Third Quantization in Bergmann-Wagoner Theory

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We present the third quantization of Bergmann-Wagoner scalar-tensor and Brans Dicke solvable toy models. In the first one we used an exponential cosmological term, for the second one we considered vanishing cosmological constant. In both cases, it is found that the number of the universes produced from nothing is very large.

1 Introduction

The Wheeler-DeWitt (WDW) equation is a result of quantization of a geometry and matter (second quantization of gravity), in this paper we consider the third quantization of a solvable inflationary universe model, i.e., by analogy with the quantum field theory, it can be done the second quantization of the universe wavefunction ψ expanding it on the creation and annihilation operators (third quantization) [1]. Because in the recent years there has been a great interest in the study of scalar-tensor theories of gravitation, owing that of the unified theories [2, 3], we choose to work with the most general scalar-tensor theory examined by Bergmann and Wagoner [4, 5], in this theory the Brans-Dicke parameter ω and cosmological function λ depend upon the scalar gravitational field ϕ. The Brans-Dicke theory can be obtained setting $\omega = \text{const}$ and $\lambda = 0$.

The WDW equation is obtained by means of canonical quantization of Hamiltonian H according to the standard canonical rule, this leads to a difficulty known as the problem of time [6]. Also, this equation has problems in its probabilistic interpretation. In the usual formulation of quantum mechanics a conserved positive-definite probability density is required for a consistent interpretation of the physical properties of a given system, and the universe in the quantum cosmology perspective, do not satisfied this requirement, because the WDW equation is a hyperbolic second order differential equation, there is no conserved positive-definite probability density as in the case of the Klein-Gordon equation, an alternative to this, is to regard the wavefunction as a quantum field in minisuperspace rather than a state amplitude [7].

The paper is organized as follows. In Section 2 we obtain the WDW equation. In Section 3 we show third quantization of the universe wavefunction using two complete set of modes for the most easy choice of factor ordering. Finally, Section 4 consists of conclusions.

2 Canonical formalism

Our starting point is the action of Bergmann-Wagoner scalar tensor theory

$$S = \frac{1}{l_p^2}\int_M \sqrt{-g}\left[\phi R^{(4)} - \frac{\omega(\phi)}{\phi}\, g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + 2\,\phi\,\lambda(\phi)\right] d^4x + \\ + \frac{2}{l_p^2}\int_{\partial M} \sqrt{h}\,\phi h_{ij} K^{ij}\, d^3x\,, \tag{1}$$

where $g = \det(g_{\mu,\nu})$, $\phi(t)$ is the conventional real scalar gravitational field, while l_p is the Planck length and $\lambda(\phi)$ is the cosmological term. The quantity $R^{(4)}$ is the scalar curvature of the Friedmann-Robertson-Walker theory, which is given, according to the theory, by

$$R^{(4)} = -\frac{6k}{a^2} - 6\frac{\dot a^2}{N^2 a^2} - 6\frac{\ddot a}{N^2 a} + 6\frac{\dot a \dot N}{N^3 a}\,. \tag{2}$$

The second integral in (1) is a surface term involving the induced metric h_{ij} and second fundamental form K^{ij} on the boundary, needed to cancel the second derivatives in $R^{(4)}$ when the action is varied with the metric and scalar field, but not their normal derivatives, fixed on the boundary. Substituting (2) in (1) and integrating with respect to space coordinates, we have

$$S = \frac{1}{2}\int \left[-Nka\phi + \frac{a\phi}{N}\,\dot a^2 + \frac{a^2}{N}\,\dot a\dot\phi - \\ - \frac{N\omega(\phi)}{6\phi}\,a^3\dot\phi^2 + \frac{N}{3}\,a^3\phi\lambda(\phi)\right] dt\,, \tag{3}$$

where dot denotes time derivative with respect to the time t, now introducing a new time $d\tau = \phi^{\frac{1}{2}}dt$ and the following independent variables

$$\alpha = a^2\phi\cosh\int\left(\frac{2\omega(\phi)+3}{3}\right)^{\frac{1}{2}}\frac{d\phi}{\phi}\,, \tag{4}$$

$$\beta = a^2\phi\sinh\int\left(\frac{2\omega(\phi)+3}{3}\right)^{\frac{1}{2}}\frac{d\phi}{\phi}\,, \tag{5}$$

$$\lambda(\phi) = 3\phi\left[\Lambda_1\cosh\int\left(\frac{2\omega(\phi)+3}{3}\right)^{\frac{1}{2}}\frac{d\phi}{\phi} + \\ + \Lambda_2\sinh\int\left(\frac{2\omega(\phi)+3}{3}\right)^{\frac{1}{2}}\frac{d\phi}{\phi}\right], \tag{6}$$