Int J Theor Phys (2009) 48: 1357–1368
DOI 10.1007/s10773-008-9907-7

Lanczos Potential for the van Stockung Space-Time

César Mora · Rubén Sánchez

Received: 19 September 2008 / Accepted: 18 November 2008 / Published online: 27 November 2008

Abstract The Lanczos Potential is a theoretical useful tool to find the conformal Weyl curvature tensor C_{abcd} of a given relativistic metric. In this paper we find the Lanczos potential L_{abc} for the van Stockung vacuum gravitational field. Also, we show how the wave equation can be combined with spinor methods in order to find this important three covariant index tensor.

Keywords Lanczos potential theory · Weyl-Lanczos equations · Lanczos coefficients

1 Introduction

The first ideas about Lanczos potential came from 1949 [1]. Nevertheless, is until 1962 when Lanczos suggested that the self-dual part of the Riemann tensor behaves as an auxiliary potential [2]. Through the covariant derivative of Lanczos potential we can obtain the conformal contribution C_{abcd} of the metric curvature. The Weyl tensor can be expressed as first order equations in terms of the Lanczos tensor L_{abc} [3–6]. The task of generating the spacetime Weyl tensor from a tensor potential is known as the Weyl-Lanczos problem and the analogous process for the Riemann curvature tensor is called the Riemann-Lanczos problem.

On the other hand, we have that the field equations of general relativity usually are written as a tensorial equation, where the left side is the Einstein tensor

$$G_{ab} = R_{ab} - \frac{1}{2} R g_{ab}, \tag{1}$$

plus a multiple of the metric λg_{ab}, and the right side is a multiple of the stress tensor $\kappa T_{\mu\nu}$ (where $\kappa = -8\pi G/c^4$). Then, the Einstein's field equations are given as

$$G_{ab} + \lambda g_{ab} = \kappa T_{ab}. \tag{2}$$

C. Mora (✉) · R. Sánchez
Centro de Investigación en Ciencia Aplicada y Tecnología Avanzada, Instituto Politécnico Nacional, Av. Legaria 694, Col. Irrigación, 11500 México, DF, Mexico
e-mail: ceml36@gmail.com