ARTICLE IN PRESS

Systems & Control Letters ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Nonstationary discrete-time deterministic and stochastic control systems: Bounded and unbounded cases

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ARTICLE INFO

Article history:
Received 13 October 2010
Received in revised form
17 February 2011
Accepted 4 April 2011
Available online xxxx

Keywords:
Discrete-time control systems
Time-nonhomogeneous systems
Time-varying systems
Nonlinear systems
Nonstationary dynamic programming

ABSTRACT

This paper is about nonstationary nonlinear discrete-time deterministic and stochastic control systems with Borel state and control spaces, with either bounded or unbounded costs. The control problem is to minimize an infinite-horizon total cost performance index. Using dynamic programming arguments we show that, under suitable assumptions, the optimal cost functions satisfy optimality equations, which in turn give a procedure to find optimal control policies. We also prove the convergence of value iteration (or successive approximations) functions. Several examples illustrate our results under different sets of assumptions.

1. Introduction

This paper concerns nonstationary (also known as time-nonhomogeneous or time-varying) discrete-time deterministic control systems of the form

$$x_{n+1} = F_n(x_n, a_n) \quad \text{for all } n = 0, 1, \ldots, \tag{1.1}$$

given an initial state x_0, where $x_n \in X_n$ and $a_n \in A_n$ denote the state and control (or action) variables at time n. The state and action spaces X_n and A_n, respectively, are supposed to be *Borel spaces*, that is, Borel subsets of complete and separable metric spaces. Therefore, (1.1) essentially includes all the cases in practical applications, namely, the cases in which the spaces X_n and/or A_n are denumerable sets, or subsets of finite-dimensional Euclidean spaces, and also infinite-dimensional spaces, as in distributed-parameter systems (see [1], for instance).

For notational ease, we first consider the deterministic system (1.1) and then we explain how to extend our results to stochastic systems

$$x_{n+1} = F_n(x_n, a_n, \xi_n) \quad \text{for all } n = 0, 1, \ldots, \tag{1.2}$$

with $x_n \in X_n$ and $a_n \in A_n$ as above, and independent random disturbances $\xi_n \in S_n$, where the S_n are Borel spaces.

Nonstationary systems such as (1.1) have been studied by several authors, but usually under restrictive assumptions, such as *linear* systems [2–5], *compact* state and/or control sets [6–8], or restriction to finite-dimensional Euclidean spaces [9,10]. For stochastic systems (1.2) either the state space is denumerable, as in [11], or they require some ergodicity conditions or strong Feller-like hypotheses [12] which do *not* hold for deterministic systems.

In this paper we study infinite-horizon optimal control problems with either bounded or unbounded cost functions. We use dynamic programming techniques to characterize the optimal cost functions and to prove the existence of optimal control policies. In fact, this research began with paper [13] in which we considered so-called *semicontinuous* control models (see Assumption 3.1), which are usually — but not necessarily — useful for control problems with *unbounded costs*. In [13] we proved Theorem 3.3(a_1) below, as well as the first part of Theorem 3.3(a_2). (See [13, Theorem 3.3].) Here, we prove the *converse* of Theorem 3.3(a_2), and, in addition, we extend these results to so-called *continuous* control models (as in our Assumption 3.2), which are specifically designed for control problems with *bounded* costs. Furthermore, we extend to time-varying systems the notion of *discrepancy* functions (see (3.5), Proposition 3.4 and Corollary 3.5), which is very useful to study problems in adaptive control, near-optimality, and many others [14–17].

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doi:10.1016/j.sysconle.2011.04.006