A Survey of Lanczos Potential

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We give a short survey of the properties of Lanczos potential, and we mention how this potential is relevant for the derivation of the Weyl curvature tensor; we give some useful examples that have been already computed elsewhere.

Key words: Weyl-Lanczos equations, Lanczos potential, Kasner space-time, Wave equation

Introduction

It was in 1962 when Cornelius Lanczos (1962) made the important observation about that for any geometry, the Weyl conformal curvature tensor, can be written as the covariant derivative of a third rank tensor L_{abc}, later called the Lanczos potential. All attempts to generalize this result for the case of the general curvature tensor of Riemann have failed. Nevertheless, the Einstein equations can be formulated in Jordan form and written in terms of the Weyl tensor. Lanczos proved that the existence of a potential for the general Riemann curvature tensor is not possible and later Bampi and Caviglia gave a completely different proof of this point (Bampi and Cavaglia, 1983). However, they did not give a method to calculate the potential L_{abc}. Afterwards, Novello and Velloso (1987) showed a method to compute the Lanczos potential tensor.

The Lanczos tensor can be derived in a covariant form and then be used to compute the Weyl tensor according to the formula:

$$W_{abcd} = L_{ab[c;d]} + L_{cd[a;b]} + \frac{1}{2}\left(L_{(ad)} g_{bc} + L_{(bc)} g_{ad} - L_{(ac)} g_{bd} - L_{(bd)} g_{ac}\right) + \frac{2}{3} L^{sl}{}_{s;l}\, g_{abcd} \qquad \ldots(1)$$

where,

$$g_{abcd} \equiv \left(g_{ac} g_{bd} - g_{ad} g_{bc}\right) \text{ and } L_{ab} \equiv L_{a\ \ s;b}^{s} \qquad \ldots(2)$$

The electric (E_{ac}) and the magnetic (H_{ac}) parts of the Weyl tensor are defined as:

$$E_{ac} = -W_{abcd} V^b V^d$$
$$H_{ac} = -W^*{}_{abcd} V^b V^d \qquad \ldots(3)$$

The Weyl tensor (W_{abcd}), the Riemann tensor (R_{abcd}), and the contracted tensor ($R^a{}_{bac} = R_{bc}$) have the following relation:

$$R_{abcd} = C_{abcd} + E_{abcd} + G_{abcd} \qquad \ldots(4)$$

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