

# **Thermal Diffusivity Measurement in Solids by means of the Photoacoustic Technique**

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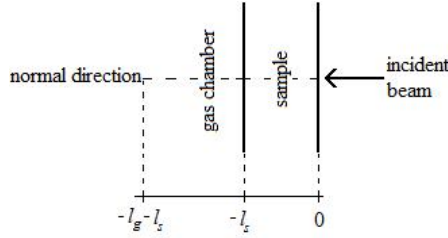
## **ABSTRACT**

We report thermal diffusivity measurements for samples of silicon, gallium arsenide and copper by means of the photoacoustic technique in a heat transmission configuration in order to obtain a comparison between the results obtained with the use of the conventional RG-model and our SP-model (based in a square periodical heat source) in the fitting process to the experimental data. Our results show that our SP-model is accurate to obtain a good fitting with the experimental data and it improves notably the results obtained with the RG-model.

## **INTRODUCTION**

The Photoacoustic (PA) technique is a non destructive test widely used for thermal characterization of solids because it's relative low cost and easy implementation; but despite the goodness of it, still there are some issues that make the PA technique to be less effective than it can be. In this technique, the Rosencwaig and Gersho model (RG-model) [1] has been widely used during more than 30 years and nowadays [2,3]. This theoretical model is based on the solutions of the heat equation considering a heat source with a sinusoidal modulation in time, and even when several authors add to the original model corrections involving other physical effects [4,5], the basic consideration of sinusoidal modulation is hardly touched [6]. This consideration approaches to the experimental conditions; however the real experimental setup normally uses a heat source obtained when a chopped light beam normally impinges on the surface of the sample. In this paper we report an experimental comparison between the thermal diffusivity obtained with the use of the RG-model and our model (SP) based in a square periodical heat source.

The PA technique in a heat transmission configuration is considered, with an open photoacoustic cell (OPC) detection [2, 3]. The monochromatic radiation normally impinges on the surface of the sample and the radiation is modulated on intensity by a mechanical chopper with a variable frequency  $\omega$ , producing a square periodical heat source on the sample surface. If the sample is considered optically opaque, the absorption of the radiation occurs only on the surface of the sample.



**Figure 1** Schematic draw of an open Photoacoustic cell (OPC) used in PA detection.

After solving the one-dimensional heat diffusion equation with a square periodical heat source, for a superficial absorption, and considering an adiabatic expansion in the PA gas chamber, we obtained the following expression for the gas pressure fluctuations [7]:

$$\Delta P = \frac{C_0}{f} \sum_{n \in \mathbb{N}} \frac{\text{Sinc}(n/2) e^{i(\phi_n - \pi/2)}}{n \sqrt{\text{Cosh}(2\nu_n) - \text{Cos}(2\nu_n)}} \quad (1)$$

Where

$$\phi_n = -\text{arcTan}\left(\frac{\text{Tan } \nu_n}{\text{Tanh } \nu_n}\right); \quad (2)$$

$$\nu_n = \sqrt{nf / f_c} \quad (3)$$

And

$$C_0 = \frac{\gamma P_0 \sqrt{\alpha_s} l_0 \beta \eta}{\varepsilon_s l_g T_0}; \quad \text{Sinc}(z) = \text{Sin}(\pi z) / \pi z$$

Here,  $C_0$  is a constant parameter, which can be determined from the following parameters: the pressure and temperature ambient in the OPC,  $P_0$  and  $T_0$  respectively; the adiabatic constant for the air  $\gamma$ ; the thermal effusivity of the sample  $\varepsilon_s$ ; the optical absorption coefficient  $\beta$ ; the light to heat conversion efficiency  $\eta$ ; the air thermal diffusivity  $\alpha_g$ ; the length of the air column in the PA cell  $l_g$ ; the incident monochromatic light flux  $I_0$  ( $\text{W}/\text{cm}^2$ ) and the modulation frequency  $f$  (Hz).

In Eq. (3),  $f_c$  is often called the characteristic frequency and, in the RG model, represents the modulation frequency at which the thermal diffusion length,  $\mu_s = (\alpha_s / \pi f)^{1/2}$ , matches the sample thickness  $l_s$ , where  $\alpha_s$  is the thermal diffusivity of the sample, and  $f_c$  is given by [8]:

$$f_c = \alpha_s / \pi l_s^2 \quad (4)$$

By fitting Eq. (1) to the experimental PA signal,  $f_c$  can be obtained. Then, by means of Eq. (4) the thermal diffusivity of the sample  $\alpha_s$  is determined if previously we know their thickness  $l_s$ .

## EXPERIMENTAL DETAILS

In our OPC experimental setup for PA technique [7-9], a 40 mW Ar<sup>+</sup> ion laser (Omnichrome 543-200 MA) is used and its monochromatic beam intensity is modulated at a frequency  $f$  by a mechanical chopper SR-540 (Stanford Research Systems) before it normally impinges on the surface of the sample. The sample is placed in contact with the front chamber of an omnidirectional electret microphone 33-3026 (RadioShack Corp.) with wide 30 - 15,000 Hz frequency response. The PA signal is detected by a lock-in amplifier SR-850 (Stanford Research Systems) interfaced to a personal computer, which permits recording the amplitude and phase of the signal as a function of the modulation frequency.

The studied samples are listed in Table I. Sample A is a semiconductor wafer of crystalline Silicon doped with arsenic; sample B consist on a semiconductor wafer of Gallium Arsenide doped with Silicon; sample C consist on a Copper sheet.

**TABLE I** List of samples and some of their characteristics.

Sample	Material	Doping	Thickness	Resistivity	Orientation
			$[\times 10^{-6} m]$	$[\times 10^2 \Omega/m]$	
A	Si	As	$500 \pm 1$	0.09 - 0.2	100
B	GaAs	Si	$466 \pm 2$	0.001 - 0.015	100
C	Cu	None	$66 \pm 2$	Not available	--

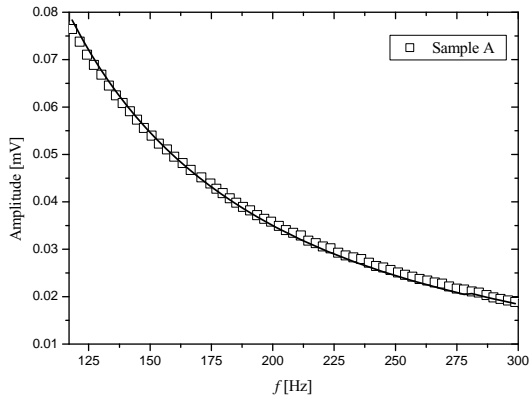
It is known that the response coefficient of conversion in a microphone depends on the frequency, but is also known that there is a (approximate) flat response in some frequency range. The manufacturer of our electret microphone estimate this flat response range approximately between 100 to 1000 Hz, but for our experimental setup, for frequencies up to 320 Hz, the noise-signal relation becomes important, so we center our analysis in a range of 120 to 300 Hz to guarantee the reliability in our results.

## RESULTS AND DISCUSSION

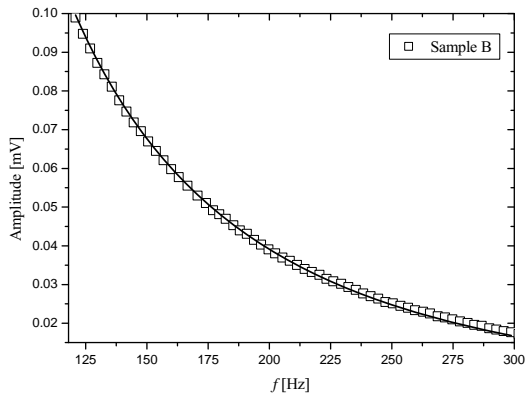
The PA signal amplitude is plotted as function of the modulation frequency, and by non-linear fitting Eq. (1) to these experimental data we can estimate the thermal diffusivity for all the samples listed in Table I. Of course, it is impossible to use the complete expression in Eq. (1); Instead of this, we consider necessary only the use of a few harmonics, until  $n = 3$  for sample B, and until  $n = 5$  for samples A and C. This consideration is correct since the individual terms in Eq. (1) decreases very fast after  $n = 5$ , for a fixed value of the modulation frequency.

### Semi-Conductors Samples

The figures 2 and 3 show the amplitude of the PA signal as a function of the modulation frequency for the samples A and B, respectively, as well as the result of the non-linear fitting with Eq. (1). As it is shown, it is obtained a very closely fitting to the experimental data, in both cases.



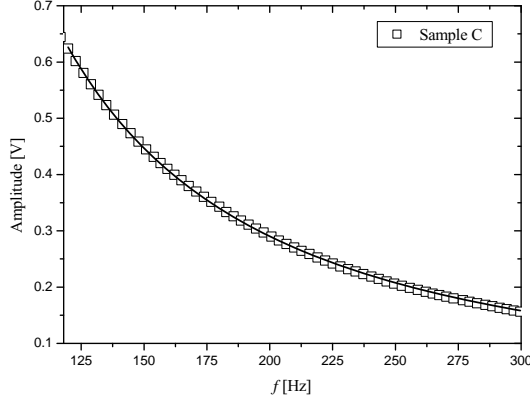
**Figure 2** PA signal amplitude for sample A; the solid line indicate the best fit by SP-model.



**Figure 3** PA signal amplitude for sample B. The solid line indicates the best fit by SP-model.

## Conductors Samples

Figure 4 show the amplitude of the PA signal as a function of the modulation frequency for the sample C, as well as the result of the non-linear fitting with Eq. (1) for  $n = 5$ .



**Figure 4** PA signal amplitude for sample C. The solid lines indicate the best fit by SP-model.

In Table II, we present the thermal diffusivity values obtained by our model and those obtained by the standard RG-model.

**Table II** Measured values for the thermal diffusivity from: SP-model, RG-model and the reported values in the literature [10].

Sample	$f_c^{\text{SPM}}$ [Hz]	$\alpha^{\text{SP-M}}$	$\alpha^{\text{RG-M}}$ [ $\times 10^{-6} \text{ m}^2/\text{s}$ ]	$\alpha(\text{literature})$
A	$114.8 \pm 0.8$	$89.4 \pm 0.6$	$161.1 \pm 21.8$	88
B	$49.0 \pm 0.1$	$33.3 \pm 0.1$	$34.0 \pm 0.2$	21 – 30
C	$8620 \pm 11$	$116.5 \pm 0.2$	$116.5 \pm 0.2$	117

From the Table II we can see that with our model the thermal diffusivity value obtained for the sample A (Si) is very near to those reported in the literature and, we improve significantly the value obtained with the RG model. For sample B (GaAs), the thermal diffusivity value obtained with our model is lightly nearer to those reported in the literature that the value obtained with the RG model. In case of sample C (Cu), we obtained exactly the same value for the thermal diffusivity with both models, which agree very well to those reported in the literature. These results can be explained as a consequence from the characteristic frequency values of the samples, which are discussed below.

For the sample A,  $f_c$  (114.8 Hz) is comparable to the frequency values in the measurement interval,  $120 \text{ Hz} \leq f \leq 300 \text{ Hz}$ , therefore, the measurement happens in an intermediate regime, between the thermally thick regime and the thermally thin regime. The marked difference between the thermal diffusivity values obtained with our model and with the RG model, show the important contribution of the harmonics ( $n \geq 3$ ) to the photoacoustic signal, which does not exist in the RG model.

In the case of sample B,  $f_c$  (49 Hz) is from three to six times smaller than the frequency values in the measurement range, therefore, the measurement happens near to the thermally thick regime, where the PA response depends on  $f^{-1}$ . Then, the harmonics ( $n \geq 3$ ) are strongly damped throughout the sample and, its contribution to the photoacoustic signal is weak. This way, the difference between the thermal diffusivity values obtained with our model and with the RG model is small.

Lastly, for the sample C,  $f_c$  (8620 Hz) is from twenty nine to seventy two times bigger than the frequency values in the measurement range, therefore, the measurement happens completely into the thermally thin regime, where the PA response depends on  $f^{-3/2}$ . In this case, the experiment is not sensitive to the modulation of the heat source, and thus it cannot distinguish between the sinusoidal and square periodical modulation. In this case and in the previous one, the relevant thermal property is the thermal effusivity, instead of the thermal diffusivity [11].

## CONCLUSIONS

We reported experimental results by means of the photoacoustic technique using the RG-model and our SP-model, in the fitting process to the experimental data, in order to obtain the thermal diffusivity in samples of silicon, gallium arsenide and copper. The agreement among the obtained values of the thermal diffusivity by our SP-model and those reported in the literature shows the utility and precision of this model for measurements of the thermal diffusivity by means of the photoacoustic technique. We show that the use of a few harmonics in our SP-model is enough to obtain a good fitting with the experimental data and improves the results obtained with the RG-model, besides it does not add a bigger complication in the data analysis.

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