like video compression for videoconferencing [Lam \& Yang, 1996].

One of the most related works for our discussion has been developed by Basu [Bassu et al., 1996]. In their work they use a 3D ellipsoidal model of the head, and keep track of it, by using regularization of the optical flow. In our work we do not use a model as general as in their work, but by concentrating on the analysis of a simple rigid object such as glasses, we can provide the same type of information. Our method can be more accurate in principle, since the detection of the glasses is more precise. than detecting just the head. Also, it is more easy and accurate to analyse the deformation experimented by a pair glasses, than a head, when it is projected on the image plane.

In the following section, we will describe the method that we have developed for glasses detection and for deriving information about its 3 D rotation in space.

## 3 Description of the Method

The method operates on gray scale images taken with a TV camera. At this stage, our implementation is operating on images of a sequence taken in advance. Thus, at this moment is not working on real time, but this is a future extension of our work. The program is written in the programming language "LAMBDA", which is part of the DIAS software. This software provide numerous routines for image processing, analysis and synthesis, as well as data analysis procedures [Towersoft, 1995].

For each image we apply the following processes:

1. A working window is defined for processing. The initial position is at the center of the image and the initial size is that of the whole image. During the further processing, the window position for the analysis of the new image is determined by the center of the parallelogram on the foregoing image (see point 8 ) and the size is slightly bigger than the area occupied by the glasses in the previous frame (Figure 1.a).
2. A gradient image is generated for locating interesting edges. The concrete type of gradient filter (Sobel or other) is not essential for further analysis. (Figure 1.b).
3. The image is searched for local maxima of the gradient magnitude every $k$ rows and columns. In our
implementation we take $k=5$, but we can take other values (Figure 2.a).
4. To reduce the number of possible start points for contour following, we select only those points that their grey value and gradient magnitude are inside suitable intervals. We take the grey value of the selected points to be close to the intensity measured at the boundary between the lens and the frame of the glasses. In our implementation, this corresponds to select "grey" points, that is, points that are not too bright or too dark. For instance, we constrain the intensity to be in the interval $[45,100]$, considering a grey scale of $[0,127]$. In the case of the gradient magnitude, we constrain it to be inside the interval [40,80] (Figure 2.b).
5. From the points with high magnitude of the gradient, a contour following algorithm is applied. We reject all contours that are too long or the area enclosed by them or form factor ${ }^{1}$ is not inside a suitable interval. Only if two contours remain, the next steps are applied, otherwise, we continue with the next frame (point 1), (Figure 3.a). Therefore some images of the sequence (approximately $5-10 \%$ in our experiments) are not analysed successfully. But with a "gliding mean" (or any other type of Kalman filtering) the orientation of such "defective" images data can be interpolated from the "neighboring" images of the sequence.
6. Using the two selected contours:
(a) A convex hull for the union of the contours is determined (Figure 3.b). It is important to note that the convex hull of a region and its deficiency are powerful tools for describing it, and its boundary. In particular, the description of a region might be based on its area and the area of its convex deficiency [Gonzalez \& Woods, 1992]. We use this idea in point 7. Algorithms for computing the convex hull can be based on morphological operations, such as the hit-or-miss transform, or in properties of $Z^{2}$ [Gonzalez \& Woods, 1992]. For an algorithm of order $O(N)$ see [Voss, 1993].
(b) the position of the working window is updated.
7. Compute the ratio of areas enclosed by the two contours and the convex hull. If the two contours are part of glasses, then the ratio must be inside a predefined interval. In our implementation, the area enclosed by the two contours is between 0.75 and

[^1]0.82 of the convex hull area. If the ratio of areas does not satisfy the constraint, then we continue with the next frame.
8. A parallelogram is fitted to the convex hull using an affine transformation of a rectangular model of the glasses (Figure 4.a and 10). This fitting can be done by the method of normalization for detecting of invariants [Rothe et al., 1996; Voss \& Suesse, 1997].
9. For each image, let's assume that the fitted parallelogram is the projection onto the image plane $\xi-\eta$ of a rectangle whose center is at the origin of the 3 D system X-Y-Z (Figure 1.b and 1.c). Because this rectangle represents the glasses in the space, we will know the viewing direction if we can find a normal vector of this rectangle. For this purpose, we suppose that initially the rectangle is on the X-Y plane, and its vertices are the points:
\[

$$
\begin{gathered}
v_{1}=(-a, b, 0), \quad v_{2}=(a, b, 0), \\
\mathrm{v}_{3}=(a,-b, 0), \quad v_{4}=(-a,-b, 0)
\end{gathered}
$$
\]

(see Figure 5.a). Then the rectangle is rotated about the origin so that its vertices are the points $w_{i}=$ $\left(x_{i}, y_{i}, z_{i}\right), \mathrm{i}=1, \ldots, 4$ (Figure 5.b).
The matrices

$$
\begin{aligned}
& R_{x(\alpha)}=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \alpha & -\sin \alpha \\
0 & \sin \alpha & \cos \alpha
\end{array}\right] \\
& R_{y(\beta)}=\left[\begin{array}{ccc}
\cos \beta & 0 & \sin \beta \\
0 & 1 & 0 \\
-\sin \beta & 0 & \cos \beta
\end{array}\right] \\
& R_{z(\gamma)}=\left[\begin{array}{ccc}
\cos \gamma & -\sin \gamma & 0 \\
\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{array}\right]
\end{aligned}
$$

represent a rotation about the $\mathrm{x}, \mathrm{Y}$, and Z axis, respectively. Let $T$ be the rotation on $\boldsymbol{R}^{3}$ that carries the rectangle from its initial position to its final position. In this way, $T$ is a composition of the above rotations. Moreover $w_{i}=T\left(v_{i}\right)$. If $u_{i}=\left(\xi_{i}, \eta_{i}\right)$, $\mathrm{i}=1, \ldots, 4$, are the vertices of the projected rectangle on the image plane $\xi-\eta$ (Figure 5.c), then $\xi_{i}=\lambda x_{i}$ and $\eta_{i}=\lambda y_{i}, \mathrm{i}=1, \ldots, 4$, for some factor $\lambda>0$ (assuming that we approximate the perspective projection by an orthographic projection).
Let

$$
\begin{gather*}
\Delta \xi_{21}=\frac{\xi_{2}-\xi_{1}}{2 a}, \quad \Delta \eta_{21}=\frac{\eta_{2}-\eta_{1}}{2 a} \\
\Delta \xi_{23}=\frac{\xi_{2}-\xi_{3}}{2 b}, \quad \Delta \eta_{23}=\frac{\eta_{2}-\eta_{3}}{2 b} \tag{1}
\end{gather*}
$$



Figure 1: (a) One of the image of the sequence with the working window superimposed. (b) Gradient image computed inside the working window.


Figure 2: (a) Computation of local maxima of the gradient. (b) Only the points that are inside a suitable grey level and gradient magnitude interval are selected.

Now we will calculate the factor $\lambda$. First, we propose that $T$ is represented by the matrix shown in (Figure 9)

## Thus

$$
\begin{aligned}
& \xi_{1}=-\lambda(a \cos \theta \cos \psi-a \sin \theta \cos \phi \sin \psi+ \\
& b \cos \theta \sin \psi+b \sin \theta \cos \phi \cos \psi) \\
& \xi_{2}=\lambda(a \cos \theta \cos \psi-a \sin \theta \cos \phi \sin \psi-b \\
& \cos \theta \sin \psi-b \sin \theta \cos \phi \cos \psi) \\
& \xi_{3}=\lambda(a \cos \theta \cos \psi-a \sin \theta \cos \phi \sin \psi+b \\
& \cos \theta \sin \psi+b \sin \theta \cos \phi \cos \psi) \\
& \xi_{4}=-\lambda(a \cos \theta \cos \psi-a \sin \theta \cos \phi \sin \psi-b \\
& \cos \theta \sin \psi-b \sin \theta \cos \phi \cos \psi) \\
& \eta_{1}=-\lambda(a \sin \theta \cos \psi+a \cos \theta \cos \phi \sin \psi+b \\
& \sin \theta \sin \psi-b \cos \theta \cos \phi \cos \psi) \\
& \eta_{2}=\lambda(a \sin \theta \cos \psi+a \cos \theta \cos \phi \sin \psi \\
& -b \sin \theta \sin \psi+b \cos \theta \cos \phi \cos \psi) \\
& \eta_{3}=\lambda(a \sin \theta \cos \psi+a \cos \theta \cos \phi \sin \psi+b \\
& \sin \theta \sin \psi-b \cos \theta \cos \phi \cos \psi) \\
& \eta_{4}=-\lambda(a \sin \theta \cos \psi+a \cos \theta \cos \phi \sin \psi-b \\
& \sin \theta \sin \psi+b \cos \theta \cos \phi \cos \psi)
\end{aligned}
$$

Substituting in (1) we obtain

$$
\begin{aligned}
\Delta \xi_{21} & =\lambda(\cos \theta \cos \psi-\sin \theta \cos \phi \sin \psi) \\
\Delta \eta_{21} & =\lambda(\sin \theta \cos \psi+\cos \theta \cos \phi \sin \psi) \\
\Delta \xi_{23} & =-\lambda(\cos \theta \sin \psi+\sin \theta \cos \phi \cos \psi) \\
\Delta \eta_{23} & =-\lambda(\sin \theta \sin \psi-\cos \theta \cos \phi \cos \psi)
\end{aligned}
$$

If we denote by

$$
\begin{array}{cc}
e_{1}=\Delta \xi_{21}+\Delta \eta_{23}, & e_{2}=\Delta \xi_{21}-\Delta \eta_{23} \\
e_{3}=\Delta \eta_{21}+\Delta \xi_{23}, & e_{4}=\Delta \eta_{21}-\Delta \xi_{23} \\
f_{1}=e_{1}^{2}+e_{4}^{2}, & f_{2}=e_{2}^{2}+e_{3}^{2}
\end{array}
$$

then

$$
\left.\begin{array}{rl}
e_{3} / e_{2} & =\tan (\theta-\psi) \\
e_{4} / e_{1} & =\tan (\theta+\psi)
\end{array}\right\}
$$

$$
f_{2} / f_{1}=\tan ^{4}(\phi / 2) \quad \Longrightarrow
$$

$$
|\phi|=2 \arctan \sqrt[4]{f_{2} / f_{1}}
$$



Figure 3: (a) Points on the two most promising contours. (b) Points that define the convex hull of the selected contours.


Figure 4: (a) The parallelogram that best fits the image of the glasses. Using an affine transformation, the rectangle of the glasses is transformed into a parallelogram. (b) Determination of the view line from the coefficients of the rotation matrix $T$.

Since $\cos |\phi|=\cos \phi$, we can substitute the values of $\theta, \phi$, and $\psi$, for example, in the expression (2) and therefore we determinate $\lambda$.
Now we assume that the rotation T is given by the composition shown in (Figure 9)
Because the rotations of the head are constrained to angles of less of 180 degrees in every direction, the angles $\alpha, \beta$, and $\gamma$ must be inside the interval $\left(-90^{\circ}, 90^{\circ}\right)$.

$$
\begin{aligned}
\xi_{1}= & -\lambda(a \cos \gamma \cos \beta+b \sin \gamma) \\
\eta_{1}= & -\lambda(a \cos \alpha \sin \gamma \cos \beta+a \sin \alpha \sin \beta \\
& -b \cos \alpha \cos \gamma) \\
\xi_{2}= & \lambda(a \cos \gamma \cos \beta-b \sin \gamma) \\
\eta_{2}= & \lambda(a \cos \alpha \sin \gamma \cos \beta+a \sin \alpha \sin \beta \\
& +b \cos \alpha \cos \gamma) \\
\xi_{3}= & \lambda(a \cos \gamma \cos \beta+b \sin \gamma), \\
\eta_{3}= & \lambda(a \cos \alpha \sin \gamma \cos \beta+a \sin \alpha \sin \beta \\
& -b \cos \alpha \cos \gamma) \\
\xi_{4}= & -\lambda(a \cos \gamma \cos \beta-b \sin \gamma), \\
\eta_{4}= & -\lambda(a \cos \alpha \sin \gamma \cos \beta+a \sin \alpha \sin \beta \\
& +b \cos \alpha \cos \gamma)
\end{aligned}
$$

From the above expressions and (1) we obtain

$$
\begin{align*}
\Delta \xi_{21} & =\lambda \cos \gamma \cos \beta  \tag{7}\\
\Delta \eta_{21} & =\lambda(\cos \alpha \sin \gamma \cos \beta+\sin \alpha \sin \beta)  \tag{8}\\
\Delta \xi_{23} & =-\lambda \sin \gamma  \tag{9}\\
\Delta \eta_{23} & =\lambda \cos \alpha \cos \gamma \tag{10}
\end{align*}
$$

Equation (9) provide us the angle $\gamma$ :

$$
\begin{equation*}
\gamma=-\arcsin \frac{\Delta \xi_{23}}{\lambda} \tag{11}
\end{equation*}
$$

and from (7)

$$
\begin{equation*}
|\beta|=\arccos \frac{\Delta \xi_{21}}{\lambda \cos \gamma} \tag{12}
\end{equation*}
$$

For determining the sign of $\beta$ we need additional information. Since the projection on the image is a perspective one, the distance $d_{51}$ from the point $u_{5}$ to $u_{1}$ isn't necessarily equal to the distance $d_{26}$ between $u_{2}$ and $u_{6}$, as is illustrated in Figure 5.b. If


Figure 5: (a) Initial position of the rectangle. (b) Final position after the rotation. (c) Projection of the glasses on the image

$$
T=R_{z(\theta)} R_{x(\phi)} R_{z(\psi)}=\left[\begin{array}{ccc}
\cos \theta \cos \psi-\sin \theta \cos \phi \sin \psi & -\cos \theta \sin \psi-\sin \theta \cos \phi \cos \psi & \sin \theta \sin \phi \\
\sin \theta \cos \psi+\cos \theta \cos \phi \sin \psi & -\sin \theta \sin \psi+\cos \theta \cos \phi \cos \psi & -\cos \theta \sin \phi \\
\sin \phi \sin \psi & \sin \phi \cos \psi & \cos \phi
\end{array}\right]
$$

Figure 6: Matrix $T$
$d_{51}>d_{26}$, the left border of the glasses is closer to the camera that the right one. So, the glasses were rotated in the positive direction, that is, $\beta>0$. Similarly, if $d_{51}<d_{26}$, then $\beta<0$.

Now, we can calculate $\alpha$ using equation (10) and (8)

$$
\begin{gather*}
\cos \alpha=\frac{\Delta \eta_{23}}{\lambda \cos \gamma}  \tag{13}\\
\alpha=\arcsin \frac{\Delta \eta_{21}-\lambda \cos \alpha \sin \gamma \cos \beta}{\lambda \sin \beta} \tag{14}
\end{gather*}
$$

We now know the angles $\alpha, \beta$, and $\gamma$, so we know the transformation $T$, and we proceed to calculate a vector parallel to the line of sight.
Since the vector $\mathbf{n}_{\mathbf{0}}=(0,0,1)^{\top}$ is normal to the rectangle $v_{1} v_{2} v_{3} v_{4}$, so the vector $\mathbf{n}_{1}=T \mathbf{n}_{0}$ is normal to the rectangle in its final position $w_{1} w_{2} w_{3} w_{4}$.

$$
\mathbf{n}_{\mathbf{1}}=\left(\begin{array}{c}
\cos \gamma \sin \beta  \tag{15}\\
\cos \alpha \sin \gamma \sin \beta-\sin \alpha \cos \beta \\
\sin \alpha \sin \gamma \sin \beta+\cos \alpha \cos \beta
\end{array}\right)
$$

The two first components of $\mathbf{n}_{\mathbf{1}}$ are used to plot a segment that indicates the viewing direction in each image (Figure 4.b and 11).

### 3.1 Experiments and Results

We took an image sequence of a user in front of monitor replicating the scenario in which he/she might work using stereoscopic glasses for visualization, as part of a


Figure 8: Some of the test images showing the working window.
virtual reality system. The head motion covered different viewing angles as in normal work (Figure 8).

The image sequence consists of 48 images of $320 \times 240$ pixels each, taken at 5 frames $/ \mathrm{sec}$. After acquisition, the image sequence was decomposed into individual images. For speeding up processing, each image was reduced to half its size in each dimension, i.e. $160 \times 120$ pixels. The mean time for the processing of one image (as mean value over the whole sequence) is 100 milliseconds when we work with a Pentium computer ( 200 MHz ). Because this time is measured when we work with the software system DIAS and the programming language LAMBDA, we are sure that the time can be decreased to 50 milliseconds by programming in the C language and using an optimizing compiler. That means that we can analyze at least 10-20

$$
T=R_{x(\alpha)} R_{z(\gamma)} R_{y(\beta)}=\left[\begin{array}{ccc}
\cos \gamma \cos \beta & -\sin \gamma & \cos \gamma \sin \beta \\
\cos \alpha \sin \gamma \cos \beta+\sin \alpha \sin \beta & \cos \alpha \cos \gamma & \cos \alpha \sin \gamma \sin \beta-\sin \alpha \cos \beta \\
\sin \alpha \sin \gamma \cos \beta-\cos \alpha \sin \beta & \sin \alpha \cos \gamma & \sin \alpha \sin \gamma \sin \beta+\cos \alpha \cos \beta
\end{array}\right]
$$

Figure 7: Rotation of Matrix $T$


Figure 9: Original and gradient image. The image on the upper left corner is part of the original sequence. The image on the lower left corner shows the gradient magnitude image. The upper right corner image and the lower right corner image, show respectively, local maxima of the gradient magnitude superimposed every $k=5$ rows and columns.
images per second.
Figure 9 shows the detection of local maxima of the gradient magnitude on one of the test images. Other main steps of the method, such as parallelogram fitting. and determination of the line of sight are display on Figure 10 and 11 respectively. The lines shown in Figure 11 are the projections of a spacial line segment which goes from the center of the glasses with a constant length $L$ in direction of the line of sight (therefore, short lines mean a small angle difference between the optical axis of the camera and the line of sight of the person). Further, the image point corresponding to the center of the glasses is shifted into the center of the image (Figure 11).

## 4 Conclusion and Future Work

We have presented a novel method for determination of line of sight with potential value for human-computer interaction. This method relies on the detection of glasses such as those used in virtual reality systems for visual-
ization. The method does not require camera calibration and is based on a simple idea which takes advantage of an affine approximation of the perspective projection. In the near future, we seek to implement our method on a continuous video signal, in real time, and as part of a virtual reality system for providing head tracking.

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Figure 10: Series of images showing the parallelogram that best fits the lenses of the glasses.

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Figure 11: Determination of the line of sight from the parallelogram that fits the glasses. The image point corresponding to the center of the glasses is shifted into the center of the image.

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[^0]:    *This work was a result of the visit of Professor Voss to the Master Program in Artificial Intelligence, UV-LANIA, during the first term of 1997.

[^1]:    ${ }^{1}$ also known as "compactness" of a region in the literature [Gonzalez \& Woods, 1992].

