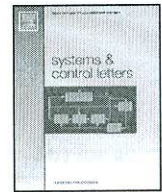




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# Nonstationary discrete-time deterministic and stochastic control systems: Bounded and unbounded cases

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## ABSTRACT

This paper is about nonstationary nonlinear discrete-time deterministic and stochastic control systems with Borel state and control spaces, with either bounded or unbounded costs. The control problem is to minimize an infinite-horizon total cost performance index. Using dynamic programming arguments we show that, under suitable assumptions, the optimal cost functions satisfy optimality equations, which in turn give a procedure to find optimal control policies. We also prove the convergence of value iteration (or successive approximations) functions. Several examples illustrate our results under different sets of assumptions.

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## 1. Introduction

This paper concerns nonstationary (also known as time-nonhomogeneous or time-varying) discrete-time deterministic control systems of the form

$$x_{n+1} = F_n(x_n, a_n) \quad \text{for all } n = 0, 1, \dots, \quad (1.1)$$

given an initial state  $x_0$ , where  $x_n \in X_n$  and  $a_n \in A_n$  denote the state and control (or action) variables at time  $n$ . The state and action spaces  $X_n$  and  $A_n$ , respectively, are supposed to be Borel spaces, that is, Borel subsets of complete and separable metric spaces. Therefore, (1.1) essentially includes all the cases in practical applications, namely, the cases in which the spaces  $X_n$  and/or  $A_n$  are denumerable sets, or subsets of finite-dimensional Euclidean spaces, and also infinite-dimensional spaces, as in distributed-parameter systems (see [1], for instance).

For notational ease, we first consider the deterministic system (1.1) and then we explain how to extend our results to stochastic systems

$$x_{n+1} = F_n(x_n, a_n, \xi_n) \quad \text{for all } n = 0, 1, \dots, \quad (1.2)$$

with  $x_n \in X_n$  and  $a_n \in A_n$  as above, and independent random disturbances  $\xi_n \in S_n$ , where the  $S_n$  are Borel spaces.

Nonstationary systems such as (1.1) have been studied by several authors, but usually under restrictive assumptions, such as linear systems [2–5], compact state and/or control sets [6–8], or restriction to finite-dimensional Euclidean spaces [9,10]. For stochastic systems (1.2) either the state space is denumerable, as in [11], or they require some ergodicity conditions or strong Feller-like hypotheses [12] which do not hold for deterministic systems.

In this paper we study infinite-horizon optimal control problems with either bounded or unbounded cost functions. We use dynamic programming techniques to characterize the optimal cost functions and to prove the existence of optimal control policies. In fact, this research began with paper [13] in which we considered so-called *semicontinuous* control models (see Assumption 3.1), which are usually – but not necessarily – useful for control problems with *unbounded costs*. In [13] we proved Theorem 3.3(a<sub>1</sub>) below, as well as the first part of Theorem 3.3(a<sub>2</sub>). (See [13, Theorem 3.3].) Here, we prove the *converse* of Theorem 3.3(a<sub>2</sub>), and, in addition, we extend these results to so-called *continuous* control models (as in our Assumption 3.2), which are specifically designed for control problems with *bounded costs*. Furthermore, we extend to time-varying systems the notion of *discrepancy* functions (see (3.5), Proposition 3.4 and Corollary 3.5), which is very useful to study problems in adaptive control, near-optimality, and many others [14–17].

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