International Journal of Control
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713393989

Nonstationary discrete-time deterministic and stochastic control systems with infinite horizon

Xianping Guo[a]; Adrián Hernández-del-Valle[b]; Onésimo Hernández-Lerma[c]
[a] The School of Mathematics and Computational Science, Zhongshan University, Guangzhou 510275, P.R. China [b] Department of Economics, Universidad Carlos III de Madrid, Spain [c] Mathematics Department, CINVESTAV-IPN, México D.F. 07000, Mexico

Online publication date: 18 August 2010

To cite this Article Guo, Xianping , Hernández-del-Valle, Adrián and Hernández-Lerma, Onésimo(2010) 'Nonstationary discrete-time deterministic and stochastic control systems with infinite horizon', International Journal of Control, 83: 9, 1751 — 1757

To link to this Article: DOI: 10.1080/00207179.2010.486044
URL: http://dx.doi.org/10.1080/00207179.2010.486044

International Journal of Control
Vol. 83, No. 9, September 2010, 1751–1757

Nonstationary discrete-time deterministic and stochastic control systems with infinite horizon

Xianping Guo[a], Adrián Hernández-del-Valle[b] and Onésimo Hernández-Lerma[c*]

[a]*The School of Mathematics and Computational Science, Zhongshan University, Guangzhou 510275, P.R. China;* [b]*Department of Economics, Universidad Carlos III de Madrid, Spain;* [c]*Mathematics Department, CINVESTAV-IPN, A. Postal, 14-740, México D.F. 07000, Mexico*

(Received 17 December 2009; final version received 12 April 2010)

This article is about nonstationary nonlinear discrete-time deterministic and stochastic control systems with Borel state and control spaces, possibly noncompact control constraint sets, and unbounded costs. The control problem is to minimise an infinite-horizon total cost performance index. Using dynamic programming arguments we show that, under suitable assumptions, the optimal cost functions satisfy optimality equations, which in turn give a procedure to find optimal control policies.

Keywords: discrete-time systems; nonlinear systems; infinite horizon problems; nonstationary dynamic programming

1. Introduction

This article concerns nonstationary (also known as nonhomogeneous or time-varying) discrete-time deterministic control systems of the form

$$x_{n+1} = F_n(x_n, a_n) \quad \text{for all } n = 0, 1, \ldots, \tag{1.1}$$

given an initial state x_0, where $x_n \in X_n$ and $a_n \in A_n$ denote the state and control (or action) variables at time n. The state and action spaces X_n and A_n, respectively, are supposed to be *Borel spaces*, that is, Borel subsets of complete and separable metric spaces. Therefore, (1.1) essentially includes all the cases in practical applications, namely, the cases in which the spaces X_n are denumerable sets, or subsets of finite-dimensional Euclidean spaces, and also infinite-dimensional spaces, as in distributed parameter systems (see e.g. Zabczyk 1974). Moreover, we allow noncompact control constraint sets $A_n(x) \subset A_n$ for each state $x \in X_n$ (see Section 2 for details).

For notational ease, we first consider the deterministic system (1.1) and then we explain how to extend our results to stochastic systems

$$x_{n+1} = F_n(x_n, a_n, \xi_n) \quad \text{for all } n = 0, 1, \ldots, \tag{1.2}$$

with $x_n \in X_n$ and $a_n \in A_n$ as above, and independent random disturbances $\xi_n \in S_n$, where the S_n are Borel spaces.

Strictly speaking, some of our results can be deduced from results by Hinderer (1970, Section 14); see also Strauch (1966). Our proof techniques, however, are straightforward, easy to follow and self-contained.

Nonstationary systems such as (1.1) have been studied by several authors, but usually under restrictive assumptions. For instance, Engwerda (1988), Schochetman and Smith (1994), and Tan and Rugh (1998) consider linear systems. On the other hand, Le Van and Dana (2003), Schochetman and Smith (2005) and Zaslavski (2006) study systems in which the state and/or the control sets are compact. For stochastic systems (1.2) either the state space is denumerable, as in Jia and Ding (2000), or they require some ergodicity condition or strong Feller-like hypotheses which do *not* hold for deterministic systems (see e.g. Guo, Liu, and Liu 2000 or Guo and Hernández-Lerma 2004). In fact, as far as we can tell, the only result that is relatively close to ours is by Keerthi and Gilbert (1985) (see also Keerthi and Gilbert (1988), Section 10). This result, however, is on Euclidean spaces and the general approach is completely unrelated to ours.

In this article we study infinite-horizon optimal control problems with *unbounded* cost functions. We use dynamic programming techniques to characterise the optimal cost functions and to prove the existence of optimal control policies.

The remainder of this article is organised as follows. In Section 2, we introduce the nonstationary control model we will be dealing with, and state the optimal control problem of interest. Our hypotheses and one of our main results, Theorem 3.3, are stated in Section 3. In Section 4, we give a proof of Theorem 3.3 based on another of our key results, namely, Theorem 4.5, which states the convergence of the

*Corresponding author. Email: ohernand@math.cinvestav.mx

ISSN 0020–7179 print/ISSN 1366–5820 online

DOI: 10.1080/00207179.2010.486044
http://www.informaworld.com